UNIVERSITY FOR DEVELOPMENT STUDIES

MODELING EXCHANGE RATE VOLATILITY USING UNIVARIATE GARCH MODELS - A CASE STUDY OF THE CEDI/DOLLAR EXCHANGE RATE.

BY

AKUMBOBE ADOMBIRE ROBERT  (B.E.D Mathematics)

(UDS/MAS/0022/12)

THESIS SUBMITTED TO THE DEPARTMENT OF STATISTICS, FACULTY OF MATHEMATICAL SCIENCES, UNIVERSITY FOR DEVELOPMENT STUDIES, IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER OF SCIENCE DEGREE IN APPLIED STATISTICS.

OCTOBER, 2015
DECLARATION

Student

I hereby declare that this thesis is the result of my own original work and that no part of it has been presented for another degree in this University or elsewhere:

Candidate’s Signature: ___________________________ Date: 27/10/15

Name: AKUMBOBE ADOMBIREE ROBERT

Supervisor

I hereby declare that the preparation and presentation of the thesis was supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies:

Supervisor’s Signature: ___________________________ Date: 28/5/2015

Name: DR. ARROWS LUCIEN

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ABSTRACT

This study examines exchange rate volatility with Generalised Autoregressive Conditional Heteroscedastic (GARCH) models using monthly exchange rate data from the bank of Ghana from January 1990 to November 2013. The data was converted to returns to enhance their statistical properties and the returns used to fit a mean equation. The ARCH LM test of the mean equation revealed the presence of conditional heteroscedasticity. The returns were therefore modeled using ARCH (3), GARCH (2,3), Exponential Generalised Autoregressive Conditional Heteroscedastic (EGARCH) (2,2) and Threshold Generalized Autoregressive Conditional Heteroscedastic (2,3). The result revealed that EGARCH (2, 2) was the best since it has the least value of AIC of (-6.2816) and SIC of (-6.1629). Diagnostic test of the EGARCH (2, 2) model residuals with the Ljung-Box and the ARCH LM tests revealed that the models were free from higher order autocorrelation and conditional heteroscedasticity respectively. The Chi-square goodness of fit test showed that the forecasted values obtained from the EGARCH (2, 2) model were not significantly different from the observed values at the 5% significance level. The leverage parameter of the EGARCH (2, 2) model was significant and positive indicating the absence of leverage effect in the returns of the cedi-dollar exchange rate. The absence of the leverage effect in the exchange rate indicates that positive shocks increases volatility than negative shocks of equal magnitude. Thus, the implication is that a strengthening dollar (weakening cedi) leads to higher period volatility than when the cedi strengthens by the same amount. It is recommended that the central bank should put in place long...
term measures to stabilise the cedi since a weakening cedi increases the uncertainty in the exchange market than a strengthening cedi.
ACKNOWLEDGEMENT

I wish to acknowledge the almighty God for his mercies and grace that saw me through these two years. My deepest appreciation goes to my supervisor Dr Albert Luguterah whose counsel, encouragement and guidance has made me pass through the storms. Many thanks also go to Mr Nasiru Suleman of the department of statistics for his advice and support over these years. To my colleagues especially Ida and Derrick, I say God bless you. Finally I want to express my sincere appreciation to my family especially my beloved wife Hellen Aduba.
DEDICATION

This piece of work is dedicated to my late dad Mr John Akugre whose wish was to see his children soar above the sky.
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ACRONYMS

BOG  Bank of Ghana
NEER  Nominal Effective Exchange rate
AR  Autoregressive
MA  Moving Average
ARMA  Autoregressive Moving Average
ARCH  Autoregressive Conditionally Heteroscedastic
GARCH  Generalized Autoregressive Conditional Heteroscedastic
TGARCH  Threshold Generalized Autoregressive Conditional Heteroscedastic
EGARCH  Exponential Generalized Autoregressive Conditional Heteroscedastic
ERP  Economic Recovery Programme
ARIMA  Autoregressive integrated moving average
SIC  Schwarz Information Criterion
AIC  Akaike Information Criterion
ARFIMA  Autoregressive Fractionally Integrated Moving average
APARCH  Asymmetric Power Autoregressive Conditional Heteroscedastic
ADF  Augmented Dicker Fuller
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<td>Partial Autocorrelation Function</td>
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<td>NSE</td>
<td>Nigerian stock exchange</td>
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<td>WACB</td>
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CHAPTER ONE

INTRODUCTION

1.1 Research Background

Exchange rate is one of the macroeconomic variables that play a central role in the management of most economies. The volume of empirical studies on the subject attests to this assertion Abuaf and Jorion, (1990); Calderon and Duncan, (2003); Cheung and Lai, (2001); Taylor, (1995). The subject becomes more imperative especially for countries that depend heavily on importation of essential commodities such as crude oil and raw materials for industrial production. Changes in exchange rates have pervasive effects, with consequences for prices, wages, interest rates, production levels, and employment opportunities, and thus with direct or indirect implications for the welfare of virtually all economic participants (Kuntomah, 2013).

The exchange rate of the Ghana cedi against, for example, the US dollar is quoted as the number of Ghana cedi required to purchase one US dollar. Accordingly, large and unpredictable changes in exchange rates present a major concern for macroeconomic stabilization policy. Given the effects that changes in exchange rates can have on economic conditions, policy makers naturally want to understand what can be done to limit exchange rate variability, and at what consequences (Kuntomah, 2013).

Countries have a choice between two basic types of exchange rate regimes (fixed or floating and variations in between). A fixed exchange rate regime is one that is administratively fixed by the government or monetary authority...
with fiscal and monetary policy deployed to maintain the fixed exchange rate. In practice, countries devalue or revalue their currencies in line with changes in the economic fundamentals. The other type of exchange rate regime is the floating exchange rate regime where the exchange rate is determined by the forces of demand and supply in the foreign exchange market (Bordo, 2003).

Volatility is a measure for variation of price of a financial instrument over time. Exchange rates are highly volatile in the short run and are very responsive to political events, monetary policy and changes in expectations. In the long run, exchange rates are determined by the relative prices of goods in different countries (Samuelson and Nordhaus, 2001).

Exchange rate is more volatile than the fundamental variables which determine the exchange rate in the long run (Gärtner, 1993). Exchange rates have become more volatile in recent years due to the abandonment of the fixed exchange rates to the floating exchange rate. It has however resulted in a massive volume in foreign exchange transactions. These transactions have grown faster than international trade and international investments flows of capital.

The risk associated with foreign exchange transactions and trading at the foreign exchange market has increased but so has also the awareness and knowledge about the subject. International private capital flows are much larger than trade flows today which indicates that exchange rates reflect mostly financial rather than trade flows, especially in the short run. However, the trade flow has a large influence upon exchange rates in the long run (Salvatore, 2004).
Exchange rate volatility is directly influenced by several macro-variables, such as demand and supply for goods, services and investments, different growth and inflation rates in different countries, changes in relative rates of return and so forth. The present floating exchange rate has been affected by previous real and monetary disturbances. Expectations about current events and future events are also important factors due to the large influence it has on exchange rate volatility.

Exchange rate volatility is said to have implications for the financial system of a country especially the tradable sector. Changes in exchange rates have pervasive effects, with consequences for prices, incomes, interest rates, manufacturing levels, and job opportunities, and thus with direct or indirect repercussions for the welfare of virtually all economic participants. When GARCH method was used as a measure of exchange rate volatility by Shipanga (2009), the result indicated that an increase in exchange rate volatility causes real export to increase.

Bawumia (2014) mention the following among others as the impact of exchange volatility (shocks); using other currencies as a store of value, increasing Inflation, declining consumer and investor confidence.

In the view of Fortura et al. (2007) a strong currency is a mixed blessing. They stated that a strong currency is good because: It lowers the prices of imports and makes trips to foreign countries less expensive; it lowers prices on foreign goods and helps to keep inflation in check. A strong domestic currency also make investment in foreign financial markets (foreign stocks and bonds) relatively cheaper. However, on the flip side, a strong currency makes
domestic exports expensive. Therefore foreigners will buy fewer goods from that country. The net effect of this trade imbalance is a fall in exports and rise in imports, Fortura et al. (2007). This study therefore investigates the exchange rate volatility between the Ghana cedi and US dollar.

1.2 Problem statement

The exchange rate and its volatility are key factors that influence economic activities in Ghana. Fluctuations in foreign exchange market have attracted considerable attention in both the economic and statistical literature. Exchange rate volatility is a swing or fluctuation over a period of time in the exchange rate. The Ghanaian economy is very sensible to fluctuations in the USD/Ghana cedi exchange rate given the fact that we generally import in US dollars. Moreover, banks as well as other financial institutions usually invest in foreign exchange instruments thus the need for accurate modelling and forecasting of volatility. There has been excessive volatility of the cedi against major exchange rates in Ghana since the adoption of flexible exchange rate regimes in 1983. Consequently sustained exchange rate volatility was thought to have led to currency crises, distortion of production patterns as well as sharp fluctuations in external reserve. Recently, currency debates have taken centre-stage with the free fall of the cedi and its accompanying effects on the Ghanaian economy.

Recently the behaviour of the Cedi (to extrapolate the future behaviour of the rate) is crucial and this has been linked largely to under development of the financial system and the exchange rates market. A huge number of studies have been done on modelling exchange rate volatility by GARCH models in
the matured capital markets, but little attention has been paid to the subject in the emerging capital markets like Ghana.

Appiah and Adetunde (2011) studied forecasting exchange rate between the Ghana cedi and the US dollar using ARIMA models which considers only the mean effect without looking at the volatility in the exchange rate. This study therefore looked at modelling exchange rate volatility between the Ghana cedi and the US dollar.

1.3 General Objectives
The main objective of the study is to model exchange rate volatility between the Ghana Cedi and United States Dollar.

1.4 Specific Objectives
In addition to the main objective, the following specific objectives will be explored.

- To develop an appropriate Univariate GARCH model for the cedi/dollar exchange rate.
- To test the adequacy of the selected model for use.
- Use the selected model for forecasting
- Evaluate the accuracy of the forecasted results.

1.5 Significance of the Study
The outcome of the research will help policy makers and the players of the exchange market in making decisions with regards to exchange rate volatility. It will also provide a basis for further research into exchange volatility in Ghana.
1.6 Organization of the study

The study is organized into five chapters. Chapter one contains the background, problem statement, general and specific objectives, significance of the study and organization of the study.

Chapter two involves introduction, structure and history of exchange rate market in Ghana, Empirical studies on exchange rate and financial time series with volatility. Chapter three outlines the introduction, source of data, the unit roots test, the autoregressive integrated moving average models, volatility of the exchange rate, the GARCH models, maximum likelihood, model diagnostics, model selection and forecasting of future returns.

Chapter four looks at introduction, preliminary analysis, further analysis, fitting the GARCH models, model diagnostics, selecting the Best fit GARCH model and discussion of the results. Chapter five is devoted to summary, conclusion and recommendations.
CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews some related literature on exchange rate and time series analysis.

2.1 Structure and history of the Exchange Rate Market in Ghana

Countries all over the world at one point or the other adopted several exchange rate regimes. From a system of fixed exchange rate prior to early 1970s, most countries today allow market forces to determine their exchange rates. At independence, Ghana was operating a fixed exchange rate regime under the colonial international economic arrangements.

The British West African Currency Board (WACB) was constituted in 1912 to control the supply of currency to the British West African Colonies. The exchange rate of the West African Pound to sterling was fixed. The country abandoned the WACB arrangement in 1963 and introduced the cedi in 1965 because the government could not follow the strict requirements of fiscal discipline that the currency board regime imposes, Bawumia (2014).

The exchange rate was £1.04/§ and Ghana continued to operate a fixed exchange rate regime. The nation however, moved away from the fixed exchange rate regime towards a floating (market determined) regime during the era of the Structural Adjustment Program (SAP) from 1983. To bridge the gap between black market and official exchange rates, foreign exchange bureaus were established in February 1988, leading to the virtual absorption of
the foreign exchange black market. The cedi exchange rate therefore became market determined with an increase in demand for foreign currency resulting in depreciation while the increase in supply of foreign currency results in appreciation of the cedi, other things being equal, (Bawumia, 2014).

The government of Ghana licensed 180 forex bureaus to operate in the country in 1990 (Dordunoo,(1994), Bhasin, (2004). Subsequently, the two-window exchange system was unified in 1992 and the market started operating an interbank whole sale system (Jebuni, 2006). The foreign exchange market is small in size with only a few active players. The central bank is the most dominant player in the market and it is responsible for 90 percent of the total amount of transactions in the market. There are currently four identifiable segments of the market. These are:

- The interbank market where banks trade foreign exchange among themselves;
- Foreign exchange bureaux which serve individuals, tourists, SMEs, etc.;
- The corporate market through which transactions between banks and their customers are conducted; and
- The unofficial market which comprises of corporations that price their products/services at their own-determined exchange rate. (source, world bank report, 2012).

There have been different exchange rate systems which countries adopted in recent years. According to Brue and McConnell (1996), the three conspicuous ones are the Gold Standard (fixed exchange rate), the Bretton Woods System, and the ‘Managed’ Floating System.
2.2 Empirical Studies on Exchange Rate

Jebuni (2006) holds that exchange rate is one of the most important economic adjustment instruments and also one of the most difficult economic policy tools because of its economic and political effects on a nation. According to Mussa (1984) factors that determine the exchange rates in an economy are largely dependent on the type of exchange rate regime (flexible or fixed) operated by the country.

According to Stockman (1978), the behaviour of exchange rates in the flexible rate system has puzzled many economists. He offered an explanation for the behaviour of exchange rates and how they are determined. In his view, foreign exchange can be considered as a derived demand because foreign exchange is demanded to purchase foreign goods and assets. He stated that the expected rate of the exchange rate should be related to anticipate changes in the terms of trade or factors associated with the terms of trade as well as to the anticipated inflation differential.

His findings were supported by Mumuni et al. (2004). They implored the monetary approach to investigate the determinants of the Cedi/Dollar rate of exchange in Ghana. They observed that, both domestic and foreign money supply conditions matter in the Cedi/Dollar dynamics. They concluded that the Cedi/Dollar rate of exchange is determined by the economic fundamentals and speculations based on the immediate past history of the rate itself.

According to Frenkel et al. (2005) changes in the exchange rate is an important source of risks for non-financial corporations. They recommended more focus on economic foreign exchange rate exposure since risk
management has become much more crucial for businesses in light of the globalization of business activities.

Kannagaraj and Sikarwar (2011) stated that countries which follow floating exchange rate regime are most vulnerable to exchange rate volatility which invariably affects their cash flows.

2.3 Review of Time Series Methods

Treating time series in a stochastic sense began in the mid-1920s (Gottman, 1981). Yule (1927) first developed an Autoregressive (AR) model when working on Wolfer's sunspot data. Slutsky (1927) first developed a Moving average (MA) model when studying a white-noise series. Box and Jenkins (1970) developed the Autoregressive Moving average (ARMA) model and gave a full account of the Integrated Autoregressive Moving average (ARIMA) model.

Mann and Wald (1943) proved a theorem to estimate the AR \( (p) \) parameters by the least squares method. Quenouille (1947) presented a simple test for AR \( (p) \) models and later extended to MA models. Also, Anderson (1971) developed a procedure to estimate the order of the AR model as well as the AR parameter. In addition, a non-linear least squares technique procedure that led to a technique of approximated likelihood solution for ARMA \( (p, q) \) models was developed by Box and Jenkins (1976).

An exact likelihood method for estimating parameters of MA \( (q) \) models and for ARMA \( (p, q) \) models was developed by (Newbold, 1974). The Box-Pierce statistics was developed by Box and Pierce (1970) and modified by (Ljung and Box, 1978).
An information criterion to assist in the assortment of an ARIMA model was proposed by Akaike (1974). A model with the smallest Akaike Information Criterion (AIC) is the best model to have minimum forecast mean square errors. On the information criterion, Schwartz (1978) indicated that AIC was not consistent when probability confronts one, and proposed a Bayesian Information Criterion (BIC).

Harvey and Phillip (1979) advanced an exact likelihood procedure to estimating parameters of an ARIMA model in State-Space form. The State-Space models are also called Structural Time Series (STS) models. Many researchers have pointed out the advantages of the State-Space form over the ARIMA models (Durbin and Koopman (2001). A time series might be characterized by trend, seasonal cycle and calendar variations, together with the effects of explanatory variables and interventions. These components can be processed separately and for different purposes for a State-Space model. On the contrary, the Box-Jenkins ARIMA model is a black-box model, which solely depends on the data without knowledge of the system structure that produces the data. The additional advantage is the recursive nature of the State-Space model that obviously allows change of the system overtime, while ARIMA models are homogenous through time, based on the stationary assumption (Box-Jenkins, 1976).

2.4 Financial Time Series with Volatility Models.

Since the global adoption of floating exchange rate system in 1973, literature on exchange rate volatility has grown tremendously and examining financial time series data with volatility models has also become very common in recent
years. One of the most important tools that characterize the changing of the variance is the ARCH model.

Engle (1982) proposes to model time-varying conditional variance with the ARCH process that use past disturbances to model the variance of the series. Early pragmatic evidence shows that high ARCH order has to be selected in order to catch the dynamics of the conditional variance.

The GARCH model of Bollerslev (1986) is a response to this issue. Several excellent reviews on ARCH/GARCH models are available in Bollerslev (1992), Bollerslev et al. (1994) and (Bera and Higgins, 1993). The maximum likelihood based inference procedures for the ARCH class of models under normality assumption are discussed in Engle (1982). Generalized Method of Moments (GMM) estimation of ARCH type models are discussed in (Mark, 1988), Glosten et al. (1991). As an alternative estimation technique, Gallant and Nychka (1987) use a semi parametric method while Robinson (1987) use a nonparametric method.

The distribution considered in ARCH and GARCH models is symmetric and fails to model the third stylized fact, namely the leverage effect. Leverage effect is another fact about financial time series which suggest that stock price movement for example is negatively correlated with volatility. To solve this problem, many extensions to GARCH models have been proposed. Among the most widely spread are Exponential GARCH (EGARCH) of Nelson (1991), the so called GJR of Glosten et al. (1993) and the Asymmetric Power ARCH (APARCH) of Ding et al. (1993).
The other well recognized volatility model is GARCH-in Mean or GARCH-M model introduced by (Engle, 1987), who considers the conditional mean equation as a function of the conditional variance. In this model, an increase in conditional variance will be associated with an increase or a decrease in the conditional mean of the process. The modelling and forecasting of exchange rates and their volatility has important implications for many issues in economics and finance. Various families of GARCH models have been applied in the modelling of the volatility of exchange rates in various countries.

West and Chow (1995) examined the forecast ability of exchange rate volatility using a number of models including ARCH using five U.S. bilateral exchange rate series. They found that generalised ARCH (GARCH) models were preferable at a one week horizon, whilst for less frequent data, no clear victor was evident. Some other studies on the volatility of exchange rates include (Meese and Rose, 1991), (Longmore and Ronbison, 2004), (Yang et al., 2006) among others.

The ARCH-type models were used by Wagala et al. (2011) to model the volatility of the Nairobi Stock Exchange weekly returns. The models applied in the study included the ARCH (p, q ), standard GARCH (p,q ), IGARCH (p,q) and TGARCH (p,q ). The results demonstrated that the ARCH (p, q) was found to be the most adequate for the NSE index, Bamburi and KQ while ARCH (p,q ) provided the best order for the NBK series. Furthermore four different orders were tested for the GARCH (p, q ), EGARCH ( p,q) and TGARCH (p,q ). These were (1,1 ),(1,2 ), (2,1 ) and (2,2 ). The order (1,1 )
was the best choice in all cases and it was consistent with results obtained from most GARCH research works.

Comparing the diagnostics and the goodness of fit statistics, the IGARCH (1,1) outperformed the ARCH, EGARCH and TGARCH models due to its stationarity in the strong sense. However, because the IGARCH model was unable to capture the asymmetry exhibited by the stock data, the EGARCH (1,1) and the TGARCH (1,1) provided the best options to describe the dependence in variance for all the four series since they were able to model asymmetry and parsimoniously represented a higher order ARCH (p).

Awogbemi and Oluwaseyi (2011) described the volatility in the consumer prices of some selected commodities in Nigerian market. The researchers examined the presence or otherwise of the volatility in their prices using ARCH and GARCH models with monthly Consumer Price Index (CPI) of five selected commodities over a period of 1997 - 2007. The results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models. The researchers also observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the 'bullish crowd') will be highly favoured in the market of the said commodity items.

Ngailo (2011) modelled financial time series with special application to modelling inflation data for Tanzania. In particular the theory of Univariate non-linear time series analysis was explored and applied to the inflation data spanning from January 1997 to December 2010. He fitted the ARCH and
GARCH models to the data. Based on the AIC and BIC values, the results revealed that the best fit models tend to be the GARCH (1, 1) and GARCH (1, 2). However after diagnostic and forecast accuracy tests were performed, the GARCH (1, 1) model was adjudged to be the best model for forecasting.

Igogo (2010) employed the ARCH family of models to measure the effect of real exchange volatility on trade flows in Tanzania for the period of 1968 to 2007. He fitted the GARCH (1, 1) and EGARCH (1, 1) models. The results indicated that GARCH (1, 1) model violated the non-negativity conditions and hence to resolve the problem, the EGARCH (1, 1) was used. The adequacy of the EGARCH (1, 1) model to measure the real exchange rate volatility was confirmed by testing for ARCH effect after running the model. Furthermore, the study revealed that it is the real exchange rate rather than its volatility that is found to have a significant effect on trade flows although the effect is larger on exports than imports. He concluded therefore that in the short run, imports are mainly affected by the domestic income while exports are mainly affected by the real exchange rate.

Asri and Mohammad (2011) proposed an alternative model for modelling the volatility of the conditional variances: A (Radial Basis Function) RBF-EGARCH Neural Networks Model. Their proposed forecasting model combines a RBF neural network for the conditional mean and a parametric EGARCH model for the conditional volatility. They used the regression approach to estimate the weight and the parameters of the EGARCH model. They carried out a simulation based on sample of Bank Rakyat Indonesia TBK stock returns and the results indicated that their proposed model is able to accurately predict 63% upward and downward
movements of future predictions. They concluded that the simulation results obtained in the forecasting performances motivates further work, which will involve comparing a different method of parameters model estimation.

Kunst (1997) studied the augmented ARCH models which encompasses most linear ARCH-type models. He considered the two basic ARCH variants for auto-correlated series; conditional variance lagged by errors (Engle, 1982) or conditional variance lagged by observations (Weiss, 1984).

He evaluated whether the restrictions evolving from these two ARCH variants are valid in practice. Time series of stock market indexes for some major stock exchanges (Standard and Poor 500 index, Stock market index for German, French, British and Japanese) were considered. For the important US Standard and Poor 500 Index and for Japanese and German stock index, the evidence indicated more or less convincingly that fourth-moments structures in financial series may be more complicated than the traditional ARCH models.

A non-parametric comparison of sample moments also supported this result. The statistical evidence presented was stronger than the weak evidence on more general structures found by Tsay (1987) in an exchange rate series. For two other countries, France and the United Kingdom, the statistical description achieved by the standard ARCH model appears to be sufficient.

Malmsten and Terasvirta (2004) used a unified framework for testing the adequacy of an estimated EGARCH model. The tests were Lagrange multiplier type tests and included testing an EGARCH model against a higher-order one and testing parameter constancy. Various existing ways of testing the EGARCH model against GARCH models were also investigated as
another check of model adequacy. This was done by size and power simulations.

Simulations revealed that the simulated LR test is more powerful than the encompassing test and that the size of the test may be a problem in applying the pseudo-score test. The simulation results indicated that in practice, the robust versions of their tests should be preferred to non-robust ones and they can be recommended as standard tools when it comes to testing the adequacy of an estimated EGARCH (p,q) model.

The stylized facts of financial time series using three popular models were studied by Malmsten and Terasvirta (2004). The models used were the GARCH, EGARCH and Autoregressive Stochastic Volatility (ARSV) models and they focused on how well these models are able to reproduce characteristic features (stylized facts) of financial series. Their study used stock returns as case of the financial series. The results showed that the GARCH model and EGARCH models were at their best when characterizing models based on time series with relatively low kurtosis and high first-order autocorrelation of squares, assuming normality of errors. However the ARSV (1) model is a better option for time series displaying a combination of high kurtosis and high autocorrelations.

Blake and Kapetanios (2005) investigated the extent of the effect of neglected nonlinearity on the properties of ARCH testing procedures. They proposed and used a new ARCH testing procedures based on neural networks which are robust to the presence of neglected nonlinearity. The neural networks were used to purge the residuals of the effects of nonlinearity before applying an ARCH test. Thus they correctly sized the ARCH test while retaining good
power for the ARCH test. Results based on Monte Carlo simulations showed that the new method alleviated the problem posed by the presence of neglected nonlinearity to a very large extent. Empirical evidence or results based on the application of the new test procedures to exchange rate data indicated substantial evidence of spurious rejection of the null hypothesis of no ARCH effects. There was also further evidence that exchange rates exhibited complicated, dynamic behaviour, with important nonlinearity and volatility effects.

Karanasos and Kim (2003) considered the moment structure of the general ARMA (r,s) -EGARCH (p,q) model and compared it with the standard GARCH model and APARCH model. In particular, they derived the autocorrelation function of any positive integer power of the squared errors and also obtained the autocorrelations of the squares of the observed process and cross correlations between the levels and the squares of the observed process assuming that the error terms are drawn from either a normal, double exponential or generalised error distributions.

Daily data on four East Asia stock indices – Korean Stock price index (KOSPI), Japanese Nikkei index (Nikkei) and the Taiwanese SE Weighted index (SE) for the period 1980 – 1997 and the Singaporean Straits Times price index (ST) for the period 1985 – 1997 were considered. They concluded that there were differences in the moment structure between the ARMA (r,s ) – EGARCH (p,q) model and the standard GARCH model. The study also concluded that, to help with model identification, results of the autocorrelations of the squared deviations can be applied to the observed data
and its properties compared with the theoretical properties of the models. Based on that, it was observed that the EGARCH model can more accurately reproduce the nature of the sample autocorrelations of squared returns than the GARCH models.

GARCH models are estimated using several error distributions, the question of which error distribution should be used over which is relative and several literature exist to support each one of the error distributions.

Hung-Chung et al. (2009) have shown that a GARCH model with an underlying leptokurtic asymmetric distribution outperforms one with an underlying Normal distribution, for modelling volatility of the Chinese Stock Market.

Wilhelmsson (2006) have demonstrated that the use of fat tailed error distributions within a GARCH (1,1) framework leads to improved volatility forecasts. The former uses nine possible error distributions to model the volatility of the Standard and Poor’s 500 with the leptokurtic distributions working out best. The author uses the Mean Absolute Error and Heteroscedasticity-adjusted MAE to evaluate the forecasts.

Chuanga et al. (2007) also finds that a student’s t distribution as distributional assumption to a GARCH model produces better forecasts as compared to the Exponential distribution and a mixture of Normal distributions. An extension of the GARCH model, the GARCH in Mean was used by Ryan and Worthington (2004) to assess the impact of market, interest rate and foreign exchange rate risks on the sensitivity of Australian bank Stock Returns.
Mandelbrot (1963) and Fama (1965), many found that the stylized characteristics of the foreign currency exchange returns are non-linear temporal dependence and the distribution of exchange rate returns are leptokurtic, such as Friedman and Vandersteel (1982), Bollerslev (1987) and Diebold (1988). Their studies have found that large and small changes in returns are “clustered” together over time, and that their distribution is bell shaped, symmetric and fat-tailed. In a study captioned “Econometric Analysis of realized Volatility: Evidence of Financial Crises”. Neokosmidis (2009) asserts that financial data have some key features, volatility clustering and leverage effects which cannot be captured by models such as the ARMA model. He proposed the use of ARCH family of models to estimate financial time series. According to Giovanis (2008) the GARCH model is able to capture volatility clustering successfully making it an appropriate model for volatility forecasting.

On the contrary, Su (2010) indicated that the EGARCH model is more fitting than GARCH. He studied the financial volatility of daily returns in China using the GARCH and EGARCH. Similarly, Jean-Philippe (2001) stated that GJR-GARCH and APARCH give better forecasts than symmetric GARCH. The study was captioned “Estimating and forecasting volatility of stock indices using asymmetric GARCH models and (Skewed) Student-t densities”. FTSE 100 and DAX 30 daily data over a 15-years period were used for the analysis.

The ARCH model was first applied in modelling the currency exchange rate by Hsieh only in 1988. In a study done by Hsieh (1989a) to investigate whether daily changes in five major foreign exchange rates contain any
nonlinearity, he found that although the data contain no linear correlation, evidence indicates the presence of substantial nonlinearity in a multiplicative rather than additive form. He further concludes that a generalized ARCH (GARCH) model can explain a large part of the nonlinearities for all five exchange rates. Since then, applications of these models to currency exchange rates have increased tremendously.

Ramasamy and Munisamy (2012) compared three simulated exchange rates of Malaysian Ringgit with actual exchange rates using GARCH, GJR and EGARCH models. For testing the forecasting effectiveness of GARCH, GJR and EGARCH the daily exchange rates of four currencies - Australian Dollar, Singapore Dollar, Thailand Bhat and Philippine Peso - were used. The forecasted rates, using Gaussian random numbers, were compared with the actual exchange rates of year 2011 to estimate errors. Both the forecasted and actual rates were then plotted to observe the synchronisation and validation. The results showed more volatile exchange rates are predicted well by the GARCH models efficiently than the hard currency exchange rates which are less volatile. Among the three models the effective model was indeterminable as these models forecast the exchange rates in different number of iterations for different currencies. The leverage effect incorporated in GJR and EGARCH models did not improve the results much.

Shamiri and Hassan (2005) examined and estimated the three GARCH(1,1) models (GARCH, EGARCH and GJR-GARCH) using the daily price data of two Asian stock indices, Strait Times Index in Singapore (STI) and Kuala Luampur Composite Index in Malaysia (KLCI) over a 14- years period. The
competing models GARCH, EGARCH and GJR-GARCH were developed based on three different distributions, Gaussian normal, Student-t, Generalized Error Distribution. The estimated results showed that the forecasting performance of asymmetric GARCH Models (GJR-GARCH and EGARCH), especially when fat-tailed asymmetric densities are taken into account in the conditional volatility, was better than symmetric GARCH. Moreover, it was found that the AR (1)-GJR model provided the best out-of-sample forecast for the Malaysian stock market, while AR(1)-EGARCH provided a better estimation for the Singaporean stock market.

Jiang (2011) examined the relationship between inflation and inflation uncertainty in China. He believed that it was worthy to investigate the inflation and inflation uncertainty relationship in China as it is commonly believed that one possible channel that inflation imposes significant economic costs is through its effect on inflation uncertainty. Jiang (2011) addressed the relationship of inflation and its uncertainty in China’s urban and rural areas separately given the huge urban-rural gaps. The GARCH(1,1) and EGARCH(1,1) models were used to generate the measure of inflation uncertainty and then Granger causality tests were performed to test for the causality between inflation and inflation uncertainty. GARCH (1, 1)-M models were also employed to further investigate the inflation-uncertainty nexus. The results provided strong statistical supportive evidence that higher inflation raises inflation uncertainty. On the other hand, the evidence on the effect of inflation uncertainty on inflation was mixed and depended on the sample period and areas examined.
Zivot (2009) provides a tour of empirical analysis of GARCH models for financial time series with emphasis on practical issues associated with model specification, estimation, diagnostics, and forecasting.

Adamu (2005) for example explored the impact of exchange-rate volatility on private investment and confirms an adverse effect. Mordi (2006) in employing GARCH model argues that failure to properly manage exchange rates can induce distortions in consumption and production patterns and that excessive currency volatility creates risks with destabilizing effects on the economy.

Adubi and Okunmadewa (1999) analysed dynamics of price, exchange-rate volatility and agricultural trade flows in Nigeria.

Bala and Asemota (2013) examined exchange rate volatility in Nigeria by application of GARCH models with exogenous breaks. Their results reveal presence of volatility in the three currencies they examined.

Chipili. (2007) looked at exchange rate volatility in Zambia. His results reveal that exchange rate are characterised by different conditional volatility base on the three GARCH models he used. He further applied principal components analysis to generate new exchange rate volatility series that capture the common underlying pattern in the estimated conditional variances, as alternative measures of exchange rate uncertainty.

Suleman et al. (2011) analysed stock market volatility in Sudan. Their empirical results show that the conditional variance process is highly persistent. Their results revealed presence of volatility in the three currencies they examined. Emenike (2010), explored the behaviour of stock return
volatility of the Nigerian Stock Exchange returns using GARCH (1,1) and the GJR-GARCH(1,1) models assuming the Generalized Error Distribution (GED) using data from the monthly all share indices of the NSE from January 1999 to December 2008.

He sought to do this by probing the NSE return series for evidence of volatility clustering, fat-tails distribution and leverage effects, because they provide critical information about the riskiness of assets on the market. He revealed that there exists volatility clustering on the NSE and used GARCH (1,1) to model that. He captured the presence of leverage effects in the series with the GJR GARCH (1,1) model. The GED shape test also revealed a leptokurtic returns distribution. By the overall results of the study, there is evidence of volatility persistence, fat-tail distribution, and leverage effects present in the NSE. He concluded that the volatility of the stock returns is persistent in Nigeria and that the shape parameter estimated from GED reveals evidence of leptokurtosis in the NSE returns distribution (Emenike, 2010).

Shipanga (2009) investigated empirically, the impact of exchange-rate volatility on the export flows of Namibia as one of the developing countries over the period 1998 – 2008. He among others evaluated exchange rate volatility using the general autoregressive conditional heteroscedasticity (GARCH), which indicated a positive and significant impact of exchange rate volatility on Namibia’s real exports. He suggested that Namibia should start exploring possibility of macro-economic policy independence and be involved in the determination of exchange rate within the CMA framework.
Whitelaw (1994) offers empirical evidence for a positive relation between a lagged volatility measure and future expected returns. For Asian stock markets, Koutmos (1999) and Koutmos and Saidi (1995) found that the conditional variance is an asymmetric function of past innovations. Positive past returns are on average 1.4 times more persistent than negative past returns of an equal magnitude.

Lee et al. (2001) examined time-series features of stock returns and volatility in four of China’s stock exchanges. They provided strong evidence of time-varying volatility and indicated volatility is highly persistent and predictable. Moreover, evidence in support of a fat-tailed conditional distribution of returns was found. By employing eleven models and using symmetric and asymmetric loss functions to evaluate the performance of these models.

Balaban et al. (2003) forecasted stock market volatility of fourteen stock markets. According to symmetric loss functions the exponential smoothing model provides the best forecast. However, when asymmetric loss functions are applied ARCH-type models provide the best forecast.

Moreover, Balaban and Bayar (2005) used both symmetric and asymmetric ARCH-type models to derive volatility expectations. The outcome showed that there was a positive effect of expected volatility on weekly and monthly stock returns of both Philippines and Thailand markets according to the ARCH model. The result was not clear if using the other models such as GARCH, GJR-GARCH and EGARCH will achieve the same outcome. For emerging African markets.
Ogum et al. (2005) investigated the market volatility using Nigeria and Kenya stock return series. Results of the exponential GARCH model indicate that asymmetric volatility found in the U.S. and other developed markets is also present in Nigerian stock exchange (NSE), but Kenya shows evidence of significant and positive asymmetric volatility. Also, they show that while the Nairobi Stock Exchange return series indicate negative and insignificant risk-premium parameters, the NSE return series exhibit a significant and positive time-varying risk premium.

Alberg et al. (2006) estimate stock market volatility of Tel Aviv Stock Exchange indices, for the period 1992-2005. They reported that the EGARCH model is the most successful in forecasting the TASE indices. Various time series methods are employed by Tudor (2008), including the simple GARCH model, the GARCH-in-Mean model and the exponential GARCH to investigate the Risk-Return Trade-off on the Romanian stock market. Results of the study confirm that EGARCH is the best fitting model for the Bucharest Stock Exchange composite index volatility in terms of sample-fit.

Bouoiyour and Selmi (2012) examined modelling exchange volatility in Egypt using GARCH models with monthly data from 1994 to 2009. Their results showed volatility clustering (i.e. Standard GARCH) and persistence which implies a mean reversion in the variance process. However, when they considered the leverage effect (i.e. Exponential GARCH) they noticed a tendency to a long memory which can be itself be a source of an explosive process.
Dima et al. (2008) looked at Estimating stock market volatility using asymmetric GARCH models with the help of Tel Aviv Stock Exchange (TASE) indices. They employed the GARCH (1,1), EGARCH(1,1), GJR(1,1) and APARCH(1,1) models estimated using GARCH 2.0 by the approximate quasi-maximum likelihood estimator assuming normal, Student-t or skewed Student-t errors. Their results showed that the asymmetric GARCH model with fat-tailed densities improves overall estimation for measuring conditional variance. The EGARCH model using a skewed Student-t distribution was discovered as the most successful for forecasting.

Forecasting exchange rate volatility using conditional variance models selected by information criteria was what Brooks and Burke (1998) sought to find in their study. They used a set of weekly continuously compounded percentage exchange rate returns on the Canadian dollar, German mark, and Japanese yen, all against the US dollar. They employed an appropriately modified information criteria to select models from the GARCH family, which are subsequently used for predicting US dollar exchange rate return volatility. They stated that the out of sample forecast accuracy of models chosen in this manner compares favourably on mean absolute error grounds, although less favourably on mean squared error grounds, with those generated by the commonly used GARCH(1, 1) model.

Shephard and Andersen (2009) on the other hand analysed the development of SV models and several volatility processes including jumps and long memory associated with equity indices, bonds, and exchange rates due to monetary policy announcements. Earlier, Andersen and Bollerslev (1998a) examined the
DM/USD intraday volatility based on a one-year sample of five minutes returns with emphasis on activity patterns, macroeconomic announcement and calendar effects. They found that market activity is correlated with price variability and that scheduled releases occasionally induce large price changes, but the associated volatility shocks appear short lived. Bollerslev (1990) proposed a multivariate time series model with time-varying conditional variances and co-variances but with conditional correlation. The validity of the model was illustrated for a set of five European/US dollar exchange rates.

Exchange rate volatility has long been a concern for academics and policy-makers, driven largely by the effect of the volatility on trade and growth (e.g., Aghion et al, 2006). In the same vein, it has been argued that uncertainty in the exchange rate always reflects the inconsistent behaviour of macroeconomic fundamentals (see, Yoon and Lee, 2008).

Accordingly, Yoon et al (2008) articulated that the amplitude of exchange rate volatility generally shows the extent to which economic agents fail to discern the direction of actual or future volatility of the exchange rate, that is, the more forecast errors are made by economic agents, the higher the trends in the volatility of the exchange rate.

Consequently, various economies have pursued different trade policies alongside institutional arrangements to ensure better stability in the foreign exchange rate in order to guarantee sustained economic growth and development. Even so, most policy-makers are enthusiastic in controlling the volatility in their exchange rate. This is evident from the International Monetary Fund’s (IMF) de facto classification of exchange rate regimes which
indicated that only 21% (40) of the 188 economies surveyed as at April 2008 allowed their exchange rate to independently float. Among the reasons for the anxiety to control exchange rate volatility, Frankel (2005) shows that finance ministers are more likely to lose their jobs after excessive depreciation. Klein and Shambaugh (2008) also report that free floating regimes have significantly higher volatility than other regimes (see, Hasan and Wallace, 1996). However, findings of Rose (2007) reveal that countries with inflation targeting regimes tend to have lower exchange rate volatility than those that do not. Bravo-Ortega and di Giovanni (2008) found higher exchange rate volatility for economies with lower openness to trade and lower per capita income.

Fundamentals account for a small proportion of the observed volatility in exchange rates. Several reasons are advanced. A weak link exists between underlying fundamentals and exchange rate volatility such that changes in exchange rates take place even when there are no detectable changes in fundamentals. The volatility of the exchange rate has increased considerably while that of the macroeconomic variables has remained virtually unchanged. Exchange rate series have non-normal distributions reflected in fat tails and excess kurtosis while fundamentals do not have similar distributional characteristics. Finally, in the short-run, exchange rates deviate from their equilibrium level implied by PPP and these deviations are large and persist for a long time due to sluggish prices.

Disequilibrium models by Dornbusch (1976), Mussa (1986) and Edwards (1987) emphasis the importance of nominal shocks with transitory effects. On
the other hand, equilibrium models like Stockman (1983) emphasise real
shocks with permanent effect which are identified as key sources of real and
nominal exchange rate fluctuations and empirical support exists (see Sfia,
2006).

According to Dornbusch (1976), real exchange rate volatility is due to slow
adjustment of commodity prices and rapid response of nominal exchange rates
(asset prices) to exogenous shocks. The nominal exchange rate overshoots its
long-run equilibrium level following immediately after the shock and this
induces volatility in real exchange rate. The implication of the sluggish
commodity price adjustment is that unlike under fixed exchange rate regime,
the nominal exchange rate is characterised by volatile behaviour under flexible
exchange rate system that in turn causes the real exchange rate to be volatile.
Stickiness in prices generates large co-variation of real and nominal exchange
rates. Hence, the more variable the money stock, the more variable will be
exchange rates and vice-versa.

De Grauwe et al. (1985) predict a non-linear positive relationship between
exchange rate volatility and variability of monetary disturbances (defined to
include money supply and inflation) based on a synthesis of stick-price and
news models of exchange rate determination. Korteweg (1980) emphasised
inflation differentials as a potential source of nominal and real exchange rate
volatility. High inflation rate erodes the purchasing power of a currency and
this causes agents to shift assets into a stronger currency and thus leads to
depreciation as the currency adjusts to accommodate inflation differential
between countries. Empirical support for domestic monetary policy as a

Engle (2003) showed how dynamic volatility models can be used to forecast volatility, options valuation and risk over a long horizon. Accordingly, Engle (2002) analysed properties of ARCH, SV, long memory and breaking volatility models by estimating the volatility of volatility and comparing it with option–implied volatilities. In terms of analysing model forecasting power, Hansen and Lunde (2005) compare 330 ARCH–type models in terms of their ability to describe the conditional variance, and finds no evidence that a GARCH (1,1) model is outperformed by more sophisticated models in their analysis of exchange rates. Teräsvirta (2006) reviews several Univariate models of conditional heteroscedasticity and reports that GARCH models tend to exaggerate volatility persistence. She also argued that if at least one of $\alpha_i > 0$ and $\beta_j > 0$, the model so considered is a genuine model.

To keep the conditional variance of the generated by GARCH (p,q) non-negative, Bollerslev imposed the nonnegative constraints on the parameters of the process, but Nelson and Cao (1992) showed that these constraints can be substantially weakened and so should not be imposed in estimation. They also provided empirical examples illustrating the importance of relaxing these constraints. They argued that the nonnegative constraint is a sufficient condition but not a necessary condition.

Kasman et al. (2011) investigated the effects of interest and exchange rate changes on Turkish bank’s stock returns and finds significant negative impact.
Their results further indicate that interest and exchange-rate volatility are the major determinants of conditional bank stock return volatility.

Giraitis, et al. (2009) examines ARCH (α) models, their stationarity, long memory properties and the limit behaviour of partial sums of their processes and their modifications like: linear ARCH, and bilinear models. In line with other theoretical studies, Ling and McAleer (2002) derive the necessary and sufficient conditions for the existence of higher order moments for GARCH and asymmetric Power GARCH models.

Thanh (2008) looked at modelling and forecasting volatility by GARCH-type models with evidence from the Vietnam stock exchange. His findings revealed the inappropriateness of asymmetric GARCH in modelling Vietnam stock return volatility. His results further provide evidence of the superiority of GARCH (1,1) and GARCH (2,1) over the other GARCH models in estimation and forecasting capabilities.
CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Introduction

This chapter presents the data and the various statistical techniques to be used in the analysis.

3.1 The exchange data

The study used monthly secondary data on exchange rate between the Cedi and the Dollar from the Bank of Ghana (BoG) from January, 1990 to November, 2013. A total of 287 data points are used in the modeling process. The data range is divided into two parts. The first part includes data from January 1990 to the end of 2012 which are used for the estimation of the models parameters. As the sample period covers 287 observations, the remaining 11 observations which are from January, 2013 to November 2013 are employed for out-of-sample forecasts. In this study, returns \( (r_t) \) were calculated as the continuously compounded returns which are the first difference in logarithm of the interbank exchange rate.

\[
r_t = \log \left( \frac{e_t}{e_{t-1}} \right)
\]

(3.1)

where \( e_t \) means Cedi/dollar exchange rate at time \( t \) and \( e_{t-1} \) represent exchange rate at time \( t-1 \). The \( r_t \) in Equation 3.1 will be used in investigating the volatility of the interbank exchange rate.

Campbell et al. (1997) gave two main reasons for using returns. First, for average investors, return of an asset is a complete and scale-free summary of
the investment opportunity. Secondly, return series are easier to handle than price series because the former have more attractive statistical properties.

3.2 Unit Root Tests

In order to make inferences on time series, they must be stationary. A time series \( \{y_t\} \) is said to be strictly stationary if the joint distribution of \((y_{t1}, y_{t2}, \ldots, y_{tk})\) is identical to that of \((y_{t1+t}, \ldots, y_{tk+t})\) for all \( t \), where \( k \) is an arbitrary positive integer and \((t_1, \ldots, t_k)\) is a collection of \( k \) positive integers. In other words, strict stationarity requires that the joint distribution of \((y_{t1}, \ldots, y_{tk})\) is invariant under time shift. A weaker version of stationarity is often assumed. A time series \( \{y_t\} \) is weakly stationary if both the mean of \( y_t \) and the covariance between \( y_t \) and \( y_{t-\ell} \) are time-invariant, where \( \ell \) is an arbitrary integer.

More specifically, \( \{y_t\} \) is weakly stationary if (a) \( E(y_t) = \mu \), which is a constant, and (b) \( \text{Cov}(y_t, y_{t-\ell}) = \gamma \ell \), which only depends on \( \ell \). A financial time series whose mean, variance and autocovariance are constant is considered to be stationary. That is autocovariance function as \( \text{cov}(y_t, y_{t+k}) \) for any lag \( k \) is only a function of \( k \) and not time, that is \( \gamma_y(k) = \text{cov}(y_t, y_{t+k}) \). However, most of the financial time series such as interest rates, foreign exchange rates, or the price series of an asset tend to be nonstationary. These series do not satisfy the requirements of stationarity so that they have to be converted to stationary processes before modelling.

Several methods have been suggested for testing the stationarity of a time series data. These include Dickey-Fuller (DF) and Augmented Dickey-Fuller test, Kwiatkowski-Phillips-Schrnidt-Shin test, Philip – Peron Test and NP test.
We shall use the Augmented Dickey-Fuller (ADF) and Philip-Peron tests to establish the stationarity or otherwise of the data. A graphical method can also be used to test for unit roots by observing the Autocorrelation function (ACF) plots. A strong and slow dying ACF will indicate deviation from stationarity.

3.2.1 The Dickey-Fuller Test

Dickey and Fuller (1979) introduced Dickey - Fuller (DF) test statistic to test whether the series contains unit root or not (A time series that is nonstationary is said to exhibit unit root). The test is performed by estimating regression models. The regression models can be fitted with constant and with constant and trend. The model with constant captures the non-zero mean under the alternative hypothesis.

The testing procedure for the ADF test is the same as for the Dickey–Fuller test but it is applied to the model;

\[ \Delta y_t = \alpha + \beta t + \pi y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_{p-1} y_{t-p+1} + \epsilon_t \]  \hspace{1cm} (3.2)

where \( \alpha \) is a constant, \( \beta \) the coefficient on a time trend and \( \rho \) the lag order of the autoregressive process. Imposing the constraints \( \alpha = 0 \) and \( \beta = 0 \) corresponds to modelling a random walk and using the constraint \( \beta = 0 \) corresponds to modelling a random walk with a drift. The null hypothesis for this test is \( H_0: \gamma = 0 \), the existence of unit root and the alternative hypothesis is \( H_1: \gamma < 0 \), the non-existence of unit root. The test statistic for the ADF test is given by

\[ ADF = \frac{\gamma}{SE(\gamma)} \]  \hspace{1cm} (3.3)
Where $\hat{Y}$ denote the Least Squares estimates of $Y$ and $SE(\hat{Y})$ is the standard error. The null hypothesis is rejected if the test statistic is greater than the critical value.

The estimation technique is ordinary Least Squares (OLS).

### 3.2.2 Philip-Perron (PP) Test

The PP test is similar to the ADF test with regards to the statement of its hypothesis. This test corrects the statistic for serial correlation and possible Heteroscedastic error terms. The test is based on the regression equation

$$\Delta Y_t = \alpha + \pi Y_{t-1} + \delta t + \varepsilon_t$$  \hspace{1cm} (3.4)

Where $Y_t$ is the time series, $\alpha$ is the intercept, $\pi$ is the coefficient of interest, $t$ is the time or trend variable and $\varepsilon_t$ is the disturbance term.

The Ordinary Least Squares standard errors are adjusted for serial correlation in the disturbance term $\varepsilon_t$. We fail to reject the null hypothesis of the existence of unit root if the test statistic is less than the critical value.

### 3.3 Autoregressive Integrated Moving Average (ARIMA) Model

One approach, advocated in the landmark work of (Box and Jenkins, 1976) is the development of a systematic class of models called autoregressive integrated moving average (ARIMA) models to handle time-correlated modelling and forecasting. It is the generalization of the ARMA $(p, q)$ model to provide adequate models for non-stationary time series variables.

The ARIMA models are generally referred to as ARIMA$(p, d, q)$ models where $p$ is the order of AR (autoregressive process), $q$ is the order of the moving average process and the $d$ is the order of the integration of the series (that is the number of times the series has to be differenced in order to make it
stationary). If the series is differenced once, then we say the series is integrated of order one and if the series is differenced d times to make it stationary, then we say the series is integrated of order d.

### 3.3.1 Autoregressive (AR) Models

A time series is said to be Autoregressive if based on its past values, future values can be predicted. An autoregressive process of order one tells us that based on the lag one \((l)\) variable of the time series, the future values of the series can be predicted. It simply tells us what will happen tomorrow only depend on what is happening today. The AR \((1)\) model is given by the equation

\[
y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t
\]

where \(\varepsilon_t\) is assumed to be a white noise process. AR \((2)\) model is also given

\[
y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t
\]

Generally an autoregressive model of order \(p\) is given by the equation

\[
y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t
\]

where \(\varepsilon_t\) is assumed to be a white noise,

\(y_t\) = response variable at time \(t\),

\(y_{t-k}\) = Observation (predictor variable) at time \(t-k\),

\(\phi_t\) = regression coefficients to be estimated.

### 3.3.2 Moving Average (MA) Models

A time series \(Y_t\) is said to follow moving average process if the current value of the observation is in terms of the past shocks or residuals. This means that based on past values of the residuals, the future values of the series can be predicted. The moving average models are always stationary because it is the
moments are time invariant. A simple MA (1) model is given by

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

(3.8)

Where $$\varepsilon_t$$ is the white noise process. An MA model of order 2 written as MA (2) is also given as:

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

(3.9)

Generally a moving average of order q is a regression model with the dependent variable, $$y_t$$ depending on the previous values of the error rather than the variable itself.

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

(3.10)

where $$y_t$$ = response variable at time t, $$\mu$$ = Constant mean of the process, $$\theta_i$$ = Coefficients of regression to be estimated and $$\varepsilon_{t-q}$$ is the error in time t-q.

ARMA models is the combination of the simple AR and MA model of order (p, q) called the autoregressive moving average model (ARMA). The p represents the order of the Autoregressive process and the q represents the order of the Moving average process. The general form of the model is given by

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

(3.11)

3.4 Volatility of the exchange rate

Volatility is the amount of price movement of a stock, bond or the market in general during a specific period. If the price moves up and down rapidly over short time periods, it has high volatility; if the price almost never changes, it has low volatility. Volatility has many other financial applications. Volatility
is not directly observable and this unobservability of volatility makes it difficult to evaluate the forecasting performance of conditional Heteroscedastic models.

Although volatility is not directly observable, it has some characteristics that are commonly seen. First, there exist volatility clusters (i.e., volatility may be high for certain time periods and low for other periods). Secondly, volatility evolves over time in a continuous manner, that is, volatility jumps are rare. Thirdly, volatility does not diverge to infinity, that is, volatility varies within some fixed range. Statistically speaking, this means that volatility is often stationary. Fourth, volatility seems to react differently to a big price increase or a big price drop, referred to as the leverage effect.

These properties play an important role in the development of volatility models. Some volatility models were proposed specifically to correct the weaknesses of the existing ones for their inability to capture the characteristics mentioned earlier. For example, the EGARCH model was developed to capture the asymmetry in volatility induced by big "positive” and “negative” asset returns.

Campbell et al. stated in 1997, “It is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time”. In some cases, the postulation of constant variance is not satisfied and this is called as the heteroscedasticity problem.

The volatility models can be divided into two main classes: deterministic and stochastic volatility models. In deterministic volatility models, the conditional
variance is a deterministic function of past observations. These are called as Autoregressive Conditionally Heteroscedastic (ARCH) type models. In stochastic case, on the other hand, the variance equation has its own innovation component which makes the process stochastic rather than deterministic.

3.5 Testing for Heteroscedasticity

One of the most significant issues to consider before applying the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) methodology is to first examine the residuals for evidence of heteroscedasticity. The Lagrange Multiplier (LM) test for ARCH effects proposed by (Engle, 1982) is applied. In summary, the test procedure is performed by first obtaining the residuals from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes; (ARMA) process. For example, in ARMA (1,1) process the conditional mean equation will be as:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$$ \hspace{1cm} (3.12)

After obtaining the residuals, the next step is regressing the squared residuals on a constant and q lags as in the following equation:

$$e_t^2 = a_0 + a_1 e_{t-1}^2 + a_2 e_{t-2}^2 + \ldots + a_q e_{t-q}^2 + v_t$$ \hspace{1cm} (3.13)

where $e_t$ is the residual.

If there exist no ARCH-effect, then it implies that the residuals of the model are Homoscedastic (have constant variance). In these models, the mean equations are written as a function of constant, autoregressive and moving
average with an error term. The Ljung Box will be used in the squared residuals to also test for ARCH effects.

3.5.1 Autoregressive Conditionally Heteroscedastic Models

Building a volatility model for a financial time series consists of four steps:

- Specify a mean equation by testing for serial dependence in the data and, if necessary, building an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence.
- Use the residuals of the mean equation to test for ARCH effects.
- Specify volatility model if ARCH effects are statistically significant and perform a joint estimation of the mean and volatility equations.
- Check the fitted model carefully and refine it if necessary.

3.6 The GARCH Models

3.6.1 The Autoregressive Conditional Heteroskedasticity (ARCH)

It is a Univariate conditionally heteroskedastic white noises, mostly useful in finance and econometrics for modelling conditional heteroskedasticity and volatility clustering. The model was developed by the Nobel Prize winner Robert Engle in 1982. The methodological innovation which sets it apart from the previous time series econometric models suggests that the variance of the error terms at the time moment \( t \) depends on the squared error terms from the past periods of time. The essence of the model was that it is much more efficient to be used simultaneously for the mean and variance of a financial time series in the case that the Conditional variance is not constant. The basic idea of ARCH models is that (a) the shock \( a_t \) of the financial instrument is serially uncorrelated, but dependent, and (b) the dependence of \( a_t \) can be
described by a simple quadratic function of its lagged values. Specifically, an ARCH \((p)\) model assumes that

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_p \epsilon_{t-p}^2, \quad \epsilon_t = \sigma_t \epsilon_t, \tag{3.14}
\]

ARCH (1) model is given as

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2, \quad \epsilon_t = \sigma_t \epsilon_t \tag{3.15}
\]

where \(\epsilon_t\), defined as \(\epsilon_t = \sigma_t \epsilon_t\) is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, \(\alpha_0 > 0\) and \(\alpha_1 \geq 0\).

The ARCH \((p)\) model according to Tsay (2005) has a lot of weaknesses some which include:

- The model assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. In practice, it is well known that the price of a financial asset responds differently to positive and negative shocks.

- The ARCH model is rather restrictive. For instance, \(\alpha_1^2\) of an ARCH (1) model must be in the interval \([0, \frac{1}{3}]\) if the series has a finite fourth moment. The constraint becomes complicated for higher order ARCH models. In practice, it limits the ability of ARCH models with Gaussian innovations to capture excess kurtosis.

- The ARCH model does not provide any new insight for understanding the source of variations of a financial time series. It merely provides a mechanical
way to describe the behaviour of the conditional variance. It gives no indication about what causes such behaviour to occur.

- ARCH models are likely to over predict the volatility because they respond slowly to large isolated shocks to the return series.

3.6.2 Generalized Autoregressive Conditionally Heteroscedastic Models

In addition to the weakness of the ARCH model mentioned above, it was also detected that high orders of the ARCH model has to be estimated before it can capture the dynamics of volatility. A generalized Autoregressive Conditionally Heteroscedastic (GARCH) model was first developed by Bollerslev in 1986. The particular feature of this model was to introduce and use the lagged conditional variance terms as autoregressive terms.

The standard GARCH \((p, q)\) process is specified as:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad \varepsilon_t = \sigma_t \varepsilon_t \quad (3.16)
\]

where again \(\varepsilon_t\) defined as \(\varepsilon_t = \sigma_t \varepsilon_t\), is a sequence of iid random variables with mean 0 and variance 1.0, \(\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0\) and \(\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1\).

GARCH \((1,1)\) model is also given below.

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (3.17)
\]

\(0 \leq \alpha_1 \beta_1 \leq 1, (\alpha_1 + \beta_1) < 1\)

The persistence of the conditional variance \(\sigma_t^2\) is captured by \(\alpha + \beta\) and covariance stationarity requires that \(\alpha + \beta < 1\).

GARCH models as pointed out above, allows the conditional variance to depend on a number of its lagged own values. The most widely model used in
practice for many financial time series is GARCH (1, 1) which contains only three parameters in the conditional variance equation. The model is very parsimonious and shown to be sufficient to capture the volatility clustering in data without the requirement of higher order models.

### 3.6.3 The Exponential GARCH Model

To overcome some weaknesses of the GARCH model in handling financial time series, (Nelson, 1991) proposes the exponential GARCH (EGARCH) model. In the basic GARCH model, since only squared residuals $\varepsilon_{t-1}^2$ enter the conditional variance equation, the signs of the residuals or shocks have no effect on conditional volatility. However, a stylized fact of financial volatility is that bad news (negative shocks) tends to have a larger impact on volatility than good news (positive shocks). That is, volatility tends to be higher in a falling market than in a rising market, Zivot (2009). For this reason, exponential GARCH (EGARCH) models were introduced.

An EGARCH $(p, q)$ model can be written as

$$
\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \frac{2}{\pi} \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \left[ \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \right] \tag{3.18}
$$

EGARCH $(1, 1)$ is given by;

$$
\ln(\sigma_t^2) = \alpha_0 + \alpha \left[ \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \frac{2}{\pi} \right] + \beta \log(\sigma_{t-1}^2) + \gamma \left[ \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right] \tag{3.19}
$$

where $\alpha_0, \alpha, \beta, j, and \gamma_k$ are constant parameters. Note that when $\varepsilon_{t-i}$ is positive ("good news"), the total effect of $\varepsilon_{t-i}$ is $1 + \gamma_i / \varepsilon_{t-i}$ while when $\varepsilon_{t-i}$ is negative ("bad news"), the total effect of is $1 - \gamma_i / \varepsilon_{t-i}$
is covariance stationary provided $\sum_{j=1}^{q} \beta_j < 1$. The EGARCH (p, q) model, unlike the GARCH (p, q) model, indicates that the conditional variance is an exponential function, thereby removing the need for restrictions on the parameters to ensure positive conditional variance. The asymmetric effect of past shocks is captured by the $\gamma$ coefficient, which is usually negative, that is, ceteris paribus positive shocks generate less volatility than negative shocks (Longmore and Robinson, 2004). The leverage effect can be tested if $\gamma < 0$. If $\gamma \neq 0$, the news impact is asymmetric.

### 3.6.4 The Threshold GARCH (TGARCH) Model

Another volatility model commonly used to handle leverage effects is the threshold GARCH (or TGARCH) model; see Glosten et al. (1994). In the TGARCH (1, 1) version of the model, the specification of the conditional variance is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$  \hspace{1cm} (3.20)

Where $d_{t-1}$ is a dummy variable, that is $d_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0, \text{ bad news} \\ 0 & \text{if } \varepsilon_{t-1} \geq 0, \text{ good news} \end{cases}$

the coefficient $\gamma$ is known as the asymmetry or leverage term. When $\gamma = 0$, the model collapses to the standard GARCH forms. Otherwise, when the shock is positive (i.e., good news) the effect on volatility is $\alpha_1$, but when the news is negative (i.e., bad news) the effect on volatility is $\alpha_1 + \gamma$. Hence, if $\gamma$ is significant and positive, negative shocks have a larger effect on $\sigma_t^2$ than positive shocks. In the general specification of this model, TGARCH (p,q), the general conditional variance equation is specified as follows:
3.7 Maximum Likelihood Estimation Approach

The study made use of the maximum likelihood estimation approach. Under the presence of ARCH effects, the OLS estimation is not efficient and the estimate of covariance matrix of the parameters will be biased due to invalids t statistics. Therefore, ARCH-type models cannot be estimated by the simple technique such as OLS. The method known as the maximum-likelihood estimation is employed in ARCH models. Maximum-likelihood is a method of estimating the parameters of a statistical model. When applied to a data set and given a statistical model, maximum-likelihood estimation provides estimates for the model's parameters. In practice it is often more convenient to work with the logarithm of the likelihood function, called the log-likelihood:

\[
\ln L \left( \frac{\theta}{x_1, \ldots, x_n} \right) = \sum_{i=1}^{n} \ln f(x_i/\theta) \tag{3.22}
\]

3.8 Model Diagnostics

After a model has been selected, it can be used to make conclusion or generalization, it is important to analyze the model to see whether there is concordance of the model with the actual world observations. Thus, we employed the Ljung-Box and the Univariate ARCH LM tests in diagnosing the developed models.

3.8.1 Ljung Box Test

The Ljung-Box test is used to test for serial correlation in the model residuals. The test statistic is given by
\[ Q_m = n(n + 2) \sum_{k=1}^{m} (n - k)^{-1}r_k^2 \approx \chi^2_m \]  

(3.23)

where

\( n \) is the number of residuals

\( r_k^2 \) represent the residual autocorrelation at lag \( k \)

\( m \) is the number of time lags included in the test

When the \( P \)-value related with \( Q_m \) is large the model is considered adequate else the whole assessment process has to start again in order to get the most adequate model.

### 3.8.2 ARCH-LM Test

The issue of conditional heteroscedasticity is a unique problem that a researcher is likely to encounter when fitting models. This happens when the variance of the residuals is not constant. To ensure that the fitted model is adequate, the assumption of constant variance must be achieved. The ARCH-LM test proposed by (Engle, 1982) was used to test for the presence of conditional heteroscedasticity in the model residuals. The test procedure is as follows;

\[ H_0: \text{There is no heteroscedasticity in the model residuals} \]

\[ H_1: \text{There is heteroscedasticity in the model residuals} \]

The test statistic is

\[ LM = nR^2 \]  

(3.24)
where \( n \) is the number of observations and \( R^2 \) is the coefficient of determination of the auxiliary residual regression.

\[
e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \ldots + \beta_q e_{t-q}^2 + \nu_t \tag{3.25}
\]

where \( e_t \) is the residual. The null hypothesis is rejected when the \( p \)-value is less than the level of significance and is concluded that there is heteroscedasticity.

### 3.9 Model Selection

There may be a tendency for two or more opposing models to compete and for that reason it is appropriate to use good model selection criteria to select the most adequate model. The most famous ones are the Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC). They serve as measures of goodness of fit that are used to select the most acceptable model. Given a data set, several contending models may be ranked according to their AIC and SIC with the one having the lowest information criterion value being the best. The information criterion tries to find the model that best explains the data with the lowest free parameters but also includes a price that is an increasing function of the number of estimated parameters. The reason behind the price is to discourage over fitting.

In general case the

\[
AIC = 2K - 2\ln(L) \tag{3.26}
\]

where \( k \) is the number of parameters in the statistical model and \( L \) is the maximized value of the likelihood function for the estimated model.
\[ SIC = \log(\sigma_e^2) + \frac{k}{n} \log(n), \quad (3.27) \]

where

- \( k \) is the number of parameters in the statistical model
- \( n \) is the number of observations in the data
- \( \sigma_e^2 \) is the error variance.

### 3.10 Forecasting Volatility

A volatility model should be able to forecast volatility. Virtually all the financial uses of volatility models entail forecasting aspects of future returns (Engle and Patton, 2001). The three main purposes of forecasting volatility are for risk management, for asset allocation, and for taking bets on future volatility (Reider, 2009). The selected GARCH type model was used in forecasting. The Chi-Square goodness of fit test was employed to evaluate the forecasted values. The Chi-Square goodness of fit test is a statistical test that measures the extent to which a set of observed sample data deviates from the corresponding set of expected values of that sample.

The null hypothesis and alternative hypothesis are given below.

- \( H_0 \): There is no significant difference between the forecasted values and the observed values

- \( H_1 \): There is significant difference between the forecasted and observed values.

The null hypothesis is rejected if the calculated value is greater than the critical value at the given significance level.
The test statistic is given as:

\[ X^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right] \]  \hspace{1cm} (3.28)

where \( O \) is the observed values and \( E \) is the expected (forecasted) values.
CHAPTER FOUR

DATA ANALYSIS AND DISCUSSION OF RESULTS

4.0 Introduction
The chapter deals with the analysis of the data and interpretation of the results. It is organized into preliminary analysis, further analysis and discussion of results.

4.1 Preliminary Analysis
Table 1 below shows summary of the statistics of the interbank exchange rate returns. The average return was 0.014756. The maximum and minimum returns were 0.147983 and -0.016216 respectively. The positive skewness of 2.655882 clearly indicates lack of symmetry in the returns. The excess kurtosis of 11.8589 indicates that the distribution of the returns were leptokurtic in nature. This implies that the returns were closely distributed around the average return. The Jacque Bera test affirms the non-normality of the returns.

Table 4.1: Descriptive Statistics for Exchange Rate Return Series

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02</td>
</tr>
<tr>
<td>Median</td>
<td>0.01</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.15</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.02</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.02</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.66</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>11.86</td>
</tr>
<tr>
<td>Jarque Bera</td>
<td>1271.46</td>
</tr>
<tr>
<td>Probability</td>
<td>0.00</td>
</tr>
<tr>
<td>Sum</td>
<td>4.22</td>
</tr>
<tr>
<td>Sum Sq. Dev.</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Figure 4.1 displays the time series plot of the returns which fluctuates about a fixed point indicating stationarity in mean and variance of the series.

![Time series plot of returns](image1)

**Figure 4.1: Time series plot of returns**

Figure 4.2 displays the ACF and the PACF of the returns. The rapid decay in the ACF and the PACF affirms the stationarity of the returns.

![ACF and PACF plots](image2)

**Figure 4.2: ACF and PACF of the returns.**
4.2 Further analysis

4.2.1 Fitting the GARCH models

The ADF test was employed to affirm the stationarity of the returns. The test performed with constant and constant with trend both affirms that the values were stationary.

<table>
<thead>
<tr>
<th>Table 4.2: Augmented Dicker-Fuller test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>ADF</td>
</tr>
<tr>
<td><strong>Critical values</strong></td>
</tr>
<tr>
<td>1%</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>10%</td>
</tr>
</tbody>
</table>

The Philip-Perrons test was further used to affirm the stationarity of the returns as shown in Table 4.3. A significant PP test statistics was obtained at the 1%, 5% and 10% critical values, affirming that, the returns were covariance stationary. This confirms the rapid decay of the ACF and PACF of the returns in Figure 4.2.

<table>
<thead>
<tr>
<th>Table 4.3: Philips-Perrons Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>PP</td>
</tr>
<tr>
<td><strong>Critical values</strong></td>
</tr>
<tr>
<td>1%</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>10%</td>
</tr>
</tbody>
</table>

Before modelling the volatility in the returns, various mean equations were fitted to the returns and the ARMA (1, 1) was selected as the best mean model based on the AIC and SIC as shown in Table 4.4. The condition mean
equation was to capture the autocorrelation caused by market microstructure effects (e.g., bid-ask bounce) as advocated by Zivot (2009).

Table 4.4: Selecting an appropriate mean equation

<table>
<thead>
<tr>
<th>ARMA(p,q)</th>
<th>AIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,1)</td>
<td>-5.1529</td>
<td>-5.1134</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
<td>-5.1463</td>
<td>-5.1067</td>
</tr>
<tr>
<td>ARMA(1,2)</td>
<td>-5.1420</td>
<td>-5.1025</td>
</tr>
<tr>
<td>ARMA(2,2)</td>
<td>-4.9528</td>
<td>-4.9131</td>
</tr>
<tr>
<td>ARMA(2,3)</td>
<td>-4.8690</td>
<td>-4.8303</td>
</tr>
<tr>
<td>ARMA(3,2)</td>
<td>-4.8313</td>
<td>-4.7915</td>
</tr>
<tr>
<td>ARMA(3,3)</td>
<td>-4.8405</td>
<td>-4.8007</td>
</tr>
</tbody>
</table>

The ARCH-LM test and the Ljung Test shown in Tables 4.5 and 4.6 respectively were used to test for the presence of ARCH effects. The results from Table 4.5 and 4.6 revealed that there was an ARCH effect in the residuals of the ARMA model (1, 1). This calls for the fitting of a variance equation (GARCH).

Table 4.5: Heteroskedasticity Test: ARCH LM Test

| F-statistic | Probability | 0.0000 |
| Obs*R-squared | Probability | 0.0000 |

Table 4.6: Test for Heteroscedasticity-Ljung Box Test

<table>
<thead>
<tr>
<th>Lags</th>
<th>Test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>86.9400</td>
<td>0.0000</td>
</tr>
<tr>
<td>12</td>
<td>104.5800</td>
<td>0.0000</td>
</tr>
<tr>
<td>18</td>
<td>105.9000</td>
<td>0.0000</td>
</tr>
<tr>
<td>24</td>
<td>110.5100</td>
<td>0.0000</td>
</tr>
<tr>
<td>36</td>
<td>119.1400</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
To determine the orders of the GARCH models, the ACF and PACF of the square returns and the square residuals were examined as shown in Figures 4.2 and 4.3 below. From both plots GARCH models of orders (1,1), (1,2), (2,1), (2,2), (3,2), (2,3) and (3,3) were fitted.

After the determination of the order of the model, the parameters of the model can now be estimated. The maximum likelihood method was employed in estimating the parameters of the model. As the order determined is usually a suggestion of the order around which the most appropriate model is found, several models of different orders were fitted and the most appropriate model selected based on the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC). The criterion is that the smaller the AIC and SIC values, the better. The first model estimated is the ARCH model. Several ARCH models were estimated and the best model selected based on the AIC.
and the SIC as shown in Table 4.7. From Table 4.7 ARCH (3) has the lowest AIC and SIC values of (-5.9823) and (-5.8900) respectively.

### Table 4.7: Selecting the best ARCH model

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(1)</td>
<td>-5.7813</td>
<td>-5.7152</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>-5.8412</td>
<td>-5.7621</td>
</tr>
<tr>
<td>ARCH(3)</td>
<td>-5.9823</td>
<td>-5.8900</td>
</tr>
</tbody>
</table>

Under the assumption of student’s t with fixed degrees of freedom of its residuals, the parameters are estimated in Table 4.8 below. Table 4.8 estimated jointly the mean equation and the variance equation. In the mean equation, the constant term was significant at the 5% significance while the AR and MA terms were significant at the 1% significance level. In the variance equation, all the parameters were significant at the 1% significance level except the second ARCH term which is significant at the 5% significance level. The significance of the ARCH parameters indicates that previous periods squared residuals have an influence on current’s variance (volatility). The sum of the coefficients of the ARCH parameters was less than one and indicates the stationarity of the model. Equation 3.14 is the ARMA (1,1) -ARCH (3) model.

\[
y_t = 0.0051 + 0.8606y_{t-1} + \varepsilon_t - 0.3239\varepsilon_{t-1},
\]
\[
\sigma_t^2 = 0.00002 + 0.4219\varepsilon^2_{t-1} + 0.1380\varepsilon^2_{t-2} + 0.4145\varepsilon^2_{t-3}, \varepsilon_t
\]
\[
= \sigma_t\varepsilon_t
\]  

(3.14)
Several GARCH models were estimated and the best model selected as shown in Table 4.9. From Table 4.9, the GARCH (2, 3) has the smallest AIC and SIC values of -6.2098 and -6.0911 respectively and hence is the most appropriate among the GARCH (p, q) models. The parameter of the estimated GARCH (2,3) were given in Table 4.10.

The parameters of the mean equation were significant at the 1% level of significance except the constant term. In the variance equation, the parameters were significant at the 1% significance level and $\sum_{i=1}^{2} \alpha + \ldots$
\[ \sum_{j=1}^{3} \beta = 0.986431 \] affirming the stationarity of the GARCH (2, 3) model. The significance of the ARCH parameters and the GARCH parameters indicates that previous periods squared residuals and previous period's residual variance have an influence on current variance of the residual.

From Table 4.10. Equations (3.16) is the ARMA(1,1)–GARCH (2, 3) model.

\[
y_t = 0.0031 + 0.8139y_{t-1} + \varepsilon_t - 0.3115\varepsilon_{t-1}, \\
\sigma_t^2 = 0.000006 + 0.3802\varepsilon_{t-1}^2 + 0.3468\varepsilon_{t-2}^2 - 0.4731\sigma_{t-1}^2 + \\
0.2356\sigma_{t-2}^2 + 0.4969\sigma_{t-3}^2, \varepsilon_t = \sigma_t\varepsilon_t 
\]

(3.16)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0031</td>
<td>0.0015</td>
<td>2.0517</td>
<td>0.0402</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.8139</td>
<td>0.0401</td>
<td>19.8520</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.3114</td>
<td>0.0776</td>
<td>-4.0126</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Variance Equation

| \(\alpha_0\) | 6.16E-06 | 1.77E-06 | 3.4708 | 0.0005 |
| \(\alpha_1\) | 0.3802  | 0.0693  | 5.4832 | 0.0000 |
| \(\alpha_2\) | 0.3468  | 0.0577  | 6.0099 | 0.0000 |
| \(\beta_1\) | -0.4731 | 0.0299  | 15.8342| 0.0000 |
| \(\beta_2\) | 0.2356  | 0.0320  | 7.3597 | 0.0000 |
| \(\beta_3\) | 0.4969  | 0.0367  | 13.5465| 0.0000 |

Table 4.11 displays several EGARCH (2, 2) models fitted to the returns. From Table 4.11, EGARCH (2, 2) was the best among the EGARCH models as it has the least values of the AIC and SIC.
From Table 4.12, the parameters of the EGARCH (2, 2) were all significant at the 1% level of significance except the second GARCH term. This means that it adds little explanatory power to the model. The sum of GARCH parameters was less than one meaning that the model is covariance stationary and persistent. The asymmetric (leverage) parameter was significant but positive indicating the absence of leverage effects. Leverage effects is said to exist in the EGARCH model if the coefficient of the leverage parameter is significant and negative. The significance of the ARCH and GARCH parameters in the EGARCH model indicates that previous periods squared residual and previous period variance of the residual have an influence on current variance of the residuals. Equation (3.18), the ARMA (1,1)—EGARCH (2,2) model can be written as:

\[
y_t = 0.0042 + 0.8586y_{t-1} + \varepsilon_t - 0.2752\varepsilon_{t-1},
\]

\[
\ln(\sigma_t^2) = -0.9122 - 0.3637 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int \frac{2}{\pi} + 0.2950 \frac{\varepsilon_{t-2}}{\sigma_{t-2}} - \int \frac{2}{\pi}
\]

\[+ 0.6807 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + 0.8013 \log(\sigma_{t-1}^2) + 0.1129 \log(\sigma_{t-2}^2), \varepsilon_t = \sigma_t \varepsilon_t \quad (3.18).
\]
Table 4.12: Parameters estimate of EGARCH (2, 2)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0042</td>
<td>0.0014</td>
<td>2.9519</td>
<td>0.0032</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.8586</td>
<td>0.0348</td>
<td>24.6600</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.2752</td>
<td>0.0538</td>
<td>-5.1100</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Variance Equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>-0.9122</td>
<td>0.2193</td>
<td>-4.1605</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.3637</td>
<td>0.1163</td>
<td>-3.1272</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.2950</td>
<td>0.0983</td>
<td>3.0019</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.6807</td>
<td>0.1036</td>
<td>6.5733</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.8013</td>
<td>0.1684</td>
<td>4.7571</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.1128</td>
<td>0.1577</td>
<td>0.7156</td>
<td>0.4742</td>
</tr>
</tbody>
</table>

Table 4.13 displays the various TGARCH models fitted to the returns. From Table 4.13, TGARCH (2,3) was the best model among the TGARCH models since it has the least values of the AIC and SIC.

Table 4.13: Selecting the best TGARCH Model

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TGARCH(1,1)</td>
<td>-6.1169</td>
<td>-6.0246</td>
</tr>
<tr>
<td>TGARCH(1,2)</td>
<td>-6.1366</td>
<td>-6.0311</td>
</tr>
<tr>
<td>TGARCH(2,1)</td>
<td>-6.0498</td>
<td>-5.9443</td>
</tr>
<tr>
<td>TGARCH(2,2)</td>
<td>-6.0154</td>
<td>-5.8967</td>
</tr>
<tr>
<td>TGARCH(2,3)</td>
<td>-6.2034</td>
<td>-6.0715</td>
</tr>
<tr>
<td>TGARCH(3,2)</td>
<td>-6.0386</td>
<td>-5.9067</td>
</tr>
<tr>
<td>TGARCH(3,3)</td>
<td>-5.9615</td>
<td>-5.8164</td>
</tr>
</tbody>
</table>

The parameters of the mean equation were significant at the 1% significance level except the constant term. In the variance equation, all the parameters were significant at the 1% significance level except the constant term which was significant at the 5% significance level. The asymmetric (leverage) parameter was significant but negative indicating the absence of leverage.
effects. Leverage effects is said to exist in the TGARCH model if the leverage parameter is significant and positive.

Equation (3.20), is the ARMA (1,1) - TGARCH (2, 3) model.

\[ y_t = 0.0020 + 0.8890y_{t-1} + \varepsilon_t - 0.5347\varepsilon_{t-1}, \]

\[ \sigma_t^2 = 0.000001 + 0.6063\varepsilon_{t-1}^2 - 0.6302d_{t-1}\varepsilon_{t-1}^2 + 0.2365\varepsilon_{t-2}^2 - 
0.2442\sigma_{t-1}^2 + 0.2121\sigma_{t-2}^2 + 0.4548\sigma_{t-3}^2, \varepsilon_t = \sigma_t\varepsilon_t \]  

(3.20)

Table 4.14: Parameters Estimate of TGARCH (2, 3) Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0020</td>
<td>0.0012</td>
<td>1.6212</td>
<td>0.1050</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.8889</td>
<td>0.0318</td>
<td>27.9376</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.5347</td>
<td>0.0635</td>
<td>-8.4082</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Variance Equation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0)</td>
<td>1.37E-06</td>
<td>5.67E-07</td>
<td>2.4074</td>
<td>0.0161</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.6062</td>
<td>0.1106</td>
<td>5.4790</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>-0.6302</td>
<td>0.1593</td>
<td>-3.9568</td>
<td>0.0001</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.2364</td>
<td>0.0508</td>
<td>4.6562</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>-0.2442</td>
<td>0.0411</td>
<td>-5.9391</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>0.2121</td>
<td>0.0374</td>
<td>5.6738</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta_3)</td>
<td>0.4548</td>
<td>0.0421</td>
<td>10.7785</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

4.2.1.1 Model Diagnostics.

Table 4.15 displays the results of Lagrange Multiplier test for ARCH effects and the null hypothesis of no ARCH effects cannot be rejected since the P-values were greater than 5%.
Table 4.15: Heteroskedasticity Test: ARCH-LM Test for ARCH (3)

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Obs*R-squared</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1870</td>
<td>2.3398</td>
<td>0.9988</td>
</tr>
</tbody>
</table>

Table 4.16 displays the result of the Ljung-Box test for diagnosing the ARCH (3) model. The probabilities values were greater than 5% level of significance for the selected lags. Therefore the null hypotheses of no autocorrelation cannot be rejected indicating that there were no remaining autocorrelation in the model residuals.

<table>
<thead>
<tr>
<th>Lags</th>
<th>test Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.7695</td>
<td>0.9400</td>
</tr>
<tr>
<td>12</td>
<td>2.3663</td>
<td>0.9990</td>
</tr>
<tr>
<td>18</td>
<td>4.0469</td>
<td>1.0000</td>
</tr>
<tr>
<td>24</td>
<td>4.4907</td>
<td>1.0000</td>
</tr>
<tr>
<td>36</td>
<td>8.8502</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

4.2.1.2 Model Diagnostics for GARCH (2, 3)

Table 4.17 displays the results of Lagrange Multiplier test for ARCH effects in the GARCH (2,3) model. The null hypothesis of no ARCHs effects cannot be rejected since the P-values are greater than 5%. There are therefore no additional ARCH effects in the residual of GARCH (2, 3).

Table 4.17: Heteroskedasticity Test: ARCH LM for GARCH (2, 3)

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Obs*R-squared</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0553</td>
<td>0.6966</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Table 4.18 displays the Ljung-Box test for the GARCH (2,3). The probabilities values were greater than 5% level of significance for the selected lags. Therefore the null hypotheses of no autocorrelation cannot be rejected.

<table>
<thead>
<tr>
<th>Lags</th>
<th>Test Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.2915</td>
<td>1.0000</td>
</tr>
<tr>
<td>12</td>
<td>0.5292</td>
<td>1.0000</td>
</tr>
<tr>
<td>18</td>
<td>1.5280</td>
<td>1.0000</td>
</tr>
<tr>
<td>24</td>
<td>1.6383</td>
<td>1.0000</td>
</tr>
<tr>
<td>36</td>
<td>1.9994</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

4.2.1.3 Diagnostics of EGARCH (2, 2)

Table 4.19 displays the results of the Lagrange Multiplier test for ARCH effects in the EGARCH (2, 2) model. The null hypothesis of no ARCH effects cannot be rejected since the P-values are greater than 5%. There are therefore no additional ARCH effects in the residual of EGARCH (2, 2).

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>0.2479</th>
<th>Probability</th>
<th>0.9954</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs*R-squared</td>
<td>3.0931</td>
<td>Probability</td>
<td>0.9949</td>
</tr>
</tbody>
</table>

Table 4.20 displays the result of the Ljung-Box test for the EGARCH (2,2) model. The probabilities values were greater than 5% level of significance for the selected lags. This affirms the null hypotheses that there is no autocorrelation in the residuals of the model.
Table 4.20: Ljung Box test for EGARCH (2, 2)

<table>
<thead>
<tr>
<th>Lags</th>
<th>Test Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.3926</td>
<td>0.9660</td>
</tr>
<tr>
<td>12</td>
<td>3.2794</td>
<td>0.9930</td>
</tr>
<tr>
<td>18</td>
<td>4.7920</td>
<td>0.9990</td>
</tr>
<tr>
<td>24</td>
<td>6.2208</td>
<td>1.0000</td>
</tr>
<tr>
<td>36</td>
<td>10.0470</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

4.2.1.4 Model Diagnostics for TGARCH (2, 3) Model.

Table 4.20 displays the results of the Lagrange Multiplier test for ARCH effects in the residuals of the TGARCH (2,3) model. The P-values were greater than 5% significance level affirming that the null hypothesis of no ARCH effects cannot be rejected.

Table 4.21: Heteroskedasticity Test: ARCH LM for TGARCH (2, 3)

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>0.0746</th>
<th>Probability</th>
<th>1.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs*R-squared</td>
<td>0.9389</td>
<td>Probability</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 4.22 displays the result of the Ljung-Box test for diagnosing the residuals of the TGARCH (2, 3) model. The probabilities values were greater than 5% level of significance for the selected lags indicating that the null hypotheses of no autocorrelation cannot be rejected.

Table 4.22: Ljung Box test for TGARCH (2, 3)

<table>
<thead>
<tr>
<th>Lags</th>
<th>Test Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.3566</td>
<td>0.9990</td>
</tr>
<tr>
<td>12</td>
<td>0.7294</td>
<td>1.0000</td>
</tr>
<tr>
<td>18</td>
<td>2.7181</td>
<td>1.0000</td>
</tr>
<tr>
<td>24</td>
<td>2.9387</td>
<td>1.0000</td>
</tr>
<tr>
<td>36</td>
<td>3.3890</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
4.2.2 Selecting the Best Fit Model

The diagnosing test for all fitted models revealed that all models fitted were adequate. A complete analysis of the models revealed that ARMA (1,1) –EGARCH (2,2) was the best model as shown in Table 4.23 since it has the least values of the AIC and SIC. An eleven month out of sample forecast was done. The forecasted values in Table 4.24 were then compared with the actual values of the returns using Chi-Square goodness of fit test. The result as shown in Table 4.25 indicates that there were no significant difference between the forecasted values and the actual values of the returns.

| Table 4.23: Selecting the most appropriate model |
|-----------------|-----------------|-----------------|
| Model           | AIC             | SIC             |
| ARCH(3)         | -5.9823         | -5.8900         |
| GARCH(2,3)      | -6.2098         | -6.0911         |
| EGARCH(2,2)     | -6.2816         | -6.1630         |
| TGARCH(2,3)     | -6.2034         | -6.0715         |

| Table 4.24: Forecast values of EGARCH (2, 2) with corresponding observed values. |
|-----------------|-----------------|-----------------|
| Year/Month      | Observed        | Forecasted      |
| 2013M01         | 0.0021           | 0.0013           |
| 2013M02         | 0.0012           | 0.0022           |
| 2013M03         | 0.0077           | 0.0019           |
| 2013M04         | 0.0061           | 0.0056           |
| 2013M05         | 0.0147           | 0.0057           |
| 2013M06         | 0.0031           | 0.0107           |
| 2013M07         | 0.0013           | 0.0054           |
| 2013M08         | 0.0033           | 0.0028           |
| 2013M09         | 0.0025           | 0.0033           |
| 2013M10         | 0.0342           | 0.0030           |
| 2013M11         | 0.0259           | 0.0214           |
4.3 Discussion of Results

The exchange rate return is positively skewed and the excess kurtosis of 11.86 shows that the returns were leptokurtic in nature. The coefficient of skewness indicates non-normality in the returns and this is supported by the Jarque Berra statistic of 1271.46 with an associated p-value of zero as shown in Table 4.1.

To provide better economic and statistical interpretation for the exchange rate data as indicated by Tsay (2005), the data was converted to returns by taking the log difference. A checked for stationarity was done using the Augmented Dicker Fuller test and Philips- Perrons test as shown in Tables 4.2 and 4.3 respectively, and both tests confirms that the data were stationary.

Several mean equations of ARMA (p, q) models were estimated and ARMA (1, 1) was selected as the best mean equation based on AIC and SIC values as shown in Table 4.4. A test for heteroscedasticity was performed on the residuals of the mean equation with the ARCH LM test and the Ljung Box test as shown in Tables 4.5 and 4.6 respectively. The order determination of the models was done by examining the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots of the squared returns and the squared residuals series.

The significance of $\alpha_i$ in ARCH(3)and $\alpha_i$ and $\beta_j$ in GARCH (2, 3) indicates that lagged conditional variance and squared disturbance has an impact on the

<table>
<thead>
<tr>
<th>Table 4.25: Chi-square goodness of fit test</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% Critical value</td>
</tr>
<tr>
<td>18.3070</td>
</tr>
</tbody>
</table>

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conditional variance, in other words this means that news about volatility from
the previous periods has an explanatory power on current volatility.

The results from the GARCH (2, 3) revealed that the volatility in the current
month’s exchange rate is explained by approximately 73% of the volatility in
the previous month’s exchange rate. Also, there was evidence of weak
stationarity in the volatility in the monthly exchange rate as the sum of the
ARCH parameters and the GARCH parameter was less than one, that is

\[ (0.380209 + 0.346804 - 0.473063 + 0.235592 + 0.496889 = 0.986431) \]

This implies that there is volatility persistence in the monthly exchange rate. The
persistence in the volatility in the monthly exchange rate means that the
impact of new shocks or information on the monthly exchange rate will last
for a longer period.

The EGARCH (2, 2) was covariance stationary since the sum of the GARCH
parameters are less than one. It also provides evidence to the effect that the
volatility in the current month’s exchange rate is perfectly explained by the
volatility in the previous month’s exchange rate.

There was the existence of asymmetric effects on the volatility of the monthly
exchange rate returns. Consequently positive shocks (news) and negative
shocks (news) would have different impacts on the volatility of the monthly
exchange rates. However, there was no evidence of leverage effects in the two
asymmetric models as the leverage parameter \( \gamma \) was positive in the EGARCH
(2, 2) and negative in the TGARCH (2, 3).
The absence of leverage effects indicates that the impact of a positive shock on the volatility of the monthly exchange rate exceeds that of a negative shock of equal magnitude. From the results, a positive shock would have an impact of 0.6119 on exchange rate in the EGARCH(2,2) and 0.8428 in the TGARCH(2,3) while a negative shock of the same magnitude would have an impact of -0.7495 in the EGARCH(2,2) and 0.2126 in the TGARCH(2,3) respectively. This is consistent with the findings of Giot (1999), Olewe (2009), Bala and Asemota (2013).

The selected model (EGARCH (2, 2) ) was diagnosed using the Univariate ARCH LM test and Ljung Box test and were found to be adequate. The evaluation of the forecasted results using the Chi-Square goodness of fit test revealed that there was no significance difference between the expected and the observed values as shown in Table 4.25.
CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter outlines brief summary, conclusion and recommendations of the study.

5.1 Summary

Modeling and forecasting the volatility of returns in exchange rate/stock markets has become a fertile field of empirical research in financial markets. This is because, volatility is considered as an important concept in many economic and financial applications like asset pricing, risk management and portfolio allocation. The volatility of the exchange rate returns have been modelled using Univariate Generalized Autoregressive Conditional Heteroscedastic (GARCH) models including both symmetric and asymmetric models that captures most common stylized facts in returns such as volatility clustering and leverage effect. These models were ARCH (3), GARCH (2, 3), EGARCH (2, 2) and TGARCH (2,3). The EGARCH (2, 2) model was selected as the best among candidate models.

Also, the presence of asymmetric effect was also observed in the exchange rate volatility. There was an absence of leverage effects as positive shock increased the volatility in the exchange rates more than a negative shock of equal magnitude. Finally the study developed a model which was used for forecasting and the forecasted values were evaluated using the Chi-Square goodness of fit test and it was found that there was no difference between the observed and the expected.
5.2 Conclusions

The conclusion of the study is stated below.

- The exchange rate volatility is persistent as the sum of the ARCH and the GARCH models was less than one.
- Positive shocks increased volatility than negative effects of equal magnitude.
- The selected model-EGARCH (2, 2) is adequate since there was no significant difference between the forecasted and observed values.

5.3 Recommendations

Based on the findings of the study, the following recommendations were made.

- This study focused on few GARCH models in modelling exchange rate returns.

However, after almost three decades, different extensions of the ARCH models have been proposed. These include multivariate ARCH, GARCH-in-mean (GARCH-M) models, Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model. It is therefore recommended that further expository studies on modelling these extensions be carried out.

- It is also recommended that the central bank should put in place long term measures to stabilise the cedi since a weakening cedi increases the uncertainty in the exchange market than a strengthening cedi as depicted by the EGARCH and the TGARCH models. In those two asymmetric models; leverage effects do not exist implying positive shocks increased volatility more than negative shocks of equal magnitude.
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Bordo M. D. (2003). Exchange Rate Regime Choice in Historical Perspective” *NBER working Paper, 9654*


`https://gupea.ub.gu.se/bitstream/2077/22593/1/
gupea_2077_22593_1`.


PUBLICATION


ACCEPTED PAPER FOR PUBLICATION

## APPENDIX

**Model diagnostic for EGARCH(2,2) Heteroskedasticity Test:**

**ARCH**

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Prob. F(12,249)</th>
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<td>Obs*R-squared</td>
<td>Prob. Chi-Square(12)</td>
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**Test Equation:**
Dependent Variable: WGT_RESID\(^2\)
Method: Least Squares
Date: 11/01/13  Time: 22:11
Sample (adjusted): 1991M03 2012M12
Included observations: 262 after adjustments

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<th>Variable</th>
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<th>Prob.</th>
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| R-squared | 0.011806 | Mean dependent var | 1.798290 |
| Adjusted R-squared | -0.035818 | S.D. dependent var | 7.533574 |
| S.E. of regression | 7.667306 | Akaike info criterion | 6.960153 |
| Sum squared resid | 14638.11 | Schwarz criterion | 7.137208 |
| Log likelihood | -898.7800 | Hannan-Quinn criter. | 7.031315 |
| F-statistic | 0.247895 | Durbin-Watson stat | 2.002504 |
| Prob(F-statistic) | 0.995429 | | |
Ljung Box test of serial correlation for EGARCH(2,2)

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