Boundary layer flow near stagnation-points on a vertical surface with slip in the presence of transverse magnetic field

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Abstract

Purpose – The purpose of this paper is to investigate the MHD boundary layer flow of viscous, incompressible and electrically conducting fluid near a stagnation-point on a vertical surface with slip.
Design/methodology/approach – In the study, the temperature of the surface and the velocity of the external flow are assumed to vary linearly with the distance from the stagnation-point. The governing differential equations are transformed into systems of ordinary differential equations and solved numerically by a shooting method.
Findings – The effects of various parameters on the heat transfer characteristics are discussed. Graphical results are presented for the velocity and temperature profiles whilst the skin-friction coefficient and the rate of heat transfers near the surface are presented. It is observed that the presence of the magnetic field increases the skin-friction coefficient and the rate of heat transfer near the surface towards the stagnation-point.
Originality/value – The presence of magnetic field increases the skin-friction coefficient and the rate of heat transfer near the surface towards the stagnation-point.

Keywords Viscosity, Magnetic field, Heat transfer, Magnetohydrodynamics, Slip, Stagnation-point
Paper type Research paper

1. Introduction

The boundary layer flow of viscous, incompressible and electrically conducting fluid near a stagnation-point is an important engineering problem due to its numerous applications in industry. It is frequently encountered in the cooling of electronic devices and nuclear reactors during emergency shutdowns. Some investigations into the stagnation-point flow problem have established the existence of dual solutions (Ishak *et al.*, 2008; Wang, 2003). Zhu *et al.* (2009) analytically investigated the stagnation-point flow problem with heat transfer over a stretching sheet using the homotopy analysis method. Hassanien and Gorla (1990) had obtained results for the stagnation-point flow of micropolar fluids over non-isothermal surfaces whilst Nazar *et al.* (2004) included its effect towards a stretching sheet. Mixed convection flow of micropolar fluid towards a stretching sheet has been reported in the literature (Lok *et al.*, 2006; Ishak *et al.*, 2007, 2008). The existence of dual solutions for both assisting and opposing flows past vertical permeable flat plates was observed by Ishak *et al.* (2010) with Wang (2003) obtaining similarity solutions of the Navier-Stokes equations for the stagnation-point flow towards a flat plate with slip. These solutions are applicable to slip regimes of rarefied gases. Wang (2006) later extended the problem further to include heat transfer

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aspect while Andersson (2002) analysed the slip flow past stretching surfaces. Mixed convection boundary layer flow near stagnation-point on vertical surfaces with slip was recently reported by Aman *et al.* (2011).

Numerical methods for solving the boundary layer equations of laminar natural convection about vertical plates can be found in Zheng *et al.* (2007). Cao and Baker (2009) investigated the slip effects on mixed convection flow and heat transfer from vertical plates with Fang *et al.* (2010) proposing a solution for the problem of slip flow over permeable shrinking surfaces (without the heat transfer aspect) using a second-order slip flow model and presented exact solutions of the governing Navier-Stokes equations.

It is worth noting that until recently, few authors considered the influence of magnetic field effects on the stagnation-point flow. Makinde and Charles (2010) presented computational dynamics of hydromagnetic stagnation point flow towards a stretching sheet whilst a numerical study of MHD generalized Couette flow and heat transfer with variable viscosity and electrical conductivity was reported by Makinde and Onyejekwe (2011). The steady two-dimensional magnetohydrodynamic stagnation flow towards a nonlinearly stretching surface was investigated by Zhu *et al.* (2010) with the computational modelling of MHD unsteady flow and heat transfer over flat surfaces with Navier slip and Newtonian heating reported by Makinde (2012). Ariel (2008) investigated the two-dimensional stagnation-point flow of an elastic-viscous fluid with partial slip.

Furthermore, Makinde and Sibanda (2011) investigated the effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation and observed that both the velocity and temperature profiles increased significantly when the heat generation parameter increases. The influence of magnetic field on liquid metal free convection in an internally heated cubic enclosure was conducted by Michele and Fabrizio (2002) whilst Rama *et al.* (1997) analysed the mixed convection in non-Newtonian fluids along a vertical plate in porous media with surface mass transfer.

The present communication highlights the effects of transverse magnetic field on the boundary layer flow near the stagnation-point on a vertical surface with slip. The paper is organized as follows: the mathematical formulation is presented in Section 2. The nonlinear differential equations are transformed in this section to ordinary differential equations using suitable similarity variables. In Section 3, the method of solution is presented. Results and discussions are presented in Section 4. The concluding remarks are given in Section 5.

2. Problem formulation

Consider a steady two-dimensional laminar boundary layer flow of a viscous, incompressible and electrically conducting fluid near the stagnation-point on a vertical surface, and assume that the velocity of the free stream is $u_e(x)$, the temperature of the plate and the ambient fluid are, respectively, $T_w(x)$ and T_∞ (see Figure 1).

The boundary layer equations describing the problem are the continuity, momentum and energy equations, respectively, represented as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma H_0^2}{\rho}(u - u_e) + g\beta(T - T_\infty) \tag{2}$$

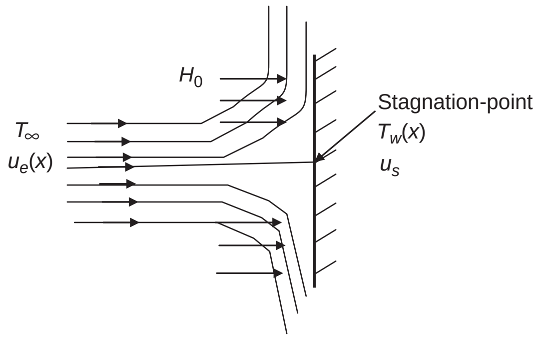

Figure 1.
Physical configuration

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma H_0^2}{\rho c_p}(u-u_e)^2 \tag{3}$$

where u and v are velocities in the x and y directions, respectively, g is the acceleration due to gravity, T is the fluid temperature, β is the thermal expansion coefficient, α is the thermal diffusivity, H_O is the transverse magnetic field strength, σ is the electrical conductivity, ρ is the fluid density, μ is the dynamic viscosity of the fluid, u_e is the free-stream velocity and c_p is the specific heat capacity at constant pressure. The boundary conditions similar to (Aman *et al.*, 2011) are assumed as:

$$\begin{aligned}&u=L\frac{\partial u}{\partial y},\ v=0,\ T=T_w+K\frac{\partial T}{\partial y},\ \text{at}\ y=0,\\&u\rightarrow u_e(x),\ T\rightarrow T_\infty\ \text{as}\ y\rightarrow\infty,\end{aligned} \tag{4}$$

where L represents the slip length, T_w represents the temperature of the wall and K is a proportionality constant. Furthermore, we make the assumption that:

$$u_e=ax,\ T_w(x)=T_\infty+bx \tag{5}$$

where a and b are constants.

We introduced the following similarity variables to transform the partial differential equations to ordinary differential equations:

$$\eta=y\sqrt{\frac{a}{v}},\ \ \psi=x\sqrt{av}f(\eta),\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where η is an independent dimensionless variable, θ is the dimensionless temperature, and ψ is the stream function defined in the usual way as:

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x} \tag{7}$$

This automatically satisfies the continuity equation given as (1). Substituting Equations (6) and (7) into Equations (2) and (3) transform the partial differential equations into higher order nonlinear ordinary differential equations:

$$f'''+ff''-f'^2+\lambda\theta+R(1-f')+1=0 \tag{8}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - f'\theta + REc(f'-1)^2 + Ecf''^2 = 0 \tag{9}$$

with the boundary conditions:

$$\begin{aligned} &f(0)=0, \ \ f'(0)=\delta f''(0), \ \ \theta(0)=1+\gamma\theta'(0), \\ &f'(\eta)\to 1, \ \ \theta(\eta)\to 0, \qquad as \qquad \eta\to\infty \end{aligned} \tag{10}$$

Here, the primes denote differentiation with respect to η, $\Pr = \nu/\alpha$ is the Prandtl number, $\delta = L(a/\nu)^{1/2}$ is the velocity slip parameter, $\gamma = K(a/\nu)^{1/2}$ is the thermal slip parameter, and $\lambda = g\beta b/a^2$ is the mixed convection parameter and $R = \sigma H_0^2/\rho a$ is the magnetic parameter. It is worth mentioning that $\lambda > 0$ corresponds to assisting flow, $\lambda < 0$ corresponds to opposing flow and $\lambda = 0$ corresponds to forced convection flow. The physical quantities of interest are the skin-friction coefficient C_f and the local Nusselt number Nu_x, which are proportional to $f''(0)$ and $-\theta'(0)$, respectively.

3. Numerical procedure

The non-linear differential Equations (8) and (9) with the boundary conditions (10) have been solved numerically using the fourth order Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method.

We let:

$$f = x_1, f' = x_2, f'' = x_3, \theta = x_4, \theta' = x_5 \tag{11}$$

Equation (11) transforms Equations (8) and (9) into systems of first order differential equations as:

$$\begin{aligned} f' &= x_1' = x_2 \\ f'' &= x_2' = x_3 \\ f''' &= x_3' = -x_1x_3 + x_2^2 - \lambda x_4 + R(x_2-1) - 1 \\ \theta' &= x_4' = x_5 \\ \theta'' &= x_5' = -\Pr x_1x_5 + \Pr x_2x_4 - \Pr Ecx_3^2 - \Pr REc(x_2-1)^2 \end{aligned} \tag{12}$$

Subject to the following initial conditions:

$$\begin{aligned} &x_1(0)=0, \ \ x_2(0)=\delta x_3(0)=s_1, \ \ x_4(0)=1+\gamma x_5(0)=s_2 \\ &x_2(\infty)\to 1, \ x_4(\infty)\to 0 \end{aligned} \tag{13}$$

In the shooting method, the unspecified initial conditions; s_1 and s_2 in Equation (13) are assumed and Equation (12) integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial condition is checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If differences exist, improved values of the missing initial conditions are obtained and the process repeated. The computations were done by a written programme which uses a symbolic and computational computer language (MAPLE). A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ to each group of parameters are determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7}. From the process of numerical computations, the local skin-friction coefficient and the local Nusselt numbers, which are, respectively, proportional to $f''(0)$ and $-\theta'(0)$ were worked out and their numerical values presented in tables.

4. Results and discussion

Numerical results are computed for the reduced skin-friction coefficient, $f''(0)$ and the rate of heat transfer, $-\theta'(0)$ at the surface. Table I shows the effect of Prandtl number (Pr), velocity slip parameter (δ), thermal slip parameter (γ) mixed convection parameter (λ), Eckert number (Ec) and the magnetic parameter (R) on the skin-friction coefficient and the Nusselt number.

It is observed that, the skin-friction coefficient increases with increasing values of the Prandtl number (Pr), mixed convection parameter (λ), Eckert number (Ec) and the magnetic parameter (R). It remained unchanged with increasing values of velocity slip parameter (δ) but decreases with increasing values of the thermal slip parameter (γ). Furthermore, the rate of heat transfer from the surface was observed to decrease with increasing values of all the controlling parameters except the velocity slip parameter (δ) which remained unchanged.

The results of the study compared to previous works for varying values of *Pr* in the absence of magnetic field parameter are shown in Tables II and III. The results show good agreement with previously published works in the literature.

Figures 2 and 3 illustrate the variation of the temperature and velocity profiles with the buoyancy parameter, λ, respectively. It is observed that increasing the buoyancy parameter increases the velocity and temperature profiles near the surface of the plate.

Figures 4 and 5 illustrate the effect of increasing the Prandtl number on the velocity and thermal boundary layers. It is observed that whilst the *Pr* decreases the temperature profiles, it has negligible effects on the velocity profiles.

The effects of the magnetic parameter on the temperature and the velocity profiles are illustrated in Figures 6 and 7, respectively. It is observed that both the temperature and velocity profiles increased near the surface when the magnetic parameter is increased.

Pr	λ	δ	γ	Ec	R	$f''(0)$	$-\theta'(0)$
0.7	0.1	0.1	1	1	1	1.63035796	0.049165934
1	0.1	0.1	1	1	1	1.63263748	−0.03102519
7	0.1	0.1	1	1	1	1.66497861	−1.20640683
0.7	1	0.1	1	1	1	2.04816411	−0.01073676
0.7	2	0.1	1	1	1	2.57555802	−0.11555708
0.7	0.1	1	1	1	1	1.63035796	0.049165934
0.7	0.1	2	1	1	1	1.63035796	0.049165934
0.7	0.1	3	1	1	1	1.63035796	0.049165934
0.7	0.1	0.1	2	1	1	1.62953697	0.034493207
0.7	0.1	0.1	3	1	1	1.62909326	0.026565681
0.7	0.1	0.1	1	2	1	1.91798321	−0.06001874
0.7	0.1	0.1	1	3	1	2.16718166	−0.15269745
0.7	0.1	0.1	1	1	2	1.65175839	−0.33577041
0.7	0.1	0.1	1	1	3	1.67339925	−0.72790459

Table I. Computations showing the variation in local skin friction and Nusselt number for varying controlling parameters

Table II.
Comparison $f''(0)$ for values of Pr when $\lambda = 1$, $\gamma = 0$, $Ec = 0$

Pr	Ramachandran *et al.* (1988)	Devi *et al.* (1991)	Lok *et al.* (2006)	Hassanien and Gorla (1990)	Aman *et al.* (2011)	Present study $R = 0$
0.7	1.7063	1.7064	1.7064	1.70632	1.7063	1.7063
1	–	–	–	–	1.6754	1.6754
7	1.5179	1.5180	1.5180	–	1.5179	1.5179
10	–	–	–	1.49284	1.4928	1.4928
20	1.4485	1.4485	1.4486	–	1.4485	1.4485
40	1.4101	–	1.4102	–	1.4101	1.4101
50	–	–	–	1.40686	1.3989	1.3989
60	1.3903	1.3903	1.3903	–	1.3903	1.3903
80	1.3774	–	1.3773	–	1.3774	1.3774
100	1.3680	1.3680	1.3677	1.3841	1.3680	1.3680

Table III.
Comparison of $-\theta'(0)$ for different values of Pr when $\lambda = 1$, $\delta = 0$, $\gamma = 0$, $Ec = 0$

Pr	Ramachandran *et al.* (1988)	Devi *et al.* (1991)	Lok *et al.* (2006)	Hassanien and Gorla (1990)	Aman *et al.* (2011)	Present study $R = 0$
0.7	0.7641	0.7641	0.7641	0.76406	0.7641	0.76406
1	–	–	–	–	0.8708	0.87078
7	1.7224	1.7223	1.7226	–	1.7224	1.7224
10	–	–	–	1.94461	1.9446	1.9446
20	2.4576	2.4574	2.4577	–	2.4576	2.4576
40	3.1011	–	3.1023	–	3.1011	3.1011
50	–	–	–	3.34882	3.3415	3.3415
60	3.5514	3.5517	3.5560	–	3.5514	3.5514
80	3.9095	–	3.9195	–	3.9095	3.9095
100	4.2116	4.2113	4.2289	4.23372	4.2116	4.2116

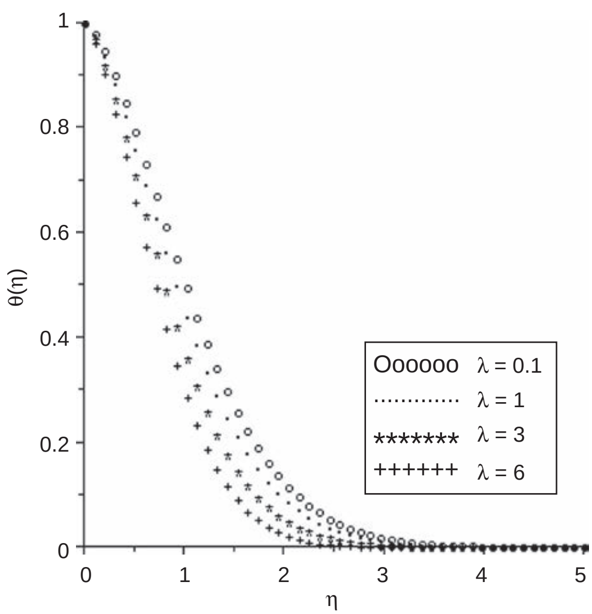

Figure 2.
Variation of temperature profiles with λ when $Pr = 0.7$, $\delta = 0.1$, $\gamma = 0.1$, $R = 1$, $Ec = 1$

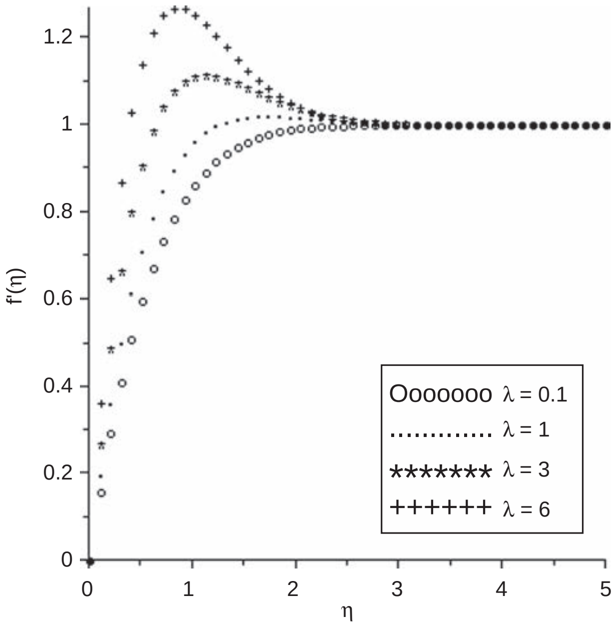

Figure 3.
Variation of velocity profiles with λ when $Pr = 0.7$, $\delta = 0.1$, $\gamma = 0.1$, $R = 1$, $Ec = 1$

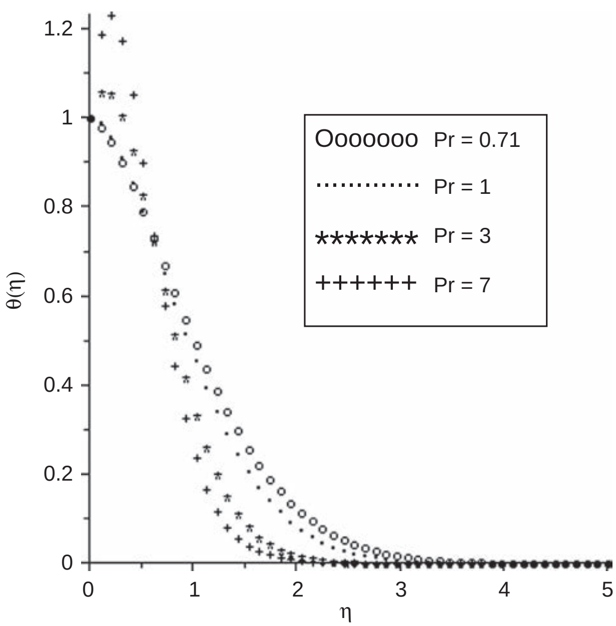

Figure 4.
Variation of temperature profiles with Pr when $\lambda = 0.1$, $\delta = 0.1$, $\gamma = 0.1$, $R = 1$, $Ec = 1$

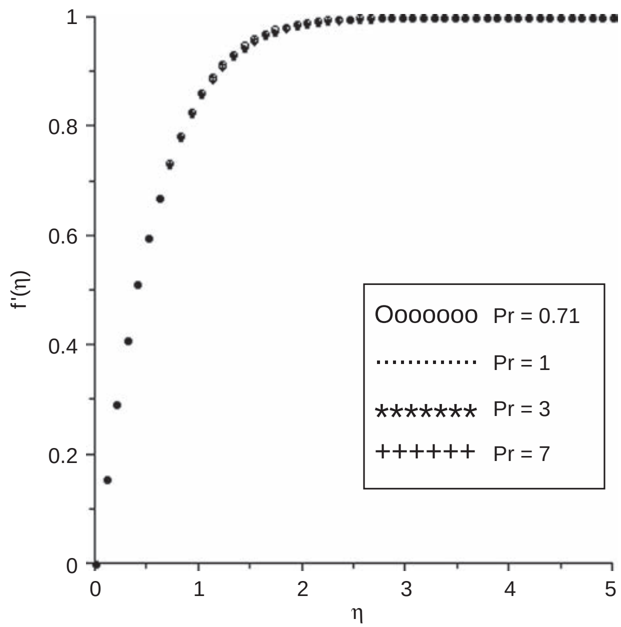

Figure 5.
Variation of velocity profiles with Pr when $\lambda = 0.1$, $\delta = 0.1$, $\gamma = 0.1$, $R = 1$, $Ec = 1$

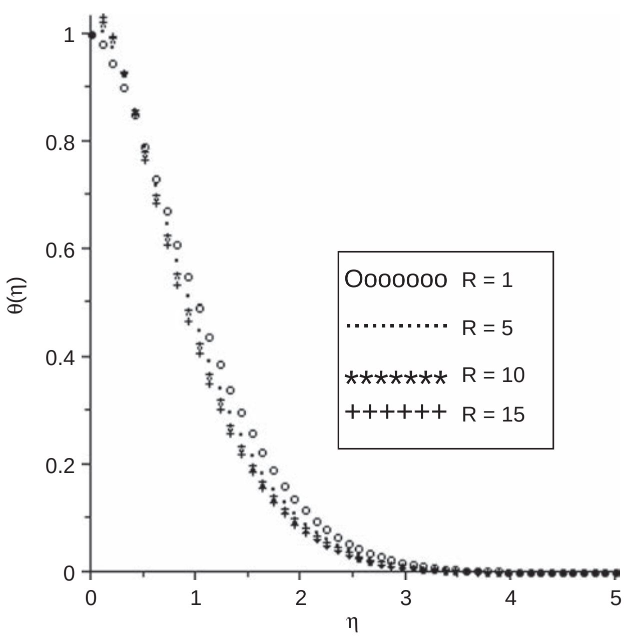

Figure 6.
Variation of temperature profiles with magnetic parameter, R, when $Pr = 0.7$, $\lambda = 0.1$, $\delta = 0.1$, $\gamma = 0.1$, $Ec = 1$

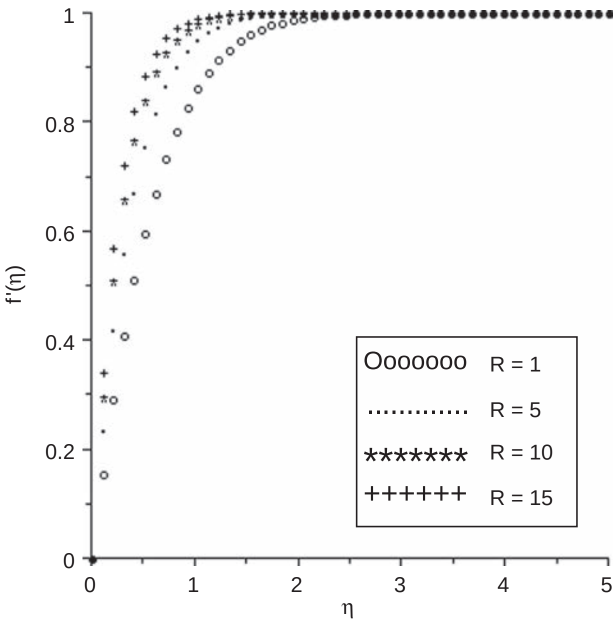

Figure 7.
Variation of velocity profiles with magnetic parameter, R, when $Pr = 0.7$, $\lambda = 0.1$, $\delta = 0.1$, $\gamma = 0.1$, $Ec = 1$

The effects of increasing the Eckert number on the temperature and the velocity profiles are depicted in Figures 8 and 9, respectively.

It is observed that the velocity profiles changes negligibly with increasing values of Eckert numbers whilst the temperature profiles increase when the Eckert number is increases.

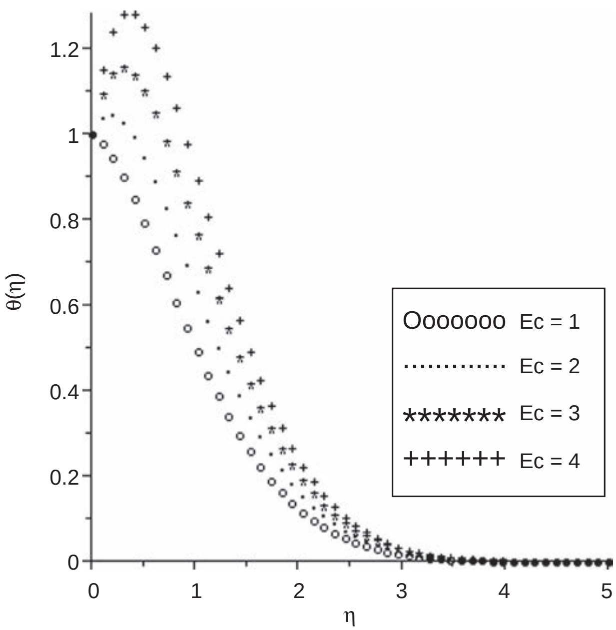

Figure 8.
Variation of temperature profiles with Eckert number (Ec) when $Pr = 0.71$, $\lambda = 0.1$, $\delta = 0.1$, $\gamma = 0.1$, $R = 1$

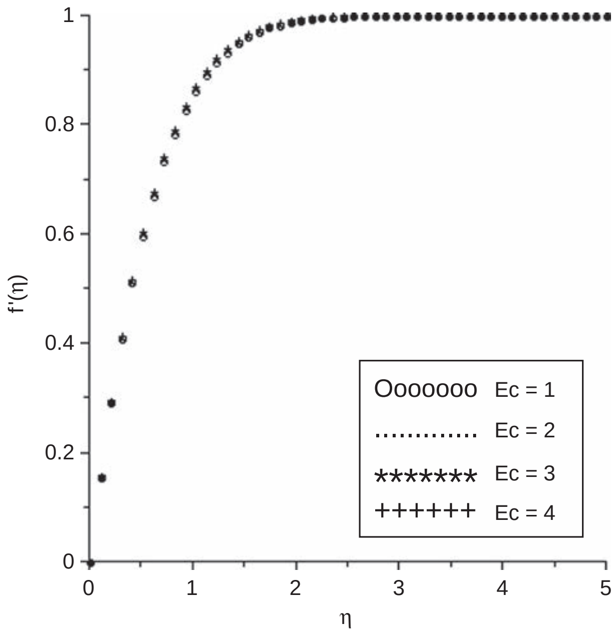

Figure 9.
Variation of velocity profiles with Eckert number (Ec) when $Pr = 0.7$, $\lambda = 0.1$, $\gamma = 0.1$, $\delta = 0.1$, $R = 1$

5. Conclusions

In this paper, the flow and heat transfer characteristics near the stagnation-point on a vertical surface with magnetic field effects on the boundary are numerically studied. The boundary layer equations governing the flow were transformed to systems of ordinary differential equations using suitable similarity variables and solved numerically. The skin-friction coefficient and rate of heat transfer from the surface were discussed. The velocity and the temperature profiles for varying controlling parameters was presented graphically and discussed. In conclusion, the buoyancy and magnetic field parameters affect the velocity profiles significantly whilst the effect of Prandtl (*Pr*) and Eckert number (*Ec*) have negligible effects on the velocity profiles but significantly influences the thermal boundary layer thickness.

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