IJFM INTERNATIONAL JOURNAL OF FLUID MECHANICS
2(2) July-December 2010; pp. 123-132

On MHD Boundary Layer Flow of Chemically Reacting Fluid with Heat and Mass Transfer Past a Stretching Sheet

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Abstract: In this paper, the boundary layer equations for the flow of a chemically reacting fluid over a stretching sheet in the presence of a magnetic field and uniform heat source is solved numerically using the most efficient numerical shooting technique with fourth order Runge-Kutta algorithm. The basic equations governing the flow in the form of partial differential equations have been reduced to a set of non-linear ordinary differential equations by applying similarity transformations. The effects of various physical parameters such as Chandrasekhar number, Prandtl number, uniform heat source/sink parameter, Schmidt number, Eckert number and the chemical reaction parameter as well as the heat transfer coefficient are tabulated and plotted in figures. Our results reveal that both magnetic field and uniform heat source have significant impact in controlling the rate of heat and mass transfer in the boundary layer region.

Keywords: Stretching sheet; Uniform heat source; Magneto-hydrodynamics (MHD); first order chemical reaction; Variable surface temperature and concentration.

1. INTRODUCTION

Many chemical engineering processes like metallurgical process and polymer extrusion process involve cooling of a molten liquid being stretched into a cooling system. In such processes the fluid mechanical properties of the penultimate product would mainly depend on two things, one is the cooling liquid used and the other is the rate of stretching as reported in [1-5]. Some of the polymer fluids such as Polyethylene oxide, polyisobutylene solution in cetane, having better electromagnetic properties are recommended as their flow can be regulated by external magnetic fields. Furthermore, boundary layer flow over a stretching sheet also arises in many practical situations such as polymer extrusion process. To name some of them; drawing, annealing and tinning of copper wires, continuous stretching, rolling and manufacturing of plastic films and artificial fibres, materials manufactured by extrusion process and heat treated materials travelling between a feed roll and windup rolls or on conveyer belts, glass blowing, crystal growing, paper production, etc.

In his pioneering work, Sakiadis [1] investigated the boundary layer flow over a continuous solid surface moving with constant speed. Crane [2] extended the works of Sakiadis to that of an extensible surface and presented an analytical solution for the boundary layer flow of an incompressible liquid caused solely by the linear stretching of an elastic flat sheet which moves in its own plane with velocity proportional to the distance from the fixed point. Tsou *et al*. [3] reported both analytical and experimental results for the flow and heat transfer aspects arising in stretching sheet problem. Gupta and Gupta [4] have investigated heat and mass transfer in magneto-hydrodynamic (MHD) fluid flow over an isothermal stretching sheet with suction/blowing effects. Chen and Char [5] extended the works of Gupta and Gupta to that of non-isothermal stretching sheet. Vajravelu and Nayfeh [6] studied the flow and heat transfer introducing the temperature dependent heat source and sink. Makinde [7] presented computational results on the boundary layer flow with heat and mass transfer past a moving vertical porous plate. The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field was reported in Makinde and Ogulu [8].

The present investigation is an extension of the work in [7, 8] to include MHD heat and mass transfer with heat generation and first order homogeneous chemical reaction over a continuous stretching sheet. The equations of conservation of mass, momentum, energy and concentration that govern the flow are coupled and solved numerically. The effects of various flow controlling parameters on the overall flow structure are presented graphically and discussed quantitatively. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible, electrically conducting viscous fluid past a flat, impermeable stretching sheet with heat generation or absorption (see Figure 1 below).

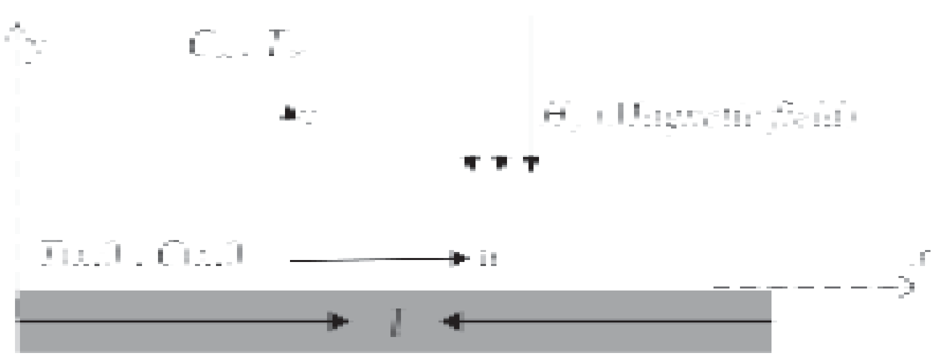

Figure 1: Schematic Diagram of the Problem

The *x*-axis is taken in the direction along which the stretching sheet is set to motion while the *y*-axis is taken perpendicular to it. The flow is generated by the action of two equal and opposite forces along the *x*-axis and the sheet is stretched in such a way that the velocity at any instant is proportional to the distance from the origin ($x = 0$). The flow field is exposed to the influence of an external transverse magnetic field of strength H_0 and the induced magnetic field is negligibly small and the cooling fluid has weak electrical conductivity so that any charge generated during the process gets accumulated on the extrusion. A chemical species diffused into the ambient fluid, initiates a first-order irreversible chemical reaction. With these assumptions the boundary layer equations governing the flow are given by;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0^2}{\rho}u\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_P}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma H_0^2}{\rho c_P}u^2+\frac{Q}{\rho c_p}(T-T_\infty)\,, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma(C-C_\infty)\,, \tag{4}$$

Equations (1), (2), (3) and (4) represents the continuity, momentum, energy and concentration equations respectively.

where u and v are the velocity components in x and y directions respectively, r is the density of the liquid, υ is the kinematic viscosity, H_0 is the strength of applied magnetic field, s is the electrical conductivity of the fluid, C is the species concentration, T is the fluid temperature, D is the mass diffusivity, γ chemical reaction coefficient, α is the fluid thermal diffusivity and c_p specific heat at constant pressure. Here we make a note that the case $Q > 0$ corresponds to internal heat generation and that $Q < 0$ corresponds to internal heat absorption. The boundary conditions for the flow problem under study are given by:

$$u=a\,x,\, v=0,\, T=T_w=T_\infty+A\left(\frac{x}{l}\right)^{\lambda_1},\, C=C_w(x)=C_\infty+B\left(\frac{x}{l}\right)^{\lambda_2} \text{ on } y=0,$$

$$u\to 0,\; T\to T_\infty,\; C\to C_\infty \text{ as } y\to\infty. \tag{5}$$

where A and B are constants, l is the characteristic length, T_w is the sheet surface temperature, C_w is the species concentration at the sheet surface, T_∞ is the temperature of the fluid far away form the sheet, C_∞ is the species concentration far away form the sheet, λ_1 and λ_2 are the variable wall temperature and concentration parameter. Equations (1) - (4) admit a self-similar solution of the form

$$\begin{gathered}u=axf'(\eta),\quad v=-\sqrt{a\upsilon}\,f(\eta),\quad \eta=\sqrt{\frac{a}{\upsilon}}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\beta=\frac{\gamma}{a},Sc=\frac{\upsilon}{D},\\ R=\frac{\sigma H_0^2}{a\rho},\quad \Pr=\frac{\upsilon}{\alpha},\quad N=\frac{Q}{\rho c_p a},Ec=\frac{a^2x^2}{c_p(T_w-T_\infty)},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.\end{gathered} \tag{6}$$

Substituting equation (6) into equations (1-4) we obtain the following nonlinear ordinary differential equations:

$$f''' - f'^2 + ff'' = Rf' \tag{7}$$

$$\theta'' + PrN\theta - Pr(\lambda_1\theta f' - f\theta') + PrEc(f''^2 + Rf'^2) = 0\,, \tag{8}$$

$$\phi'' - \beta Sc\phi - Sc(\lambda_2\phi f' - f\phi') = 0 \tag{9}$$

where prime denotes differentiation with respect to η and R, Pr, N, Sc, Ec, β represent the Chandrasekhar number, Prandtl number, uniform heat source/sink parameter, Schmidt number, Eckert number and the chemical reaction parameter respectively. The boundary conditions in equation (5) then become

$$f(h)=0,\ f'(\eta)=1,\ \theta(\eta)=1,\ \phi(\eta)=1,\ \text{at}\ \eta=0,\ f'(\eta)\to 0,\ \theta(\eta)\to 0,\ \phi(\eta)\to 0,\ \text{as}\ \eta\to\infty \tag{10}$$

Chang [9] and Rao [10] have obtained closed form solutions of equation (7) which clearly reveals that the solution is not unique. However, we choose an appropriate solution among them. Using the realistic solution, the velocity components are given by

$$f(\eta) = \frac{1-e^{-m\eta}}{m},\ u = axe^{-m\eta},\ v = -\sqrt{a\upsilon}\left(\frac{1-e^{-m\eta}}{m}\right), \tag{11}$$

where $m = \sqrt{1+R}$. The local skin-friction coefficient or the frictional drag coefficient is given by

$$C_f = \frac{\tau_w}{\rho xa\sqrt{a\upsilon}} = m \tag{12}$$

Other physical quantities of interest in this problem namely; the local Nusselt number (Nu) and the Sherwood number (Sh) can be easily computed. These quantities are defined in dimensionless terms as $Nu = -\theta'(0)$ and $Sh = -\phi'(0)$.

3. COMPUTATIONAL METHOD

The ordinary differential equations (7-9) with boundary conditions in equation (10) can be solved by using Newton–Raphson shooting method along with fourth-order Runge–Kutta integration algorithm. Let $\theta = x_1$, $\theta' = x_2$, $= \phi = x_3$, $\phi' = x_4$. Equations. (7-9) are then transformed into a system of first order differential equations as follows;

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -\Pr Nx_1 + \Pr(\lambda_1 x_1 e^{-m\eta} - \frac{x_2(1-e^{-m\eta})}{m}) - \Pr Ec(m^2 e^{-2m\eta} + \mathrm{Re}^{-2m\eta}) \\ x_3' &= x_4 \\ x_4' &= \beta Scx_3 + Sc(\lambda_2 x_3 e^{-m\eta} - \frac{x_4(1-e^{-m\eta})}{m}) \end{aligned} \quad ,\ (13)$$

subject to the following initial conditions,

$$x_1(0) = 1,\ x_2(0) = s_1,\ x_3(0) = 1,\ x_4(0) = s_2. \qquad (14)$$

The unspecified initial conditions; s_1 and s_2 are guessed systematically and equation (13) is then integrated numerically as an initial value problem to a given terminal point. The procedure is repeated until we get the results up to the desired degree of accuracy: namely 10^{-7}. A code is written in MAPLE package [11] and solutions are presented graphically. The value of η_∞ was found to each iteration loop by the assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters *R, Pr, N*,β, *Sc* and *Ec* is determined when the values of unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7}.

4. NUMERICAL RESULTS AND DISCUSSION

An MHD boundary layer flow and heat transfer of a chemically reacting fluid over a stretching sheet with power law variable surface temperature and concentration in presence of uniform heat source is investigated. Numerical computation have been carried out to study the effect of various physical parameters such as Chandrasekhar number *R*, Prandtl number *Pr*, Schmidt *Sc*, heat source/sink parameter *N* and the power law variable surface temperature and concentration parameters (λ_1 and λ_2) on the boundary layer. The value of *Pr* is taken to be 0.71 which corresponds to air and the values of *Sc* are chosen in such a way that they represent the diffusing chemical species of most common interest in air like H_2, H_2O, NH_3 and Propyl Benzene whose *Sc* values are 0.24, 0.6, 0.78 and 2.62 respectively. Results for wall temperature and concentration gradients are tabulated in Table 1 below. Analyzing this table, we infer that the wall heat flux increases with increasing λ_1 and decreases with increasing *R* while the rate of mass transfer at the sheet surface increases with increasing values of λ_2 and *R.*

Table 1
Computations Showing the Wall Heat Transfer Rate (N=0.1,Pr =0.71, Sc = 0.6)

R	Ec	β	λ_1	λ_2	$-\theta'(0)$	$-\phi'(0)$
0.0	1.0	1.0	1.0	1.0	0.44783	**1.10530**
0.1	1.0	1.0	1.0	1.0	0.45482	**1.11080**
0.5	1.0	1.0	1.0	1.0	0.47043	**1.13855**
0.1	1.5	1.0	1.0	1.0	0.30684	**1.11080**
0.1	2.0	1.0	1.0	1.0	0.15886	**1.11080**
0.1	1.0	1.5	1.0	1.0	0.45482	**1.24756**
0.1	1.0	2.0	1.0	1.0	0.45482	**1.36862**
0.1	1.0	1.0	2.0	1.0	0.77928	**1.11080**
0.1	1.0	1.0	3.0	1.0	1.04679	**1.11080**
0.1	1.0	1.0	1.0	2.0	0.45482	**1.29899**
0.1	**1.0**	**1.0**	**1.0**	**3.0**	**0.45482**	**1.47091**

The effect of transverse magnetic field on velocity and temperature profiles are depicted in Figures 2 and 3. From these plots it is observed that the transverse magnetic field contributes to the reduction in the velocity profile and thickening of thermal boundary layer. This is evident form the fact that applied transverse magnetic field produces a body force, to be precise the Lorentz force, which opposes the motion. The resistance offered to

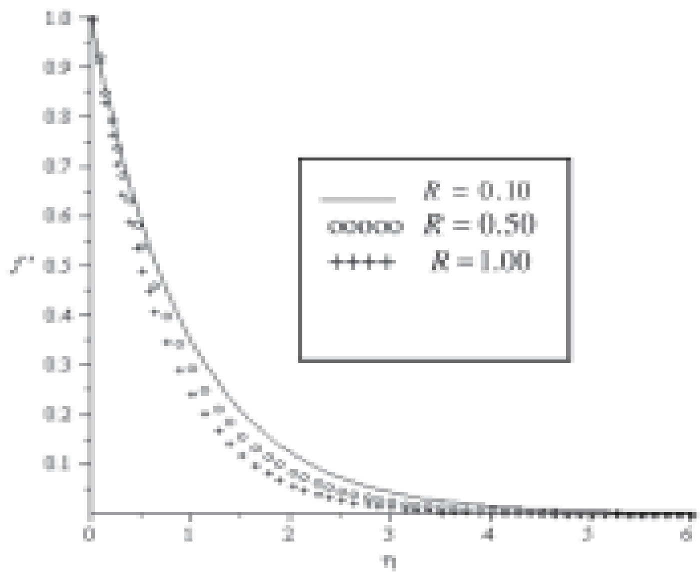

Figure 2: Variation of the Dimensionless Velocity Profiles with Increasing Magnetic Field Strength

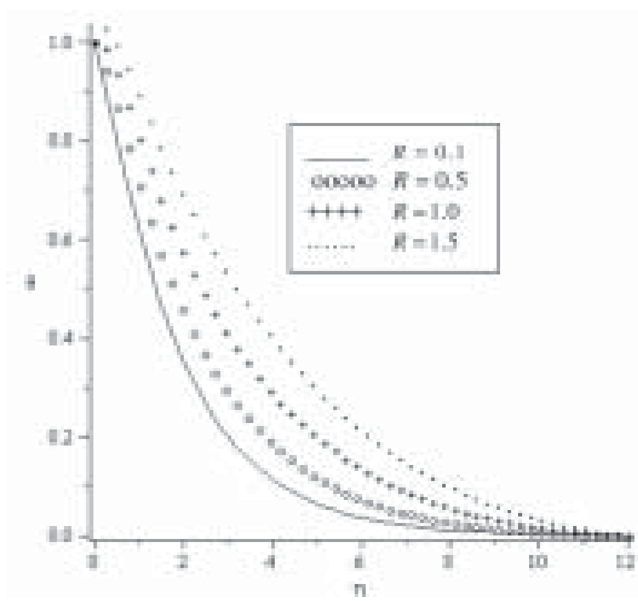

Figure 3: Variation of the Dimensionless Temperature Profiles with Increasing Magnetic Field strength when λ_1 = Ec = 1, Pr =0.71, N = 0.1

the flow is responsible in enhancing the temperature. Figure 4 illustrates the effect of Ec on the fluid temperature. From this plot it is evident that large values of Eckert number due to increasing viscous dissipation results in thickening of thermal boundary layer. Furthermore, it is noteworthy that the fluid temperature increases with an increase in surface temperature as shows in Figure 5. The chemical species concentration profiles are depicted

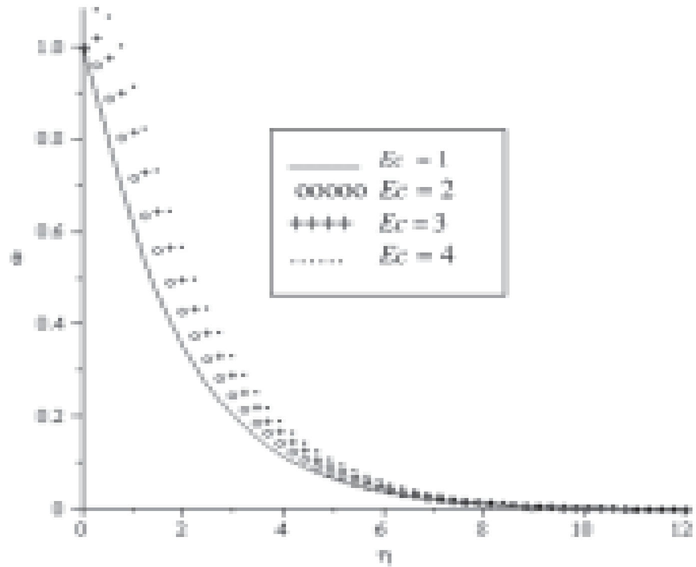

Figure 4: Variation of the Dimensionless Temperature Profiles with Increasing Eckert Number when λ_1 = 1, Pr = 0.71, N = 0.1, R = 0.1

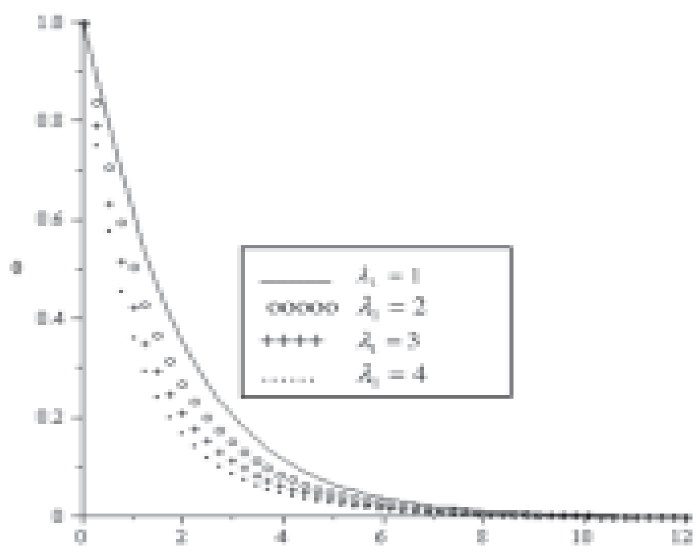

Figure 5: Variation of the Dimensionless Temperature Profiles with Increasing Wall Temperature Exponent λ_1 when N = 0.1, R = 0.1, Pr = 0.71, Ec = 1.

in Figures 6 to 8. We observed a general exponential decrease in the concentration profile. Furthermore, it is interesting to note that concentration boundary layer decreases with increasing values of the Schmidt number (*Sc*), heat absorption parameter (*N*) and reaction rate parameter (β).

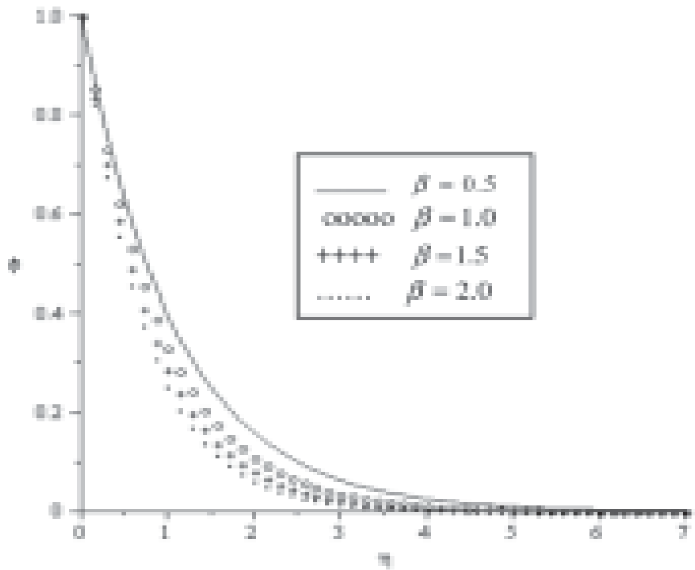

Figure 6: Variation of the Dimensionless Concentration Profiles with Increasing Reaction Parameter when Sc = 0.6, λ_2 = 1, *R* = 0.1

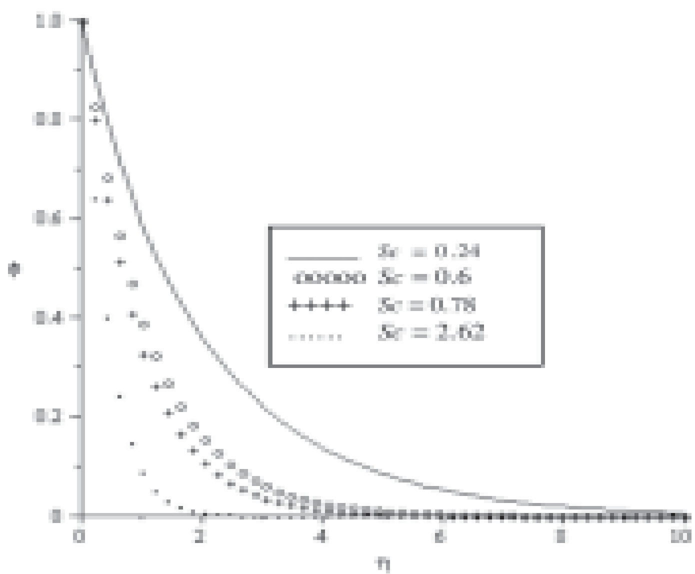

Figure 7: Variation of the Dimensionless Concentration Profiles with Increasing Schmidt Number when β = 0.5, λ_2, = 1, *R* = 0.1

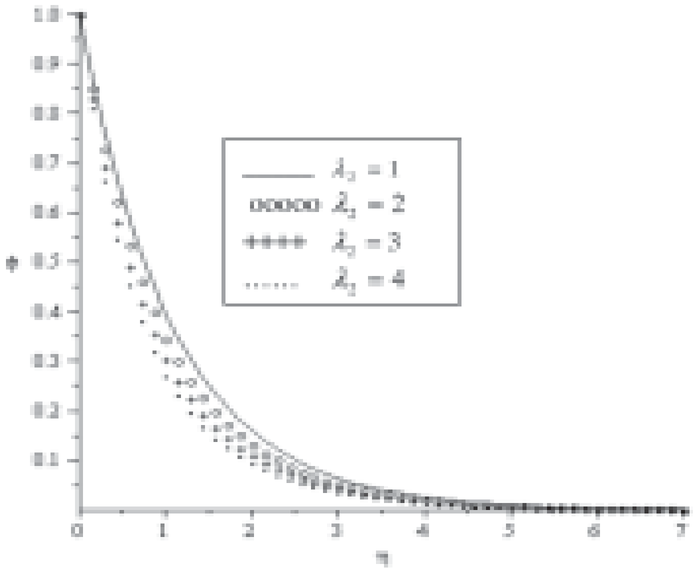

Figure 8: Variation of the Dimensionless Concentration Profiles with Increasing Wall Concentration Exponent Parameter when Sc = 0.6, β = 0.5, R = 0.1

5. CONCLUSIONS

In this paper an analysis has been carried out to study the MHD boundary layer flow of chemically reacting fluid with heat and mass transfer past a stretching sheet. Numerical solutions are obtained for temperature and concentration boundary layer equations. The effect of several parameters controlling the velocity and temperature profiles are shown graphically and discussed. The study revealed that the cooling rate of a stretching sheet in an electrically conducting fluid, subject to a magnetic filed and chemical reaction can be controlled and a final product with desired characteristics can be achieved.

Acknowledgements

The authors gratefully acknowledge the financial support from the National Council for Tertiary Education under the TALIF funded project.

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