Assessing the Performance of ELL and EBP Models in Estimating District Level Poverty Indices in The Presence of Outliers in the Northern Region of Ghana.

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Abstract

The objective of this study was to assess the performance of the Elbers, Lanjouw and Lanjouw (ELL) and the Empirical Best Predictor (EBP) Small Area Estimation (SAE) models in estimating the Foster-Greer-Thobecke (FGT) poverty indices for the Northern Region of Ghana in the presence of outliers. The sixth round of the Ghana Living Standard Survey (GLSS6) data and the 2010 Population and Housing Census (2010 PHC) data were used for the study. The performances of these SAE models under normality and non-normality assumptions were evaluated by computing and comparing their Absolute Relative Biases and Relative Root Mean Squared Errors values under both conditions by conducting a model-based simulation study in the absence and presence of outlier contaminated data. Results from the study showed that no matter the level of contamination, the EBP model is a better performer and more stable than the ELL model in estimating all the FGT poverty indicators for the Region. Therefore, it was recommended that in future poverty estimating exercises, the EBP model be used to estimate the FGT poverty indicators for the Northern Region of Ghana.

Keywords: *Foster-Greer-Thobecke, Ghana Living Standard Survey, Households, Poverty Line, Population and Housing Census, Small Area Estimation*

Introduction

In Ghana, geographically, there have been large differences in poverty and other social indicators. However, poverty profiles based on household surveys tend to be limited to broad areas because survey sample sizes are too small to allow analysts to construct reliable estimates of poverty at local levels. Meanwhile, policymakers often request for finely disaggregated information at the neighbourhood, town or village level to implement anti-poverty programmes. The demand for reliable small area information has led to the development of a range of estimation techniques referred to as “Small Area Estimation” methods. Small Area Estimation (SAE) is a statistical technique that combines survey and census data to estimate welfare indicators for disaggregated geographical units such as districts, municipalities, metropolitans or even communities (Rao, 2003). There are two types of Small Area Estimation models: unit and area-level models. The unit-level models use household-level population census data on household units. Examples of these models include the Battesse et al. (1988), ELL and EBP models. On the other hand, the area-level small area estimation models use community-level averages instead of household-level data. An example of an area-level model is the Fay-Herriot (1979) model. Both types of models have been used by researchers at the World Bank and other research centers. Generally, small area estimation techniques allow the production of more reliable small area estimates without adding much burden to the limited resources of most statistical agencies (Marissa, 2010). According to Elbers et al. (2003), estimation of poverty statistics is dominated by the ELL model, a model proposed by the World Bank for all developing countries to use for their poverty estimation exercises. However, in recent times, researchers in SAE have extensively studied the World Bank method and have proposed alternative small area estimation models for estimating poverty statistics at the district levels, such as the M-Quantile approach by Chambers and Tzavidis (2006), the Empirical Best Prediction (EBP) approach by Molina and Rao (2010) and Tzavidis et al. (2010). Research is still ongoing to identify the SAE techniques that are easy and efficient to implement, with the estimation of small area poverty indicators being an outstanding problem. Though, some studies such as Molina and Rao (2010), Tzavidis et al. (2013), Molina et al. (2014), and Souza et al. (2015), have been done to identify the most efficient SAE models in estimating poverty statistics by comparing some SAE methods, clear advice on how to select an appropriate SAE method on a study such as this, is not yet available (Das, 2016). Moreso, in most of these studies, the residual terms are assumed to be normally distributed, with a few studies comparing the performance of the models when the error terms are not normally distributed (see Molina & Rao, 2010; Sinha & Rao, 2009). This implies a knowledge gap that may affect decision-making, especially to policymakers and other stakeholders who try to reduce poverty by relying on models/methods that do not reflect the real target populations/groups on the ground. In this paper, the researchers sought to address this gap by carrying out a detailed numerical assessment of the performance of the ELL and EBP models in estimating the poverty indices for the Northern Region of Ghana in the presence of outlier contaminated data. The rest of the paper is organized as follows. Section 2 explores some relevant literature. Theories of ELL and EBP models, estimation procedures, as well as sources of data and criteria for selection of variables are discussed in section 3. Section 4 contains results and discussions of the ELL and EBP models in the absence and presence of outliers, and section 5 covers conclusion and recommendations.

Literature review

A few studies have compared some SAE models simultaneously. Betti et al. (2007) undertook a simulation experiment using two real datasets from Albania to compare the performance of the ELL and MQ models. The study indicated that the ELL model provided more biased estimates than the MQ model. However, there were problems in the MQ estimates when the small areas are not covered in the survey. Tzavidis et al. (2013) also used the Tuscany Poverty data to conduct an empirical study and compared the performance of ELL and MQ models and also concluded that the MQ model was more efficient than the ELL model supporting the findings of Betti et al. (2007).

Molina and Rao (2010) compared a number of SAE models through a simulation study. They considered only area-specific random effects in the population model and assumed very poor predictive power of the regression model. They concluded that the ELL model could provide worse results than the direct estimator in such a situation. They however, did not include the square poverty gap, something this study sought to add.

Souza et al. (2015) also compared the ELL and the EBP models using data from the 2010 Census of Minas Gerais state of Brazil where information on household per-capita income is collected. They conducted a simulation study by selecting 400 samples of the 2008-2009 Consumer Expenditure Survey (CES) from the known population and then applied the ELL and EBP models to estimate only the incidence of poverty and poverty gap at the municipal levels. They concluded that the ELL performed better than the EBP estimator in terms of relative bias (RB) and relative root mean squared errors (RRMSE).

Chambers and Tzavidis (2006) proposed an M-quantile approach that is robust to the presence of outliers. Chambers and Chandra (2013) developed a procedure to fit a linear mixed model using a random effect block bootstrap, and Chambers et al. (2014) proposed a bias-corrected version of the M-quantile estimators and provided analytical MSE estimators for these robust methods. Weide and Elbers (2013) studied normal mixture models on the area effects but assumed that the errors were normal. Their results showed that the normality-based EBP method is robust provided the errors remain normal. Diallo and Rao (2014) derived EBP estimators of small area poverty indicators under the skewed normal distribution model. Results from their study were in line with that of Van der Weide and Elbers (2013). Salvati et al. (2011) compared the finite-sample performance of the small area estimator based on penalized splines M-quantile regression with the linear M-quantile small area estimator of Chambers and Tzavidis (2006) and with Empirical Best Linear Unbiased Predictor (EBLUP) estimators based on Battese et al. (1988) model and on nonparametric regression model of Opsomer et al. (2008). It is seen from the review that literature for the assessment of SAE methods are scanty, and most studies were done globally with the assumption that the errors and data are normally distributed with a few studies such as Sinha and Rao (2009) and Molina and Rao (2010) evaluating the performance of the models when the errors are not normally distributed.

Methodology

This study was conducted in the then Northern Region of Ghana, which is now made up of 3 regions, namely the Northern, North-East and the Savannah regions. The regions are located within the savannah belt of Ghana. The area has a total population of 2,479,461 (2010 PHC). The region has the largest population of the poor in the country, constituting 26% of the country's total poverty (GSS, 2014).

sources of data

Data for this study was sourced mainly from the Ghana population and housing census 2010 and the sixth round of Ghana living standard survey (GLSS6). The census provided information on the population of interest and the GLSS6 provided information on consumption of the households.

Criteria for selecting Variable

The Small Area methodology involves selecting a set of common and comparable variables in the survey and census data. These selected variables are then used to estimate a per capita consumption regression model based on the survey data. The set of parameter estimates obtained from the consumption model is applied to similar variables in the census to obtain the predicted per capita consumption for each census household in the district. Only variables whose census mean fell within 95% confidence interval of the survey means were included in the model. However, because the survey and census years were not the same, the means of some correlates were not statistically equivalent and so were not selected. The unit of analysis is the household, focusing on the head of the household.

Theoretical framework

Poverty indicators

There are several poverty indicators, however, for this study, the FGT family of poverty indicators are adopted (Foster et al., 1984) as the FGT measures can be disaggregated for population sub-groups and the contribution of each sub-group to poverty can be calculated. These indicators include the Head Count Ratio (HCR), Poverty Gap, and Squared Poverty Gap. The FGT poverty measure is represented mathematically as:

$$P_\alpha = \frac{1}{N}\sum_1^P \left(1 - \frac{x_i}{l}\right)^\alpha I(x_i < l), \quad (\alpha \geq 0), \quad \alpha = 0, 1, 2. \qquad (1)$$

Where α is the amount of the sensitivity of the index, l is the poverty line, x_i is the per-capita consumption for the ith person's household and $I(x_i < l)$ is an indicator function that takes value 1 when household i is poor and 0 otherwise.

If $\alpha = 0$, we have the HCR, when $\alpha = 1$, we have the poverty gap and when $\alpha = 2$, we have the squared poverty gap or poverty severity.

The Elbers, Lanjouw and Lanjouw (ELL) Model

The ELL (World Bank) method assumes a unit-level linear mixed model for a log-transformation of the variable measuring welfare of individuals with random effects for the sampling clusters or primary sampling units. If we further assume that the sampling clusters are the areas, then the model becomes the nested error regression model of Battese *et al.* (1988) for the log-transformation of the welfare variables, that is, $Y_{ah} = \log(E_{ah})$ is assumed to be linearly related with a p-vector of auxiliary variables X_{ah} which may include unit-specific and area-specific covariates and includes random area effects u_d as follows:

$$Y_{ah} = X_{ah}^T\beta + u_a + e_{ah} \qquad (2)$$

$$h = 1,2 \ldots. N, \qquad a = 1, 2, \ldots.. A,$$

Where β is a p-vector of regression coefficients, $u_a \sim N(0, \sigma_u^2)$, $e_{ah} \sim N(0, \sigma_e^2)$ u_a and e_{ah} are independent and are assumed to follow a normal distribution with a constant variance component.

Poverty indicators are functions of the distribution of Y_{ah} not $X_{ah}^T\beta$ and so ELL uses a bootstrap procedure to regenerate the conditional distribution of Y_{ah} by adding simulated values of cluster-effect u_a^* and household-specific e_{ah}^* to each estimated fitted value $\left(X_{ah}^T\hat{\beta}\right)$. The ELL method provided unbiased estimates of poverty measures with their standard errors by means of a parametric procedure as outlined in the steps below.

1) We fitted model (2) to the sample data (Y_s, X_s) to obtain $\hat{\beta}$, $\widehat{V}\left(\hat{\beta}\right)$, $\hat{\sigma}_u^2$ *and* $\hat{\sigma}_e^2$ by using the Restricted Maximun Likelihood (REML) method.
2) We then generated the values of the regression parameters from the corresponding parametric distribution as $\beta^* \sim N(\hat{\beta}, \hat{V}(\hat{\beta}))$.
3) The cluster-specific and household-specific random errors were generated independently and identically from the corresponding parametric distribution as $u_a^* \sim N(0, \hat{\sigma}_u^2)$ and $e_{ah}^* \sim N(0, \hat{\sigma}_e^2)$ respectively for $h = 1, 2, \ldots . D$. The random errors were also generated from the corresponding empirical distribution to relax the parametric assumptions of random errors.
4) Generated B $(say\ B = 1000)$ independent and identically distributed bootstrap population income values $\{Y_{ah}^*,\ h = 1, 2, . . . A\}$ via the bootstrap super-population model

$Y_{ah}^{*} = X_{ah}^{T}\beta^{*} + u_{a}^{*} + e_{ah}^{*}$, $h = 1,\ 2, \ldots N,\qquad a = 1,\ 2,.$ (3)

The parameter of interest for a particular area was calculated by aggregating the generated income values belonging to the small area. The FGT poverty measures $F_{\alpha h}^{*(a)}$ were then calculated from each bootstrap population and then the ELL estimates with their mean squared errors were calculated as:

$F_{\alpha h}^{ELL} = A^{-1}\sum_{1}^{A} F_{\alpha h}^{*(a)}$, where $F_{\alpha h}^{*(a)} = \frac{1}{A}\sum_{1}^{A}\left(1 - \frac{x_i}{l}\right)^{\alpha} I(x_i < z)$ and

$$MSE\left(\hat{F}_{\alpha h}^{ELL}\right) = A^{-1}\sum_{1}^{A}\left\{F_{\alpha h}^{*(a)} - \hat{F}_{\alpha h}^{ELL}\right\}^{2} \qquad (4)$$

Where l is the poverty line and x_i is the per-capita consumption of household h.

The Empirical Best Predictor (EBP) Model

The Empirical Bayes model is a unit-level mixed model based on the nested error linear regression model. It is known in SAE literature as the nested error linear regression model (Battese et al., 1988).

The model has a random area-specific effect and a unit-level error term that assumes that the population variable, y_{ik} follow the nested error regression model (5) with the random effects,

u_i and errors, ε_{ik} , being normally distributed.

$$y_{ik} = X_{ik}^{T}\beta + u_i + \varepsilon_{ik} \qquad (5)$$

$$i = 1, 2, \ldots, D \qquad k = 1, 2, \ldots, N \qquad \mu_i \sim N(0, \sigma_{\mu}^{2})$$

$$\varepsilon_{ik} \sim N(0, \sigma_{\varepsilon}^{2})$$

Where u_i and ε_{ik} are area and household-specific random errors.

This model, like the ELL model, also works by using two data sources, namely, a sample survey dataset, which is used to fit the nested error linear regression model, and a population census dataset which is used in predicting (under the model) the synthetic values of the outcome for the entire population. Both datasets must share identically defined variables, but the target variable (consumption or income) is only available in the sample survey dataset. In the case of EBP model, the FGT estimator is divided into sampled and out-of-sampled parts as

$F_{\alpha i} = N_i^{-1}\left[\sum_{k\in s_d} F_{\alpha ik} + \sum_{k\in r_d} F_{\alpha ik}\right]$ for $\alpha = 0, 1, 2$ and the out-of-sampled part is then estimated from a Monte Carlo calculation. The estimator that will minimize the squared error loss called the best estimator of $F_{\alpha i}$ is then given by

$\hat{F}_{\alpha i} = E_{y_{ri}}[F_{\alpha i} I y_{si}] = \int F_{\alpha i} f(y_{ri} I y_{si})\, y_{ri}$, where $f(y_{ri} I y_{si})$ is the joint density of

y_{ri} (vector of out-of-sample *y* in area *i*) given

y_{si} (vector of sample *y* in area *i*). The basic procedure of the EBP method to obtain the estimate of $F_{\alpha i}$ can be explained briefly as:

1) According to Rao (2003) the nested error regression model (5) is fitted to the survey data (Y_s, X_s) by the Restricted Maximum Likelihood (REML) method to obtain the estimated regression parameters , $\hat{\beta}, \hat{\sigma}_\mu^2, and\ \hat{\sigma}_\varepsilon^2.$
2) L Independent realization of $y_{ri}\left\{y_{ri}^{(l)},\ l = 1, 2, \dots, L\right\}$ were then generated from the conditional distribution $f(y_{ri} / y_{si})$ which is assumed to follow a normal distribution with mean $\mu_{irIs} = X_{ri}\beta + \sigma_\mu^2 1_{N_i - n_i}\mathbf{1}^T V^{-1}(y_{si} - y_{si}\beta)$ and variance-covariance matrix $V_{irIs} = \sigma_\mu^2(1-\gamma_i)1_{N_i-n_i}1_{N_i-n_i}^T + \sigma_\varepsilon^2 I_{N_i-n_i}$, where $\gamma_i^{-1} = \sigma_\mu^{-2}\left(\sigma_\mu^2 + n_i^{-1}\sigma_\varepsilon^2\right)$ $X_{si}\ and\ X_{ri}$ are matrices of sampled and out-of-sample values of explanatory variables belonging to the i^{th} area. The Monte Carlo (MC) approximation method is simplified by generating observations from $y_{ik}^{(l)} = X_{ik}^T\hat{\beta} + V_i^{*(l)} + \varepsilon_{ik}^{*(l)}$ where $ik \in r_i$ where $V_i^* \sim N(0, \hat{\sigma}_\mu^2(1-\hat{\gamma}_i))$ is independent of $\varepsilon_{ik}^* \sim N(0, \hat{\sigma}_\varepsilon^2)$
3) $F_{\alpha i}^{(l)}$ is then calculated using the vector $y_i^{(l)} = \left\{y_{si}^T,\ y_{ri}^{T(l)}\right\}$.
4) This is then averaged over *L* replicates to obtain the EBP estimates as

$$\hat{F}_{\alpha i}^{EBP} = L^{-1}\sum_{l=1}^{L} F_{\alpha i}^{(l)}, \qquad (6)$$

Quality measures of SAE Methods

Though there are several quality measures such as AIC, BIC, Likelihood ratio test, RRMSE and ARB in the SAE literature. In order to compare this study with other works, we have focused only on the ARB and RRMSE of each indicator as they are the quality measures used by most researchers in SAE studies.

Absolute Relative Bias

Suppose $\hat{y}$ is any value of the estimators and y is the corresponding known value.

Then for each area i, the bias is defined as

$$Bias(\hat{y}_i) = \frac{1}{N}\sum_{n=1}^{N}(\hat{y}^n{}_i - y_i^n)$$

$$RB(\hat{y}_i) = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{\hat{y}_i^n - y_i^n}{y_i^n}\right) \qquad (7)$$

$$ARB = |RB|$$

Relative Root Mean Squared Error

The Root Mean Squared Error (RMSE) for each area is defined as

$$RMSE(\hat{y}) = \left[\frac{1}{N}\sum_{n=1}^{N}(\hat{y}_i^n - y_i^n)^2\right]^{1/2}$$

$$RRMSE(\hat{y}) = \frac{RMSE\,(\hat{y})}{\sum_1^N \hat{y}_i^n} \qquad (8)$$

Simulation Study

In order to evaluate the performance of EBP and ELL estimators in the presence of outliers, a model-based simulation study was conducted to determine which of these models is resistant to the presence of outliers. The procedure applied is the model-based simulation set-up as in Molina & Rao (2010) in which the data are generated at the unit-level following the nested error regression model of Battesse et al. (1988) (see Molina & Rao, 2010; for details)

$$Y_{ah} = X_{ah}^T\beta + u_a + e_{ah}, \qquad (9)$$

$$h = 1, 2, \dots\dots.. N_a \ , a = 1, 2, \dots\dots.. A$$

A poverty line of GHS 1,314.00, was used for this study as this is the official definition used in Ghana (GSS, 2013).

Contamination of models with Outliers

In order to create artificial outliers, the errors, e_{ah}, were generated from a mixture of two normal distributions with different variances. Specifically, the model errors are generated

$$e_{ah} \sim (1-\varepsilon)N(0,\sigma_e^2) + \varepsilon N(0,\delta\sigma_e^2) \qquad (10)$$

where $N(0,\sigma_e^2)$ is the distribution of the non-contaminated part of the data,

$\varepsilon N(0,\delta\sigma_e^2)$ is the distribution of the contamination, and ε is the contaminated level and is generated as $\varepsilon \sim Bern(p)$ as applied in Molina & Rao (2010).

Simulations were implemented in the statistical software environment R (R development core team 2013) using the package lme4 (Batesse et al., 2014), which fits Gaussian linear and nonlinear mixed-effects models, and the package sae (Molina & Marhuenda 2015), which has functions for small area estimation.

Results and Discussion

Empirical Results from the ELL Model

Table 1 is the results from the OLS of the ELL model. From the results, it can be seen that household size, marital status, education, roof, floor, and phone are all significant (P-Value <0.01) covariates of consumption. However, it is seen that the employment status of the head of the household is not significant (p-value > 0.10), which rather deviates from literature. This could be so because most of the districts in the Northern Region are rural where white colour jobs are the only ones considered as forms of employment, so if one is working on his/her farm, he/she does not consider that as a form of employment hence its insignificance. The overall performance of the model indicates that it has an MSE of 0.285, RMSE of 0.5341, R-sq of 0.528, R-sq(adj.) of 0.517, and an F-ratio of 46.206. R-sq(adj.) of 0.517 means that the model is able to explain 51.70% of the variability of the response data around its mean. An F-ratio of 46.206 means we have to reject the null hypothesis that all the regression coefficients are zero in favour of the alternative hypothesis that at least one of the coefficients is not zero.

Table 1: Parameter Estimates of the ELL model

variables	Coefficient.	Std. Err.	t	P>t
hhsize	0.086	0.007	12.720	0.000
sex	-0.204	0.078	-2.600	0.586
age	0.001	0.001	0.870	0.384
marst	0.462	0.070	6.630	0.000
edu	0.476	0.073	6.725	0.000
roof	0.332	0.044	7.530	0.000
walls	0.378	0.053	7.160	0.000
floor	0.447	0.160	2.790	0.005
phone	0.769	0.375	2.050	0.040
empst	0.109	0.102	1.070	0.284
_cons	6.074	0.099	61.690	0.000
MSE = 0.285	RMSE = 0.5341	R-sq = 0.528	R-sq(adj) = 0.517	F-ratio = 46.206

Source: Authors' own estimation

Checking for Model Assumptions

Residual Analysis of the ELL model

The cluster-level had a Sig-Eta value of 0.00870, var of sigma-Eta-square of 0.00003 and Ratio of variance-of-Eta over MSE of 0.03051. The ratio of variance-of-Eta over mean squared error tells how much of total variation (measured by MSE) can be interpreted by the cluster effect. A negative value means that the residual plot is homogeneous across all clusters and for that matter there may not be any cluster effect. However, in this our case, there is a ratio of 0.03051 meaning that the model can interpret 3.051% of total variation due to the cluster effect. This means that the residuals are not homogeneous across all clusters. The Anderson-Darling test was also run to establish the normality distribution of the residuals. The results of the test showed an A-Squared value of 2.560 which is an indication that at ⊠ levels greater than 0.005, there is evidence that the data do not follow a normal distribution. However, in real-life applications such as this study, an exact fit to a distribution is barely met, and so we can therefore say that the residuals are normally distributed with a mean 0.000 and standard deviation 0.530.

District level Estimates of the FGT measures of poverty under normality assumption

Table 2 displays the district level estimates of the ELL model's FGT measures of poverty and their MSEs. With a regional HCR of 44.2%, it is seen from Table 2 that ten (10) of the districts have their HCR values higher than 44.2%. Of those with HCRs greater than 44.2%, six (6) of the districts are in the Gonjaland, three (3) from Mamprugu and one (1) from Dagbon. It is further observed that Mamprugu/Mogduri has the highest HCR of 87.3%, followed by Bole District (74.5%) and then East Gonja District (72.4%). The districts with the least HCRs are Tatale/Sanguli (3.82%), Zabzugu (10.41%) and Nanumba North (16.74%). For the Poverty gap (FGT1), it is observed that nine (9) of the districts had their FGT (1) values higher than the Regional average of 15.5%. Tatale/Sanguli, Zabzugu and Nanumba North Districts had the lowest poverty gap values of 0.72%, 2.20% and 4.47%, respectively while Maprugu/Mogduri (39.7%), Bole (36.08%) and East Gonja (33.65%) Districts had the highest values. In the case of the squared poverty gap, eleven (11) districts have their FGT(2) values higher than the regional value of 7.4%.

This finding is in line with the finding of Ghana Statistical Service (GSS, 2013).

Table 2: District level Estimates of FGT measures and their MSEs by the ELL model

District	Sample size	FGT(0)	MSE(0)	FGT(1)	MSE(1)	FGT(2)	MSE(2)
Bole	30	0.7450	0.0310	0.3610	0.0179	0.2150	0.0155
Sawla/Tuna/Kalba	72	0.5150	0.0322	0.1750	0.0152	0.0810	0.0091
West Gonja	30	0.3910	0.0342	0.1280	0.0138	0.0600	0.0077
Gonja Central	75	0.4930	0.0295	0.1760	0.0144	0.0840	0.0088
East Gonja	85	0.7240	0.0245	0.3370	0.0179	0.1930	0.0136
Kpandai	82	0.6490	0.0268	0.2600	0.0147	0.1340	0.0095
Nanumba South	45	0.2160	0.0229	0.0580	0.0074	0.0230	0.0036
Nanumba North	83	0.1670	0.0166	0.0450	0.0056	0.0180	0.0028
Zabzugu	44	0.1040	0.0175	0.0230	0.0050	0.0080	0.0023
Yendi	61	0.4100	0.0189	0.1460	0.0095	0.0720	0.0062
Tamale Metro	156	0.2390	0.0272	0.0720	0.0107	0.0310	0.0056
Tolon	60	0.2990	0.0292	0.0910	0.0114	0.0400	0.0065
Savelugu/Nanton	100	0.3650	0.0254	0.1150	0.0097	0.0520	0.0053
Karaga	45	0.3170	0.0400	0.0860	0.0136	0.0350	0.0065
Gusheigu	89	0.3140	0.0218	0.1020	0.0098	0.0460	0.0055
Saboba	45	0.3050	0.0288	0.0870	0.0109	0.0360	0.0056
Chereponi	30	0.3050	0.0302	0.0940	0.0120	0.0420	0.0064
Bunkpurugu/Yunyoo	89	0.5150	0.0252	0.1760	0.0118	0.0820	0.0072
Mamprusi East	75	0.4260	0.0208	0.1460	0.0110	0.0690	0.0068
Mamprusi West	88	0.5300	0.0212	0.1800	0.0109	0.0840	0.0069
North Gonja	15	0.4430	0.0406	0.1470	0.0170	0.0670	0.0101
Kumbungu	59	0.3160	0.0444	0.0970	0.0175	0.0430	0.0093
Sagnarigu	97	0.2240	0.0187	0.0640	0.0073	0.0270	0.0038
Mion	85	0.5220	0.0256	0.1880	0.0133	0.0910	0.0085
Tatale/Sanguli	31	0.0380	0.0112	0.0070	0.0028	0.0020	0.0011
Mamprugu Mogduri	31	0.8730	0.0262	0.3970	0.0257	0.2150	0.0202

Source: Authors' own estimation

Parameter Estimates of the EBP model under normality assumption

The results of running the model is shown in Table 3. With the exception of sex, age and employment status (empst), all other covariates are significant at less than 1%. The standard deviation of the random area effect is 0.125, and that of the error is 0.275. The log likelihood ratio test statistic compared with a simple linear regression model with no area random effect is -268.80. This means that there is evidence of the presence of extra structure in the data which the linear function of the variables has not explained. The model also had AIC value of 561.60 and BIC value of 626.81.

Table 3: Table showing parameter estimates for EBP Model

variables	Estimate	standard error	Degree of freedom	t-value	p-value
intercept	8.358	0.074	1667	112.455	0.000
hhsize	0.166	0.020	1667	8.250	0.000
age	0.003	0.008	1667	0.400	0.691
edu	0.158	0.024	1667	5.800	0.000
empst	-0.030	0.067	1667	-0.447	0.654
floor	0.389	0.094	1667	4.108	0.000
marst	0.185	0.022	1667	8.374	0.000
phone	0.436	0.124	1667	3.502	0.001
roof	0.121	0.016	1667	7.384	0.000
sex	-0.032	0.026	1667	-1.245	0.213
walls	0.057	0.018	1667	3.085	0.002
AIC = 561.60	BIC = 626.81				

Source: Authors' own estimation

Checking departures from model assumptions

A check for departures from model assumptions showed that both the household-level errors and the random-effects have not deviated so much from the normality assumption as confirmed by the skewness [for the error (0.163) and random effect (0.152)], kurtosis [3.023 for the Error and 2.912 for the random effect] and Sharpiro-Wilks normality test [Shapiro of 0.856 and 0.345 for W and P respectively for the error and 0.978 and 0.196 for W and P respectively for the Random Effect. The Cook's distance measure was also used to determine the presence of outliers in the data set and it was observed that only three of the dataset were identified as outliers. However, they were found not to be influential because all their distances were less than one and so did not warrant deletion or expulsion from the analysis.

District level Estimates of FGT measures by the EBP model

The district-level estimates of the FGT poverty measures were calculated and presented in Table 4. From Table 4, thirteen (13) of the districts have their incidence of poverty (HCR) being more than the Regional level of 44.2%. This finding is in line with the findings of GLSS 6 (2015). A headcount ratio (HCR) of 44.2% means that 44.2% of the sampled households have their per capita consumption being less than the poverty line of GHS1,314.00. The results show that the highest incidence of poverty (HCR) is observed in the North Gonja District (84.8%), closely followed by Bole District (80.1%) whilst, Nanumba North District (19.9%), Sagnarigu District (15.2%) and Tamale Metropolitan Assembly (9.9%) have the lowest incidence of poverty in the region. By traditional settings, it was observed that out of the thirteen districts whose HCRs were above the 44.2%, seven (7) of them are in the Gonjaland, four (4) in Dagbon and two in Mamprugu with none in Nanung. In terms of the poverty gap, it was observed that nine (9) districts had their rates higher than the regional average of 15.5%. Tatale/Sanguli District (0.72%) has the least rate, followed by Zabzugu District (2.3%) with Mamprugu/Mogduri (39.7%), Bole District (36.1%) and East Gonja District (33.7%) had the highest poverty gaps. It was again observed from Table 4 that poverty severity is highest in the Bole and Mamprugu/Mogduri Districts of 21.5% each. In comparison, the least severity is observed in Tatale/Sanguli (0.22%) and Zabzugu (0.76%) Districts.

Table 4: District level Estimates of FGT measures and their MSEs by the EBP model

District	sample size	FGT(0)	MSE(0)	FGT(1)	MSE(1)	FGT(2)	MSE(2)
Bole	30	0.8010	0.0007	0.3600	0.0007	0.2150	0.0013
Sawla/Tuna/Kalba	72	0.5410	0.0008	0.1750	0.0007	0.0810	0.0009
West Gonja	30	0.7160	0.0008	0.1280	0.0004	0.0600	0.0007
Gonja Central	75	0.5680	0.0007	0.1750	0.0005	0.0830	0.0008
East Gonja	85	0.5980	0.0006	0.3360	0.0004	0.1930	0.0006
Kpandai	82	0.6750	0.0008	0.2590	0.0004	0.1340	0.0006
Nanumba South	45	0.3670	0.0010	0.0580	0.0003	0.0230	0.0006
Nanumba North	83	0.1990	0.0006	0.0440	0.0004	0.0180	0.0006
Zabzugu	44	0.5660	0.0008	0.0220	0.0005	0.0070	0.0010
Yendi	61	0.3060	0.0006	0.1460	0.0003	0.0710	0.0003
Tamale Metro	156	0.0990	0.0001	0.0710	0.0003	0.0310	0.0002
Tolon	60	0.6410	0.0010	0.0900	0.0007	0.0400	0.0011
Savelugu/Nanton	100	0.2810	0.0004	0.1140	0.0002	0.0510	0.0006
Karaga	45	0.3750	0.0012	0.0860	0.0007	0.0340	0.0013
Gusheigu	89	0.4500	0.0008	0.1010	0.0005	0.0450	0.0014
Saboba	45	0.4400	0.0013	0.0860	0.0005	0.0350	0.0008
Chereponi	30	0.1690	0.0015	0.0930	0.0007	0.0420	0.0011
Bunkpurugu/Yunyoo	89	0.5150	0.0008	0.1760	0.0008	0.0820	0.0011
Mamprusi East	75	0.3150	0.0010	0.1450	0.0005	0.0680	0.0007
Mamprusi West	88	0.3650	0.0005	0.1800	0.0002	0.0830	0.0004
North Gonja	15	0.848	0.0030	0.146	0.0016	0.066	0.0032
Kumbungu	59	0.374	0.0006	0.096	0.0005	0.042	0.0013
Sagnarigu	97	0.152	0.0001	0.063	0.0005	0.026	0.0005
Mion	85	0.557	0.0011	0.187	0.0004	0.091	0.0007
Tatale/Sanguli	31	0.246	0.0009	0.007	0.0007	0.002	0.0007
Mamprugu Mogduri	31	0.617	0.0020	0.397	0.0008	0.215	0.0017

Source: Authors' own estimation

Comparison of performances of EBP and ELL in estimating the FGT indicators

Table 5 displays averages of ARB and RRMSE for all the three (3) poverty estimates. It is observed from the table that EBP consistently exhibits larger ARB values compared to the small values for the ELL model. It is observed that as poverty index ( ) is increasing the biases of both models increase however that of EBP are more pronounced. Thus as the poverty index increases, the EBP becomes more biased than the ELL. This observation supports that of Guadarrama (2017) where it was observed that the ARB for the poverty indicators increases as one is estimating the poverty indicators even though their study ended at the poverty gap. This, (as discussed in Guadarrama 2017) is because both poverty gap and severity depend to a greater extent on the extreme of the left tail of the consumption distribution which is difficult to estimate correctly from a finite sample. From this, one can conclude that the ELL performs better than the EBP model under the normality assumption in terms of biasedness. These results support that of Das (2016) and Souza et al. (2015). It however contradicts the works of Guadarrama (2017) and Molina et al. (2014) which states that the EBP exhibits lower ARB than the ELL in estimating the poverty indicators under the normality assumptions. It is seen from Table 5 that the ELL model exhibits larger RRMSE values for all the three poverty indicators. Based on the rule of thumb of Despotovic et al. (2016), one can say that the EBP model performs consistently better than the ELL in terms of RRMSE as all its RRMSE values across the three indicators are all less than 10%. This means that the EBP yields higher precision and can provide greater stability for the estimates in terms of precision than the ELL and so the EBP is the most efficient estimator between the two estimators under consideration. This finding is supported by that of Molina, et al. (2014), Natalia (2015) and Guadarrama, et al. (2015) but contradicts Souza, et al. (2015).

Table 5: Table showing percent of Absolute Relative Bias (ARB) and Relative Root Mean Squared Error (RRMSE) for ELL and EBP estimates of Poverty incidence, Poverty Gap and Poverty Severity under normality assumption

Model	Absolute Relative Bias (ARB)			Relative Root Mean Square Error (RRMSE)		
	Poverty Indicators			Poverty Indicators		
	HCR P_o	Poverty Gap P_1	Square Poverty Gap or Severity P_2	Head Count Ratio P_0	Poverty Gap P_1	Square Poverty Gap P_2
ELL	19.79	25.93	31.57	16.30	11.00	8.60
EBP	20.55	25.95	32.58	3.00	2.30	3.50

Source: Authors' own estimation

Results of Simulation

In this section, we present the results of the simulations under the nested error model with outliers. Two outliers were considered: less frequent or mild outliers and the more frequent or extreme outliers.

Preliminary analysis of the data indicated that there were some outliers in the data. However, by the Cook's distance criteria, the outliers were seen not to be influential because all their Cook's distances were found to be less than one. The data were then artificially contaminated by introducing some outliers and examining the effect of the contamination on the performance of the models. The performance of the estimators were then evaluated by computing for each small area (District) the absolute relative bias (ARB) and the relative root mean squared errors (RRMSE).

Results of analysis from less frequent contaminants

Table 6 shows the average percent ARB and RRMSE for all the FGT poverty indicators for the mild contaminants. It is seen from the Table that in all the indicators, the ARB values for EBP model are smaller compared to that of the ELL model.

In terms of the RRMSE it is seen that the EBP model has smaller RRMSE values compared to the ELL model. As the poverty index increases, the RRMSE increases, an observation also made by Guadarrama et al. (2014).

Table 6: Mean percentage Absolute Relative Bias (ARB) for EBP and ELL estimates of Poverty incidence, Poverty Gap and Poverty Severity with less frequent contaminants

Model	Absolute Relative Bias (ARB)			Relative Root Mean Square Error (RRMSE)		
	Poverty Indicators			Poverty Indicators		
	HCR P_o	Poverty Gap P_1	Square Poverty Gap or Severity P_2	Head Count Ratio P_0	Poverty Gap P_1	Square Poverty Gap P_2
ELL	23.51	28.11	33.89	25.30	28.81	33.31
EBP	23.01	27.60	31.74	24.83	28.84	31.86

Source: Authors' own estimation

Comparing these results to that of no outliers, it was seen that the percent ARB of the EBP model increased by 3.22% for HCR, 1.67% for PG and 0.17% for SPG, whilst that of the ELL model increased by 2.96% for HCR, 2.16% for PG and 1.32% for SPG. It was also seen from the differences that, apart from the HCR, the differences in the corresponding indicators show that the ELL estimators are the worst performers. This means that the EBP model performs better in terms of biasedness in the presence of mild contaminants than the ELL model. In terms of biasedness, it was observed from the results that, with the exception of HCR, the PG and SPG results all indicate that the ELL estimators perform relatively poorer than the EBP compared to the case of no contamination. From these analyses, it was seen that, in the nested error model with random effects, the ELL method performs worse than the EBP model. This, as stated in Guadarrama et al. (2014), could be because the ELL does not account for unexplained between-area variations. This simulation study results support the study that was carried out by Molina & Rao (2010), Nandram et al. (2014), Guadarrama et al. (2014), Natalia (2015) and Guadarama et al. (2015) as well as Marhuenda, Molina et al. (2017). It however contradicts Berg and Chandra (2015), Das (2016) and Souza et al. (2015). We can therefore conclude from this simulation study that the EBP estimators track the true values better than the ELL estimators since it was the EBP estimators that have smaller average model bias as well as the smaller average RRMSE values in the presence of mild outliers.

Results of analysis from more frequent contaminants

For the more frequent outliers, Table 7 displays the average percent ARB and the mean percentage Relative Root Mean Squared Errors for all estimators of poverty indicators (HCR, PG and SPG). It is seen from Table 7 that in all the indicators the average percentage ARB values for EBP are small (24.51% for HCR, 29.35% for poverty gap and 33.61% for poverty severity) as against (27.43% for HCR, 76.34% for poverty gap and 201.50% for poverty severity) for the ELL. It is again seen from Table 7 that the EBP estimators have the smallest RRMSE values (27.58% for HCR, 31.60% for PG and 33.36% for SPG) compared to the ELL estimators (28.04% for HCR, 31.86% for PG and 34.89% for SPG).

Table 7: Mean percentage Absolute Relative Bias (ARB) and Mean percentage Relative Root Mean Squared Error (RRMSE) for EBP and ELL estimates of Poverty incidence, Poverty Gap and Poverty Severity with more frequent contaminants

Model	Absolute Relative Bias (ARB)			Relative Root Mean Square Error (RRMSE)		
	Poverty Indicators			Poverty Indicators		
	HCR P_0	Poverty Gap P_1	Square Poverty Gap or Severity P_2	Head Count Ratio P_0	Poverty Gap P_1	Square Poverty Gap P_2
ELL	27.43	76.34	201.50	28.04	31.86	34.89
EBP	24.51	29.35	33.61	27.58	31.60	33.36

Source: Authors' own calculation

The simulation study generally observed that as the poverty index increases, the biasedness also increases in all the scenarios considered for this study. The results indicated that the ELL estimators performed relatively poorer than the EBP compared to the case of no outliers and mild outliers. From the analysis, it was also seen that in the nested error model with random effects, the EBP method performed better than the ELL model an observation that is supported by Guadarrama et al. (2014). This simulation study results support the works of Molina & Rao (2010), Nandram et al. (2014), Guadarrama et al. (2014), Natalia (2015) and Guadarama et al. (2015) as well as Marhuenda et al. (2017). It, however, contradicts Berg and Chandra (2015), Das (2016) and Souza et al. (2015) in terms of biasedness.

For the RRMSE, it is observed that the EBP estimators on average, again had smaller values than the ELL estimators, as shown in Table 7. Like the ARB, it is observed that as the poverty index is increasing, the RRMSE increases, an observation also made by Guadarrama et al. (2014).

It was also seen that the EBP estimators show stable distributions of relative bias and RRMSE for all the poverty indicators than the ELL estimators in this study. This shows that the RRMSE of the models is affected more when the data contamination is extreme. We can conclude from this study that the EBP model is more robust than the ELL in estimating the poverty indexes in the Northern Region of Ghana.

Conclusion and Recommendation

The objective of this study was to assess the performance of ELL and EBP models in estimating District level poverty statistics in the Northern Region of Ghana in the presence of outliers. The study concluded that both EBP and ELL are sensitive to the presence of outliers in the distribution of the response variable; however, the ELL model is more sensitive than the EBP. The study revealed that the EBP model performed better in estimating the various poverty indices than the ELL and also proved to be more stable than the ELL in the presence of outliers and that no matter the level of contamination the EBP still performed better as the EBP had smaller ARB and RRMSE values than the ELL in all the poverty indexes. It is also observed that as the poverty index is increasing, the RRMSE increases. In evaluating the models' performances, it can be concluded that the EBP model is superior to the ELL in estimating the poverty indices in the Northern Region irrespective of the level of contamination since it was the EBP model that had the smallest ARB and RRMSE values. We can also say that the EBP estimators track the true values of the poverty indexes better than the ELL estimators since it is the EBP estimators that have the smallest average relative bias as well as the smallest average RRMSE values in the presence of outliers. Based on the findings, it is recommended that the EBP model be used for any future poverty estimation exercise in Ghana as a developing country. The comparison between model-based estimators should be restricted to only precision and not bias, as the model model-based components could introduce a potential bias. In terms of policy making, the study has revealed a significant variations in the incidence and depth of poverty among the districts in the Region and would assist policy-makers and other stakeholders to design and implement effective monitoring and evaluation policies as well as resolve the growing needs of micro-level planning and poverty estimation.

References

Battese, G. E., Harter, R. M. and Fuller, W. A (1988). An Error-Components Model for Prediction of County Crop Areas Using Survey and Satellite Data.

Berg, E. and Chandra, H. (2015). Small Area Estimation for Unit-Level Lognormal Model. Computational Statistics and Data Analysis, 78, 159–175.

Betti, G., Neri, L., and Salvati, N. (2007). Comparing M-quantile and ELL methods for poverty mapping. IASS Satellite Conference on Small Area Estimation, Pisa: University of Pisa.

Chambers, R. and Tzavidis, N. (2006). M-quantile Models for Small Area Estimation, Biometrika, 93, 255-268.

Chambers, R and Chandra, H. (2013). A random Effect Block Bootstrap for Clustered Data, Journal of Comp. Graph. Stat. 22(2), 452-470

Chambers, R., Chandra, H., Salvati, N., and Tzavidis, N. (2014). Outlier robust small area estimation. Journal of the Royal Statistical Society. Series B: Statistical Methodology, 76(1), 47–69.

Das, S. (2016). Robust Inference in Poverty Mapping. Retrieved from http://ro.uow.edu.au/theses/4822/

Despotovic, M., Nedic, V., Despotovic, D., and Cvetanovic, S. (2016). Evaluation of Empirical Models for Predicting Monthly Horizontal Diffuse Solar Radiation, Renewable and Sustainable Energy Reviews, Vol. 56, 246-260, ISSN 1364-0321.

Diallo, M. S., and Rao, J. N. K. (2014). Small Area Estimation of Complex Parameters Under Unit-level Models with Skew-Normal Errors. Proceedings of the Survey Research Methods Section. 970-984. American Statistical Association.

Elbers, C, Lanjouw, J. O. and Lanjouw, P. V. (2003). Micro-Level Estimation of Poverty and Inequality. Econometrica, 71(1), 355-364.

Fay, R. E. & Herriot, R. A. (1979). Estimation of income for small places: An application of James-Stein procedures to census data. Journal of American Statisticians Association, 74, 269-277.

Foster, J., Greer, J. and Thorbecke, E. (1984). A class of decomposable poverty measures, Econmetrica, 52, 761-765.

Ghana Statistical Service (2002). 2000 Population and Housing Census: Summary Report of Result of Final Results, Accra

Ghana Statistical Service. (2013). National Analytical Report 2010 Population and Housing Census.

Ghana Statistical Service. (2014). Poverty Profile in Ghana (2005-2013), GLSS 6.

Ghosh, M. & Rao, J. N. K. (1994). Small Area Estimation: an appraisal, Statistical Science, 9, 55-93.

Giusti, C., Marchetti, S., Pratesi, M., and Salvati, N. (2009). Small Area Estimation : Recent methodological developments for the estimation of poverty measures SAMPLE project.

Guadarrama, M., Molina, I., and Rao, J. N. K. (2014). A Comparison of Small Area Estimation Methods for Poverty Mapping. Statistics in Transition New Series, 1(17), 41–66.

Gudarrama, M. (2017). Small Area Estimation Methods under Complex Sampling Design, PhD Thesis, UC3M, Universidad Carlos III de Madrid.

Harold, C. and Quentin, W. (2007). African development Indicators: The West and Central Africa Poverty mapping Initiative, World Bank.

Marhuenda, M. Y. (2015). Package ' sae .'

Marhuenda, Y., Molina, I., Morales, D. and Rao, J. N. K. (2017). Poverty Mapping in Small Areas Under a twofold Nested Error Regression Model. Journal of Royal Statistical Society: Series A (Stat. Soc.), 180(3), 1111-1136

Marissa, C. I. (2010). Intercensal Updating of Small Area Estimates. Unpublished PhD Thesis, Massey University, New Zealand.

Molina, I and Rao, J. N. K. (2010). Small Area Estimation of Poverty Indicators, Canadian Journal of Statistics, 38: 369-385.

Molina I, Nandram. B. and Rao. J. N. K. (2014). Small Area Estimation of General parameters with Application to Poverty Indicators: A Hierarchical Bayes Approach, The Annals of Applied Statistics, Vol 8, No. 2, 852 – 885.

Molina, I., and Rao, J. N. K. (2014). A Comparison of Small Area Estimation Methods for Poverty Mapping, Statistics in Transition, New Series and Survet Methodology, Joint Issue: Small Area Estimation, 17(1), 41–66.

Opsomer, J. D., Claeskens G., Ranalli, M. G., Kauermann, G. and Breidt, F. G. (2008). Non Parametric Small Area Estimation Using Penalized Spline Regression. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 70(1), 265–286.

R Core Team (2014). R A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/

Rao, J. N.K . (2003). Small Area Estimation. Hoboken, New Jersey: John Wiley & Sons, Inc

Sinha, S. K. & Rao J. N. K.(2009). Robust Small Area Estimation Under Unit Level Models, Canadian Journal of Statistics, 27–38.

Souza, D., Brendolin, N., Quintaes, V., Pessoa, D and Onel, S. (2015). Comparing Small Area Estimation Methods for Poverty Indicators in the Municipalities of

Brazilian State of Minas Gerais. ISI Satellite meeting on Small Area Estimation. Santiago: University.

Srivastava, A. K., Sud, U. C. and Chandra, H. (2007). Small Area Estimation- An Application to National Sample Survey Data, Journal of Indian Society of Agricultural Statistics, New Delhi, 61(2): pp 249- 254

Stephen J. H, M. C. I. and G. J. (2010). Comparison of Survey regression techniques in the context of small area estimation of poverty, Component of statistics, Canada.

Sud U.C, Bhatia V. K., Chandra H., and Srivastava, A. K. (2011). Crop Yield Estimation at the District level by Combining ICS data and Census data, Rio de Janeiro, Brazil.

Tzavidis, N., Marchetti, S. and Donbavand, S. (2013). Outlier Robust Semi-Parametric Small Area Methods for Poverty Estimation. In G. Betti & A. Lemmi, Poverty and Social Exclusion: New Methods of Analysis. Oxon: Routledge.

Tzavidis N, Salvati, N., Pratesi, M. and Chambers, R. (2008). M-quantile Models with Applications to Poverty Mapping, Statistical Methodology and applications, 17, 393-411.

Tzavidis N. and Chambers R. (2006). Robust Prediction of Small Area Means and Distributions, Working Papers.

Van der Weide, R. and Elbers, C. (2013). Estimation of Normal Mixtures in a Nested Error Model with Application to Small Area Estimation of Welfare. Presented at the SAE Conference 2013, Bangkok, Thailand.

Zhao, Q. and Lanjouw, P. (2003). Using PovMap2, A User's Guide. The World Bank.