UNIVERSITY FOR DEVELOPMENT STUDIES

CLASSICAL AND BAYESIAN SWITCHING VOLATILITY MODELS FOR ANALYSING STOCK RETURNS IN GHANA

EDWARD AKURUGU

2021

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UDS/MAS/0003/19

THESIS SUBMITTED TO THE DEPARTMENT OF STATISTICS, FACULTY OF MATHEMATICAL SCIENCES, UNIVERSITY FOR DEVELOPMENT STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF PHILOSOPHY DEGREE IN APPLIED STATISTICS

SEPTEMBER, 2021

DECLARATION

I hereby declare that this thesis is my original work and that no part of it has been presented for another degree in this university or elsewhere.

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Supervisors

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of the thesis laid down by the University for Development Studies.

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ABSTRACT

The focus of this study was to model and forecast stock returns of Ghana Commercial Bank on the Ghana Stock Exchange using classical and Bayesian switching volatility models. Due to the presence of stylised facts in stock returns, this study finds it imperative to identify an appropriate risk model that best describes these features. The data utilised in this study are the stock prices of Ghana Commercial Bank transformed into monthly averages of daily closing prices covering 138 months. The study applied the two-state Markov-Switching GARCH models in deciding on the appropriate models to forecast the stock returns under the classical and Bayesian perspectives. In choosing the substantive models, selection criteria's such as log-likelihood, Akaike Information Criterion, Bayesian Information Criterion were considered under the classical estimation and Deviance Information Criteria was considered under the Bayesian estimation. Based on the selection criteria's under both estimations, E-GARCH variance specification with skewed student-t conditional distribution (innovation) was found appropriate modelling the stock returns. The estimates under both approaches find the first regime to possess the features of "turbulent market conditions" while regime two exhibit "tranquil market conditions". However, comparative risk analysis finds the Bayesian perspective to generally perform better in estimating VaR and ES at both the 1% and 5% respectively as compared to the classical perspective. Investors should invest in Ghana Commercial Bank due to the good returns associated with the stocks and where there is the existence of "turbulent market conditions", the recovery rate is shorter for these stocks.

ACKNOWLEDGEMENTS

I wish to show my appreciation to the Almighty God, for his abundant benison bestowed upon me resulted in the successful completion of this thesis. Numerous people have contributed in diverse ways to make my education come this far. While time may not permit me to enumerate the names of all these people, my heartfelt thanks go to my supervisors; Dr. Suleman Nasiru and Ms. Irene Dekomwine Angbing for their countless constructive criticisms and words of inspirations that got me through the successful end of this thesis. Special mention goes to Mr. Abdul-Majeed Ayebire, Mr. Sadat Banbil, Ms. Gifty Atambila, Ms. Georgina Akariyanebire and Ms. Victoria Asakesira Awuni; for their support financially, words of encouragement and advice towards my education have contributed immensely in making academia the way it is expected to be on my part. I also extend my appreciation to my parents for constantly defying all odds just to make me achieve greater heights in education. I am grateful for such assistance and it is my fervent prayer that the Almighty God grant all their heartfelt desires. Also, my special thanks to the families of Abesiko and Atubila for their prayers and continual support in all phases of my endeavours.

I wish to also acknowledge the presence of my friends for extending their support in time of need and all I will say is may the good Lord bless you all abundantly.

DEDICATION

I dedicate this thesis to my treasured parents; Mr. Simon Anibire and Ms. Akasale Atubila, my brothers and sisters; Samuel Akurugu, Elizabeth Akurugu, Joseph Asucam, and Sandra Asucam.

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARCH-LM	Autoregressive Conditional Heteroscedasticity Lagrange Multiplier
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
CDF	Cumulative Density Function
CNY	China Yuan Renminbi
DAX	Deutscher Aktienindex
D-DREAM	Discrete Differential Evolution Adaptive Metropolis
DF	Dickey-Fuller
DIC	Deviance Information Criterion
E-GARCH	Exponential Generalised Autoregressive Conditional Heteroscedasticity
ES	Expected-Shortfall
EWMA	Exponential Weighted Moving Average
FTSE	Financial Times Stock Exchange
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GARCH-M	Generalised Autoregressive Conditional Heteroscedasticity in Mean

GCB	Ghana Commercial Bank
GED	Generalised Error Distribution
GJR	Glosten-Jagannathan Runkle
GJR-GARCH	Glosten-Jagannathan Runkle Generalised Autoregressive Conditional Heteroscedasticity
GSE	Ghana Stock Exchange
JPY	Japanese Yen
KPSS	Kwiatkowski-Philips-Schmidt-Shin
KRW	Korean Won
LL	Log-Likelihood
MA	Moving Average
MCMC	Markov Chain Monte Carlo
MLE	Maximum Likelihood Estimation
MRS-GARCH	Markov Regime-Switching Generalised Autoregressive Conditional Heteroscedasticity
MRS-ZD-GARCH	Markov Regime-Switching Zero-Drift Generalised Autoregressive Conditional Heteroscedasticity
MS	Markov-Switching
MS-GARCH	Markov-Switching Generalised Autoregressive Conditional Heteroscedasticity
MYR	Malaysia Ringgit
PDF	Probability Density Function

Q-GARCH	Quadratic Generalised Autoregressive Conditional Heteroscedasticity
RS	Regime-Switching
RS-GARCH	Regime-Switching Generalised Autoregressive Conditional Heteroscedasticity
SGD	Singapore Dollar
SGED	Skewed Generalised Error Distribution
SST	Skewed Student-t
ST	Student-t
S&R	Standard and Poor
SET	Stock Exchange of Thailand
SR-GARCH	Single-Regime Generalised Autoregressive Conditional Heteroscedasticity
SV	Stochastic Volatility
T-GARCH	Threshold Generalised Autoregressive Conditional Heteroscedasticity
T-GARCH-M	Threshold Generalised Autoregressive Conditional Heteroscedasticity in Mean
TSE	Tehran Stock Exchange
USD	United States Dollar
VaR	Value-at-Risk
ZD-GARCH	Zero-Drift Generalised Autoregressive Conditional Heteroscedasticity

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

The daily stock return is noted for uncertainties that could arise during periods of closures. Such uncertainties become evident during periods of political instability, financial crisis, natural disasters (pandemics), market speculations, government policies (such as interest rate and tax policies) among others. For instance; the recent financial clean-up in the banking sector of Ghana and its attendant closure of financial institutions could render the financial market volatile. Volatility in the context of stock returns refers to the variability associated with stock price changes for some time (Guris and Sacildi, 2016). The basis on which these variations in the random component of stock returns can be measured is the variance or standard deviation.

In the area of financial applications, the approach of linear time series models such as the Autoregressive (AR), Moving Average (MA) and the mixture of AR and MA components known as the Autoregressive Moving Average (ARMA) has been utilised extensively. Persio and Vettori (2014) indicated that the drawback to these models is their inability to describe non-linear dynamic patterns such as asymmetry and volatility clustering owing to variances that change with time. To address this shortcoming, Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model to handle changes associated with variances that are time-dependent (such as low or high volatility). More recent studies in the area of stock returns that applied ARCH models can be found in Degiannaakis and Xekalaki (2004) and Khan et al. (2018). However, the ARCH model is deficient since it fails to recognise that stock returns respond to positive and negative shocks differently.

Another extension to the ARCH model is the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model which combines an AR component with a MA component such that the conditional variance is expressed as a linear function of both past squared observations and conditional variances (Bollerslev, 1986) unlike the former model of Engle which expresses the conditional variance as a linear function of past observations. Chou (1998) indicated that the GARCH model is associated with excessive persistence attributable to the unpredictability of financial variables. Several studies have all pointed to the high persistence of volatility in the GARCH model as a result of the possible presence of structural breaks in the variance (Diebold, 1986; Lamoureux and Lastrapes, 1990; Bauwens et al., 2014). Also, factors such as dramatic events concerning the stock market crash might inhibit the application of GARCH models in modelling volatile financial time series (Schwert and Seguin, 1990; Nelson, 1991; Engle and Mustafa, 1992). Andersen and Bollerslev (1998) contended that GARCH models certainly provide good volatility forecasts and in-sample fit, hitherto the performance of the model in terms of forecasting is very poor. The poor forecasting performance of the model might be ascribed to structural changes (structural breaks) in the data generating process. This is buttressed in studies by Bauwens et al. (2014) and Lamoureux and Lastrapes (1990) pointing to the fact that the precision of volatility forecast can be affected due to structural changes in the volatility dynamics of financial assets.

From the foregoing, a more general class of GARCH models are needed towards allowing for regime shifts of the data generating process so as to obtain more robust estimates of the conditional volatility. The Markov-Switching GARCH (MS-GARCH) model first introduced by Hamilton (1989) also known in the literature as the Regime-Switching GARCH (RS-GARCH) model helps to address this issue. This model permits switches in the conditional volatility between discrete latent (unobservable) states, with the transition between states followed by a hidden and finite-order Markov chain. Also, the probability of volatility switching thus expected duration of each regime is determined from the transition probability of the Markov process. Similarly, risk predictions improve with the application of the MS-GARCH models since they can adjust quickly to variations in the levels of unconditional volatility (Marcucci, 2005; Ardia, 2008; Gregoriou and Pascalau, 2011).

Another approach that can handle structural breaks in the volatility dynamics is the Bayesian approach. In the Bayesian paradigm, both parameter and model uncertainties are taken into account that is latent regime variables are treated as part of the model parameters. This approach has expanded in time with numerous researchers focusing attention on this area. For instance; Carlin et al. (1992) made use of the Bayesian approach to analysing the state-space model. Jacquier et al. (1994) investigated Stochastic Volatility (SV) models with Bayesian Markov Chain Monte Carlo (MCMC).

Based on this background, the study, therefore, employs the classical and Bayesian approach to model the stock returns of a selected company on the Ghana Stock Exchange (GSE) using MS-GARCH models. In this study, the classical and Bayesian techniques were used to address the frequent structural breaks associated with the standard GARCH models.

1.2 Problem Statement

The GSE since its inception has witnessed several companies being listed on the financial market. Investors and companies in this sector of the national economy on daily basis monitor the performance and movement of stock returns to make financial decisions. However, investors and companies sometimes find it difficult to predict the stock returns due to the volatility associated with the financial market variables (Rachev et al., 2008; Billio et al., 2016). Because of this, several researchers have developed models and their variants to handle financial time series variables. Popular among these models is the GARCH model which can forecast stock returns volatility even if observed stock returns series possess structural breaks which tend to produce inaccurate estimates and hence affect the forecasting performance of the model.

The purpose of this study is to apply another form of GARCH models that address stylised facts (such as volatility clustering, heavy-tailed distribution among others) accompanying stock returns data by employing an appropriate risk model (MS-GARCH) which allows specifically the conditional mean and variance to switch from one GARCH process to the other (Bauwens et al., 2010). The classical and Bayesian techniques of the MS-GARCH were considered in this study due to their added advantage of modelling structural breaks of the volatility dynamics of stock returns.

1.3 General Objective

To model and forecast the stock returns of Ghana Commercial Bank (GCB) on the GSE using Markov-Switching (MS) volatility models.

1.4 Specific Objectives

i. To model the returns using MS volatility models.

ii. To compute the Value-at-Risk (VaR) and the Expected Shortfall (ES) of GCB.

iii. To compare the volatility forecasts of GCB.

1.5 Research Questions

The study sought to provide answers to the following research questions;

i. Do the MS volatility models fit the data?

ii. What are the VaR and ES estimates of GCB?

iii. Which estimation approach best forecasts the volatility of GCB?

1.6 Significance of the Study

Financial time series variables such as stock returns play a crucial role in the national economy of Ghana and hence the quest to search for appropriate risk volatility models that well predict and provide a justification on concrete financial decisions for investors and stakeholders in the financial sector towards risk management.

The research adds to the financial literature since there seems to be little work that has been done in Ghana on the GSE using MS-GARCH models and its Bayesian counterparts. The majority of the work done in Ghana on the GSE concerning volatility modelling and forecasting is limited to ARCH and GARCH specifications. The MS-GARCH and its Bayesian models introduced in this study have the added advantage of addressing the frequent structural breaks in the volatility dynamics of financial assets.

1.7 Limitations of the Study

There are several companies listed on the GSE however due to the constraints of time, only stock returns of GCB were considered which could possibly narrow down the scope of the study. Also, a single Markov chain was used in assessing the convergence of the parameters under the Bayesian technique which could serve as limitation to the study.

1.8 Organisation of the Study

The study is organised into five chapters. The first chapter is made up of the background of the study, problem statement, research objectives and questions, the significance of the study, limitation of the study and organisation of the study.

Chapter two constitutes the literature review. This chapter reviews relevant studies conducted by several writers and authors in the area of finance with respect to the stated objectives.

Chapter three comprises of the research methodology. This chapter outlines the various ways and methods utilised in achieving the research objectives and questions.

Chapter four discusses the findings of the results presented and lastly, the summary of findings, conclusions and recommendations form chapter five of the study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

In this chapter, several previous studies on financial applications of GARCH and MS-GARCH models and their variants were reviewed and discussed respectively. Similarly, classical MS-GARCH and Bayesian MS-GARCH models were also delved into and discussed.

2.1 Application of GARCH Models in Finance

The past few decades have witnessed researchers apply financial statistical models to address volatilities subject to financial time series variables. Due to the complexities of financial time series, several models are being developed and others modified from existing ones with the quest to examine the fit and performance of such models under the area of financial applications.

Bollerslev (1986) pioneered the GARCH model as an extension to the ARCH model. This model has developed in time over the past years and is generally accepted for measuring volatility just as it is the case of the ARCH model masterminded by Engle (1982). The GARCH model can respond to positive and negative shocks differently which is seen to address such deficiency found to be associated with the ARCH model. Several GARCH models exist with each model having to address a specific purpose in financial time series.

Engle and Bollerslev (1986) in their study developed the Integrated GARCH process to include integration properties and asymmetric GARCH model permitting the modelling of data associated with asymmetric effects of positive and negative shocks (Engle, 1990).

Nelson (1991) introduced the Exponential GARCH (E-GARCH) model that has grown in time towards modelling asymmetries to examine the relationship between return and volatility. Glosten et al. (1993) in their study found the relevance of asymmetry triggered by good and bad news in a volatile series. They suggested including past positive and negative innovations with identity function that resulted in the conditional variance following different processes owing to asymmetry. The finding from their study directed towards the issue of negative shocks having a larger impact on volatility. Thus, their finding can be interpreted to mean bad news have a larger influence in comparison to the conditional dynamics of volatility followed after good news. Because of this, various asymmetric GARCH models have expanded in the literature.

Franses and Dijk (1996) resorted to the application of linear (GARCH) model and two other non-linear modifications of the GARCH models (that is the Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) and Quadratic GARCH (Q-GARCH)) that describe stock market indices possessing the features of negative skewness. The forecasting performance of the weekly stock market indices showcased the Q-GARCH model significantly could result in an improvement on the GARCH model and best for the estimation of sample data that do not exhibit features of extreme observations (outliers). The study also as part of the recommendation indicated the GJR-GARCH model not to be utilised for forecasting purposes.

Chong et al. (2002) made use of the GARCH models and their various modifications which aided in investigating the volatility of currency exchange rates. The authors resorted to the approach of Maximum Likelihood in estimating the parameters of the models, estimation of the within-sample assessed using several measures of fit statistics, and the Mean Square Error utilised to assess the accuracy of both the out-of-sample and one-step-ahead forecasts. The study revealed volatility persistency in the exchange rate. The study supported the usefulness of the application of GARCH models in estimating the within-sample and in furtherance, at least the within-sample found to be rejected for the constant variance model. Also, the fit statistics (that is Q-statistic and Lagrange Multiplier) test showcased that preference should be given to long memory GARCH model instead of short memory and high order ARCH model. Besides, inconsistent outcomes were obtained based on the various goodness-of-fit statistics where the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) recommended the GARCH models as the best towards modelling within-sample while the Mean Square Error supported the GARCH in Mean (GARCH-M). However, the out-of-sample and one-step-ahead forecasts, the stationary GARCH-M model performed better than the other GARCH models.

Garcia et al. (2003) adopted GARCH models to forecast day-ahead electricity prices. The models utilised in their study relied on historical data of both the Spanish and California markets to obtain the forecasts of the twenty-four clearing prices for the subsequent day. The study found the adopted predicted technique to perform better.

The accuracy of these models was tested via statistical measures such as Mean Week Error and Forecast Mean Square Error. It became evident that results were quite reasonable since the errors associated with the model was not above the threshold of 10% of which the average errors were 7% for the Spanish market and almost close to 4% for the California market.

Assis et al. (2010) made use of the different univariate time series models in comparing the forecasting performance of Bagan Datoh cocoa beans prices. The time series models considered in the study were Exponential Smoothing, ARIMA, GARCH and the mixed ARIMA/GARCH. The model selection criteria's such as Root Mean Squared Error, Mean Absolute Percentage Error, Mean Absolute Error and Theil's Inequality or U-Statistics) served as the basis to determine the best forecasting model. The study indicated that the series under consideration was influenced by a positive linear trend factor and the results from the regression test presented the non-existence of factors of seasonality. Also, the study from the ex-post forecasting found the GARCH model to outperform all the other types of models.

Aikaeli (2007) considered studying the dynamics of money and inflation in Tanzania by narrowing down on the time lag between changes in money supply and inflation rate response. The study focused on the application of seasonally adjusted monthly data covering the period 1994-2006. The GARCH model via the Maximum Likelihood Estimation (MLE) method was devised towards investigating the relationship between inflation and money and the significance of the lagged value. The estimates of the GARCH model were considered appropriate and the forecast horizon of seven months ahead indicated that a change in money supply tends to affect the rate of inflation substantially.

In Bulgaria, Patev et al. (2009) modelled and forecasted volatility with a focus on thin emerging stock markets. The study employed varied models that are Risk Metrics, Exponential Weighted Moving Average (EWMA) with t conditional distribution and EWMA with Generalised Error Distribution (GED) conditional distribution. The authors concluded in the study that, EWMA with t innovations and EMWA with GED innovations passably estimate the risk of the Bulgarian stock market.

In a much more recent study, Usman et al. (2017) also employed different specifications of the GARCH models to study the Nigerian stock market. The study fitted eleven competing GARCH models for the returns data covering the period of January 1996 to December 2015. Based on the fit statistics (Log-Likelihood (LL), Schwarz Bayesian Criterion and AIC), the identified models in the study were not the same by subjecting the data to training and testing periods. The identified model under the training period was the Component GARCH (1, 1) and E-GARCH (1, 1) and that of the testing period was the ARCH (1) and GARCH (2, 1). In furtherance, the study found the extreme model classes (E-GARCH (1, 1) and GARCH (2, 1)) to denote the best and worst groups respectively.

Maqsood et al. (2017) applied different GARCH models in estimating volatility for the daily returns of the Kenyan Stock Market specifically the Nairobi Securities Exchange. In the study, both asymmetric and symmetric models were utilised to capture the presence of these stylised facts (leverage effect and volatility clustering) commonly associated with the stock markets. The finding from the study found the volatility process to be highly persistent suggesting evidence of risk premium existence for the index return. The finding ascertained thereby supported the hypothesis of a positive correlation between volatility and expected stock returns. The study also justified the presence of leverage effects in the series of the Nairobi Securities Exchange by showcasing a better fit of the asymmetric GARCH models to the empirical data as compared to the symmetric GARCH models.

Abonongo et al. (2016) studied some stylised facts specifically asymmetry and persistent stock returns on the GSE. They employed a variant of the GARCH model known as the Threshold GARCH in Mean (T-GARCH-M (1, 1)) model and half-life measure of daily returns from the period 02/01/2004 to 20/12/2014. The findings in their study showcased all stocks exhibited volatility persistency (explosive process). Also, extending this volatility persistency through using the half-life measure of the stocks, except Fan Milk Limited almost all the stocks had strong mean reversion and short half-life measure. They posited in the study that all returns displayed a positive leverage effect parameter confirming that bad news affected volatility than the good news of equal magnitude.

In another study, Li et al. (2018) developed the first order Zero-Drift GARCH (ZD-GARCH (1, 1)) model. The study by the authors considered extending the classical GARCH (1, 1) model by getting rid of the drift term in the conditional variance equation. The newly developed model (ZD GARCH (1, 1)) can study both heteroscedasticity and conditional heteroscedasticity, and with an interesting feature of always non-stationary irrespective of the sign accompanying the Lyapunov exponent, unlike the classical GARCH (1, 1) model. However, the study went further to indicate that the stability of the model is seen if the Lyapunov exponent is zero hence over time, the sample path is expected to oscillate randomly between zero and infinity. The finite sample performance of the developed estimators and tests was obtained for the stable ZD-GARCH (1, 1) model, and compared to the non-stationary classical GARCH (1, 1) model via simulations. Evidence from the comparative study finds the stable ZD-GARCH (1, 1) model more suitable to the non-stationary classical GARCH (1, 1) model.

Several researchers have indicated that GARCH models can be utilised rapidly for most financial time series, however, the deficiencies in these models have also been studied. For instance; Perez-Quiros and Timmermann (2001) in their study of stock returns asymmetries under the business cycle concentrated on the conditional distributions of financial returns and indicated that recessionary and expansionary period have different characteristics, though the parameters of a GARCH model are assumed to be constant for the entire period. Similarly, other studies pinpointed the high volatility persistence coupled with standard GARCH models due to structural breaks in the variance (Diebold, 1986; Lamoureux and Lastrapes, 1990; Bauwens et al., 2014) affecting the precision of volatility forecast as a result of structural changes in the volatility dynamics of financial assets. To address these drawbacks of structural breaks in the volatility dynamics with the GARCH model have led to researchers proposing regime switches.

2.2 Application of MS-GARCH Models in Finance

More recently, literature has focused attention on MS or Regime-Switching (RS) models with the view of characterising financial time series properties in different regimes.

Hamilton (1989) introduced the MS-GARCH model as an extension to the GARCH model and its variants with the view of addressing regime changes in the conditional variance dynamics of time series. Hamilton found it expedient to develop the model to describe the United States business cycle which was found to be characterised by periodic shifts from recessions to expansions and vice versa.

Cai (1994) and Hamilton and Susmel (1994) applied the original idea of RS parameters developed by Hamilton (1988, 1989 and 1990) into an ARCH specification such that possible structural breaks can be accounted for. In their respective studies, the problem of infinite path dependence was avoided as a result of the use of ARCH specification rather than GARCH specification.

Furthermore, Ramchand and Susmel (1998) found it practical to examine the connection between correlation and variance in a RS-ARCH model. They relied on weekly stock returns data for the United States and a few other major markets to examine such a relationship. They established in their study that correlations between the United States and other world markets are 2 to 3.5 times higher when the United States market is in a state of high variance.

Klaassen (2002) focused on improving GARCH volatility forecasts with RS-GARCH by resorting to about twenty years of daily data on three United States dollar exchange rates. In the study, volatility persistence was evident and to address that, a more flexible GARCH model that distinguishes two regimes with different levels of volatility; GARCH effects were not restricted within each regime. Also, Klaassen (2002) indicated in the study that RS-GARCH produces significantly better forecasts of volatility as compared to Single-Regime GARCH (SR-GARCH). The relative outperformance was quantified to be 22% and 58% for the one-day and ten-day horizon respectively.

In a much recent study, Ang and Bekaert (2002a) focused on international asset allocation using RS models. They considered bivariate and trivariate RS models that can capture asymmetric correlations in volatile and stable markets and typified a United States investor's optimal asset allocation under constant relative risk aversion. The findings from their study indicated that the effect of doing away with regimes is trivial, however; increase conditionally when a risk-free asset can be held.

Marcucci (2005) focused on modelling and forecasting United States stock return precisely Standard and Poor (S&P) 100 by employing Markov Regime-Switching GARCH (MRS-GARCH) models and standard GARCH models. In the study, parameters in the model were permitted to switch between regimes of low and high volatility. His study went further to subject the residuals of the conditional distribution on the assumption of fat-tailed and Gaussian, and a state-dependent degree of freedom adopted to model time-varying kurtosis. The empirical analysis from the study showed that for a broad collection of statistical loss functions, the MRS-GARCH models performed better than all standard GARCH models in forecasting volatility at shorter horizons and with longer horizons, standard asymmetric GARCH models performed best.

Sajjad et al. (2008) estimated VaR for both long and short positions of S&P500 and Financial Times Stock Exchange (FTSE) 100 using a MS-GARCH model of asymmetry. The model was seen as an improvement on already existed VaR methods through the incorporation of both regime change and skewness or leverage effects. For instance; Sajjad et al. (2008) from exceptions and regulatory-based tests point to the fact that VaR estimation for both long and short FTSE100 positions using the MS-GARCH specifications performed better than other models and as well do quite well for the positions of S&P500. They concluded in their study that disregarding skewness and regime changes have the repercussion of imposing larger than necessary conservative capital requirements.

Chang et al. (2008) and Haas (2010) in their studies permitted different distributions with the focus of achieving forecast accuracy. Finding relevant from their studies is that, in terms of forecast accuracy, the return series can be modelled efficiently by permitting regime densities to follow a skew-normal distribution with Gaussian tail features.

Guidolin and Timmermann (2008b) investigated the effects of international asset allocation using time-variations of higher-order moments of stock returns such as skewness and kurtosis. Findings from their study indicated that with a large number of assets, the proposition of a new tractable method towards resolving the problem of asset allocation under MS is of grave relevance.

In South Africa, Babikir et al. (2012) studied structural breaks relevance towards forecasting stock return volatility through the exploration of tests of in-sample and out-of-sample, and daily returns for the Johannesburg Stock Exchange All-Share Index from 07/02/1995 to 08/25/2010. The study found evidence of structural breaks in the unconditional variance of the stock returns, with a high degree of persistence and the GARCH (1, 1) model possessing parameter estimates variability across sub-samples defined by the structural breaks. Also, as part of the study, Babikir et al. (2012) noted that the combination of forecasts from various benchmarks and competing models that accommodate structural breaks in volatility improves the precision of volatility forecast for out-of-sample tests. Furthermore, the study showed that in terms of asymmetry associated with stock return volatility, the MS-GARCH model seems to capture better as compared to GJR-GARCH (1, 1) model which suited better for longer horizons. However, the study in totality found GARCH (1, 1) model to perform better than models of asymmetric.

Reher and Wilfling (2011) developed an integrative MS-GARCH model which had the capacity of specifying complex GARCH equations in two distinct Markov states (regimes) and modelling GARCH equations of various forms across the two Markov states (regimes). For specificity, the developed MS was flexible and could estimate E-GARCH in the former and a standard GARCH specification in the latter Markov state (regime). Reher and Wilfling (2011), as an extension to their study, derived MLE and utilised the unifying MS-GARCH model on daily surplus returns of the German stock market index known as Deutscher Aktienindex (DAX). Among others in their study, estimation outcomes confirmed the developed unifying MS-GARCH model to outperform all accepted MS-GARCH models so far found to be estimated in financial literature. Also, their study found the German stock market to possess significant MS with considerable differing volatility structures across the states (regimes).

Zhang et al. (2015) investigated crude price volatility for various data frequencies and time horizons intending to evaluate the forecasting performance of the SR-GARCH models (specifically the standard linear GARCH and the non-linear GJR-GARCH and E-GARCH models) and the two-regime MRS-GARCH model. Finding from the study with the use of most model evaluation criteria emphasised better performance in respect of in-sample estimation for the MRS-GARCH models as compared to the SR-GARCH model, even though it showcased some form of inferiority in few cases for other evaluation criteria. Also, the finding found superiority in terms of daily data providing a more accurate forecast of volatility for the two-regime MRS-GARCH model however such a superiority wades off for weekly and monthly data. Further study on three-regime GARCH model types of the linear and non-linear GARCH type models yielded greater accuracies in volatility forecast for the latter (non-linear GARCH) models considering longer time horizons of daily data. Besides, the linear SR-GARCH model performs better overall in forecasting VaR as compared to the non-linear GARCH model forms considered in the study.

In other jurisdiction, Maiyo (2018) focused on investigating the efficiency of the MRS-GARCH model in comparison to that of the classical GARCH models by utilising prices of tea traded for some time horizon. The data used in the study was weekly covering the period 2010 to 2017. The study provided both the in-sample and out of sample forecasts for the competing models under the MRS-GARCH and the SR GARCH models respectively. Comparative analysis was achieved with the adoption of the statistical loss functions of which the in-sample forecast was performed using the goodness of fit tests while the out-of-sample was conducted on the premise of the forecast accuracy. Evidence from the study pinpointed that the high persistence associated with GARCH models can be overcome with the use of the MRS-GARCH models confirming regime clustering in the empirical data. Also, the MRS-GARCH models performed better than the SR-GARCH models for forecasting out-of-sample but this dominance is seen to disappear for a longer horizon of time.

Ardia et al. (2018) conducted a large-scale data-based study that sought to compare the forecast performances of the SR and MS-GARCH models in the area of risk management. The model developed in their study yielded more accurate VaR, ES and left-tail distribution forecasts than their SR counterparts.

Caporale and Zekokh (2019) modelled on four most populous cryptocurrencies (Bitcoin, Ethereum, Ripple and Litecoin) through the estimation of several GARCH models to the log-returns of these exchange rates under consideration. In the study, a rolling window basis was applied in estimating the one-step-ahead VaR and ES respectively; substantive models selected for these risk analysis (VaR and ES) through the backtesting approach and Model Confidence Set adopted for the statistical loss functions. The findings affirmed that standard GARCH may produce inaccurate VaR and ES and hence recommended mitigating these inaccuracies in risk analysis by allowing asymmetries and RS to be incorporated into the standard GARCH model.

In other studies, Shi (2020) developed the Markov Regime-Switching Zero-Drift GARCH (MRS-ZD-GARCH) model as an extension to the ZD-GARCH model of Li et al. (2018). This model addressed the issue of stability in the ZD-GARCH model due to the presence of structural changes. To this effect, Shi (2020) in the model development of MRS-ZD-GARCH, permitted regime switches within the framework of the ZD-GARCH. The estimators of the new model were derived, and the stability test also conducted using simulation studies for with and without the presence of structural changes using three stocks (S&P500, NASDAQ and Apple returns). The result from the test finds the MRS-ZD-GARCH model to perform better than the ZD-GARCH model.

In a recent study, Nunian et al. (2020) focused on modelling quarterly exchanges rates of Singapore Dollar (SGD), Korean Won (KRW), China Yuan Renminbi (CNY), Japanese Yen (JPY) and the United States Dollar (USD) against Malaysia Ringgit (MYR). The empirical data was modelled through the utilisation of MS and MS-GARCH models and the analysis took the form of a comparative study. Based on the model selection criteria's (LL, AIC and BIC), the MS was favoured and considered the best model. The results found JPY and SGD with probabilities of 0.96 and 0.84 respectively to be associated with a highly persistent trend on the first regime while CNY, KRW and USD with probabilities of 0.99, 0.95 and 0.82 respectively to be accompanied with high persistent trends on the second regime.

Zolfaghari and Hoseinzadi (2020) employed a variant of MRS-GARCH models with different innovations to measure the impact of exchange rate uncertainties on the Industry Index Return of the Tehran Stock Exchange (TSE) over six years (2013-

2019). The study modelled Industry Index Return by differentiating between two different regimes for both the conditional mean and variance. From the study findings, MRS E-GARCH-M under the conditional distributions (GED and student-t) performed best towards modelling Industry Index Return volatility. Also, the study by Zolfaghari and Hoseinzadi (2020) showcased evidence of Iran's stock market possessing RS behaviour. In furtherance, the findings through the adoption of the Autoregressive Distributed Lag model established that fluctuations in foreign exchange rates tend to impact significantly and distinct on the uncertainty of Industry Index Return across the different regimes.

2.3 Classical and Bayesian MS-GARCH Models

The past few years have witnessed researchers apply the Bayesian approach in numerous fields such as finance (Chen et al., 2009); agriculture (Shiferaw, 2018); energy (Billio et al., 2018) among others. This approach can be developed and estimated for volatility models. Carlin et al. (1992) utilised state-space models in the Bayesian framework paving way for literature to enlarge in time. Jacquier et al. (1994) were the first to pioneer the Bayesian MCMC algorithm to study SV models. Gweke (1994) in a study made use of Bayesian importance sampling Monte Carlo Method. Chib and Greenberg (1995) used the independence chain Metropolis-Hastings algorithm and accept-reject M-H algorithm methods which helped in the simulation of volatility because the conditional density of volatility was not in standard form.

The focus of this study sought to compare the classical and Bayesian approach toward modelling stock returns using risk models of which several researchers have delved into. For instance; Ardia (2009) proposed a RS threshold asymmetric GARCH model of specifications based on MS with student-t innovations and K separate Glosten-Jagannathan Runkle (GJR (1, 1)) processes whose asymmetries are located at free non-positive threshold parameters. The study also proposed a novel MCMC scheme that permitted an automatically full Bayesian estimation. Ardia (2009) in his study concludes no differences in the results of the posterior concerning asymmetries and their thresholds when periods of high volatility are compared with milder ones. Also, comparisons with that of the SR specification demonstrated a finer in-sample fit and that of a forecasting performance for the MS model.

Furthermore, Chen et al. (2009) employed a MS heteroscedastic model with a distribution of fat-tailed error to study the asymmetric effects conditioned on both the mean and volatility of financial time series. They proposed a model that simultaneously combines MS in the mean and variance because, with the MS-GARCH model, the switching variable is assumed to be an unobservable first-order Markov process. Competing models were developed and compared using the Bayesian VaR forecast. The results in the study suggested the proposed Double MS-GARCH model with exogenous variables was favourable.

In another study, Chen et al. (2012) forecasted Pre and Post Global Financial Crisis VaR through a computational Bayesian framework with an Adaptive MCMC method. Comparative analysis of the parametric models was considered including the standard, threshold nonlinear and MS-GARCH specifications. Furthermore, standard and nonlinear SV models with a focus on four error probability distributions: Gaussian, Student-t (ST), Skewed-t (SST) and GED were equally considered. The findings from the study indicated in almost all instances, GARCH models performed better than SV models. Also, pre-crisis supports the use of asymmetric volatility models; while at the 1% level of pre and post-crisis, models with the skewed-t errors were ranked best for a one (1) day horizon, while at the 5% level Integrated GARCH models were favourable. Also, all models used forecast VaR less accurate and non-conservatively post-crisis.

Bauwens et al. (2011) made inferences into GARCH models by subjecting to an unknown number of structural breaks at unknown dates. They based the study on a method of differential evolution MCMC to make inferences on model estimation and forecasting. The study also used simulations and comparisons with existing algorithms of MCMC to demonstrate the rapidity and efficiency of the algorithm of Discrete Differential Evolution Adaptive Metropolis (D-DREAM) through the application of seven financial time series daily returns purposely to ascertain the presence of structural breaks. The study concluded that all the studied series exhibit at least three structural breaks. However, the study still confirmed structural breaks but less under student-t innovations. Besides, an assessment of the forecasting ability using Copula-GJR-GARCH models with and without recurrent regimes showcased better forecasts than the fixed parameter GJR-GARCH model in about 25% of cases.

In a recent study, Boonyakunakorn et al. (2019) utilised Bayesian MS-GARCH models in VaR estimation to inquire into the forecasting ability of volatility of the Stock Exchange of Thailand (SET) return. Specifically, their studies utilised the Deviance Information Criterion (DIC) to reach a reasonable conclusion. They found the Bayesian two regime MS-GJR-GARCH models with a GED to fit best to the empirical data based on the DIC. The model supported the fact that the two regimes are different in both unconditional volatility levels and persistence of the volatility process. The results from VaR backtesting at the 5% level of risk corroborated that Bayesian two regime MS-GARCH models perform better than their SR counterpart. The statistics obtained from their study generally goes to confirm that Bayesian two regime MS-GARCH models are found to improve the forecasting ability of SET volatility.

Nkemnole and Ebomese (2019) utilised methods of Maximum Likelihood and MCMC under the classical and Bayesian frameworks respectively to obtain parameter estimates of MS-GARCH models for a single regime, second regime and three regimes. The study made use of monthly exchange rate data of Bureau de Change for 18 years to forecast volatility. Also, the study adopted an information criteria mechanism under the classical and Bayesian frameworks to ascertain the best performing models. It was apparent that based on the Maximum Likelihood of the classical approach, the three RS-GARCH models performed best as compared to the single and two RS models. Also, the MCMC estimation under the Bayesian framework established evidence of the two RS model to outperform the other RS models.

2.4 Value-at-Risk (VaR) and Expected-Shortfall (ES) Model

VaR is very crucial in modelling financial time series. Investors and stakeholders will want to measure or quantify the risk of loss for investments. Also, it can be seen to mean the odds of losing money. In the area of statistics, the upper percentile of the distribution of losses is said to mean VaR. Siaw (2014) stated a classical example of a

95% VaR to be the upper estimate of the losses which is found to be exceeded with a probability of 5%.

Yamai and Yoshiba (2002) found the methods of estimation employed for standard VaR models to have issues measuring extreme price movements. They went further to define VaR to mean a measure of the distributional quantile and disregarded extreme loss beyond the level of VaR. This means that VaR in their view may ignore relevant information concerning the tails of the causal distribution. In brief, this problem can be identified as tail risk (CGFS, 2000).

Artzner et al. (1997) addressed the problem associated with VaR through the proposition of the usage of ES. They explained ES as the conditional expectation of the loss given that the loss is beyond the level of VaR. Yamai and Yoshiba (2002c) indicated in their study that ES does not have tail risk under more relaxed conditions than VaR.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter focused on the data, methods and techniques deployed in meeting the stated research objectives and questions.

3.1 Data and Source

The data utilised in this study are the closing stock prices of GCB on the GSE which was converted into monthly stock returns. This company had enough data points that covered the period of interest. The period considered for the study ranges from 07/2009 to 12/2020 covering 138 months. The data was obtained from the GSE database (www.gse.com.gh).

3.2 Returns

The stock volatility is utilised as an indicator of the uncertainty of returns on stocks. In this respect, the volatility of financial markets is measured using the standard deviation, σ or variance, σ^2 (Lim and Sek, 2013). In this study, the variance was computed as:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \mu)^2 . \qquad (3.1)$$

From equation (3.1), define μ to be the mean return and r as the returns. It is imperative to note that a smaller variance $\left(\sigma^2\right)$ value indicates lower volatility and lower risk and vice versa.

According to Lim and Sek (2013), returns can be said to mean a total loss or gain from an investment over a specified period. In financial applications, monthly stock returns are denoted as $\{r_t\}_{t=1}^{T}$. Hence the monthly log-stock returns can be determined from the monthly stock prices p_t on month t through utilising:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right). \tag{3.2}$$

3.3 Unit Root Test

In financial time series analysis, it is very essential to determine the presence or otherwise of unit root purposely to establish the nature of the process that underlies the observed series. In this study, two unit root test that is Augmented Dickey-Fuller (ADF) and Kwiatkowski-Philips-Schmidt-Shin (KPSS) were deployed to better appreciate the nature of the process depicting the log-return series.

3.3.1 Augmented Dickey-Fuller (ADF) Test

Dickey and Fuller (1979) developed the ADF stationary test as an improvement over the Dickey-Fuller (DF) test. The ADF test employed in this study seeks to examine whether the log-return series possess unit root or equivalently, the log-return series follows a random walk. In the ADF test, a unit root process and a stationary process respectively is given as:

$$\begin{aligned} \rho_t &= \phi_1 \rho_{t-1} + \varepsilon_t \\ \rho_t &= \phi_0 + \phi_1 \rho_{t-1} + \varepsilon_t \end{aligned} . \tag{3.3}$$

The hypothesis for the ADF test is given by:

$$\begin{aligned} H_0 &: \phi_1 = 1 (\textit{imply } \log - \textit{return series possess a unit root and is not stationary}) \\ H_1 &: \phi_1 \neq 1 (\textit{imply } \log - \textit{return series does not possess a unit root and is stationary}) \end{aligned} .$$

The test statistic associating the test of the hypothesis is also given by:

$$ADF = \frac{\hat{\phi}}{SE\left(\hat{\phi}\right)}, \tag{3.4}$$

where $\hat{\phi}$ is the estimate of ϕ and $SE\left(\hat{\phi}\right)$ is the standard error least-square estimate of $\hat{\phi}$. It is important to note that if the test statistic of the ADF test is greater than the critical value, we reject the null hypothesis that log-return series possess unit root.

3.3.2 Kwiatkowski-Philips-Schmidt-Shin (KPSS) Test

Another test of stationarity is the KPSS test proposed by Kwiatkowski et al. (1992). The test of the hypothesis is seen as a complement of the ADF test. The test of the null and alternative hypotheses of the data generating process is given respectively as:

$$\begin{aligned} H_o &: I(0) \ (\log - \textit{return series is stationary}) \\ H_1 &: I(1) \ (\log - \textit{return series is not stationary}) \end{aligned} .$$

The test statistic for the KPSS test is stated as:

$$KPSS = \frac{1}{T^2}\sum_{t=1}^{T}\frac{S_t^2}{\hat{\sigma}_\infty^2}, \tag{3.5}$$

where T is the number of observations, $S_t = \sum_{j=1}^{t}\left(Y_t - \bar{Y}\right)$ and $\hat{\sigma}_\infty^2$ is an estimator of the long-run variance of the white noise process. The test is seen to be rejected under the null hypothesis for large KPSS values.

3.4 Model Diagnostics

The usage of any model to conclude or make an informed decision require that; it is subjected to model diagnostic to verify whether it conforms to real-world situations. Hence, some of the model diagnostic approaches adopted in this study is the trace plots, density plots and ACF plots.

3.5 Single-Regime GARCH (SR-GARCH) Models

The first framework to be utilised in modelling volatility is the ARCH model developed by Engle (1982) and is given by:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2, \tag{3.7}$$

for $k = 1, 2, ..., K$. In this case, we define the parameter $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}\right)^T$. In achieving the condition of positivity, then set $\alpha_{0,k} > 0$ and $\alpha_{1,k} \geq 0$ while covariance-stationary in each regime is attainable if and only if (iff) $\alpha_{1,k} < 1$. Also from equation (3.7), $h_{k,t}$ can be seen to be the conditional variance, y_{t-1}^2 is the past squared error term (past squared monthly return residuals). The ARCH model is noted for its failure to respond to positive and negative shocks differently.

Another extension to the ARCH model is the GARCH model introduced by Bollerslev (1986). This model reduces the number of parameters that define sufficiently the volatility process in the ARCH model. The GARCH model proposed by Bollerslev (1986) is stated as:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{3.8}$$

for $k = 1, 2, ..., K$. In this instance, we define the parameter $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \beta_k)^T$. In obtaining the positivity condition, it is imperative to set $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$ and $\beta_k \geq 0$ while stationary condition associated with the covariance in each regime is attainable iff $\alpha_{1,k} + \beta_k < 1$. Also from equation (3.8), let $h_{k,t}$ be the conditional variance, $h_{k,t-1}$ as lagged conditional variance and y_{t-1}^2 as the past squared error term (past squared monthly return residuals).

Nelson (1991) pioneered the E-GARCH model to capture some stylised facts in financial time series besides leptokurtic returns such as asymmetric effects between positive and negative asset returns which is a limitation of the GARCH model. This means that the E-GARCH model specification considers the leverage effect where past negative observations tend to have a larger impact on the conditional volatility than past positive observations of the same magnitude. In the view of Nelson (1991), the E-GARCH model is given by:

$$\ln(h_{k,t}) = \alpha_{0,k} + \alpha_{1,k} \left(|\eta_{k,t-1}| - E\left[|\eta_{k,t-1}| \right] \right) + \alpha_{2,k} \eta_{k,t-1} + \beta_k \ln(h_{k,t-1}), \tag{3.9}$$

for $k = 1, 2, ..., K$, where the expectation, $E\left[\left|\eta_{k,t-1}\right|\right]$ is taken with respect to (wrt) the distribution conditional on the regime (state) k. In this case, let the parameter $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^T$ and stationarity condition of the covariance in each regime be achievable iff $\beta_k < 1$. Also define $h_{k,t}$ to be the conditional variance, $h_{k,t-1}$ as the lagged conditional variance, $\eta_{k,t-1}$ to depict the underlying conditional distribution (such as ST, SST, GED and GED) and $\alpha_{2,k}$ to depict the degree of asymmetry. If $\alpha_{2,k} = 0$ is an indication of a perfect symmetric model while if $\alpha_{2,k} < 0$ indicate that past negative returns influences conditional volatility much more as compared to past positive returns of the same magnitude. It is imperative to note that the E-GARCH model specification does not need parameters restriction since the equation is on the log variance rather than the variance itself, the positivity of the variance is automatically ensured.

The GJR-GARCH model was proposed by Glosten et al. (1993) as an additional volatility model that handles leverage effects in financial time series. They further stated the ability of the GJR-GARCH to capture asymmetry in the conditional volatility process and hence the model is given as:

$$h_{k,t} = \alpha_{0,k} + \left(\alpha_{1,k} + \alpha_{2,k} I\left\{y_{t-1} < 0\right\}\right) y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{3.10}$$

for $k = 1, 2, ..., K$, where $I\{\bullet\}$ is the indicator function which assumes a value of unity if the condition holds and zero otherwise. In this case, define $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^T$ as parameters of the GJR-GARCH model, $h_{k,t}$ as the conditional variance, $h_{k,t-1}$ as the lagged conditional variance and y_{t-1}^2 as the past squared error term (past squared monthly return residuals). From equation (3.10), the extra parameter $\alpha_{2,k} \geq 0$ is seen to control the degree of asymmetry concerning the conditional volatility response to the past shock in regime k.

Also, Zakoian (1994) proposed the T-GARCH model with the conditional standard deviation (volatility) as the dependent variable rather than the conditional variance. The T-GARCH model can be written as:

$$h_{k,t}^{1/2} = \alpha_{0,k} + \left(\alpha_{1,k} I\left\{y_{t-1} \geq 0\right\} - \alpha_{2,k} I\left\{y_{t-1} < 0\right\}\right) y_{t-1} + \beta_k h_{k,t-1}^{1/2}, \qquad (3.11)$$

for $k = 1, 2, ..., K$. In this instance, define the parameter $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^T$. Also define $h_{k,t}^{1/2}$ as the conditional standard deviation, $h_{k,t-1}^{1/2}$ as the lagged conditional standard deviation, y_{t-1} as the past error term (past monthly return residuals) and $I\{\bullet\}$ as an indicator function. From equation (3.11), it can be realised that we require setting $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} > 0$ and $\beta_k \geq 0$ to attain positivity.

Furthermore, in each regime (state), covariance-stationary can be obtained by setting

$$\alpha_{1,k}^2 + \beta_k^2 - 2\beta_k\left(\alpha_{1,k} + \alpha_{2,k}\right) E\left[\eta_{t,k} I\left\{\eta_{t,k} < 0\right\}\right] - \left(\alpha_{1,k}^2 - \alpha_{2,k}^2\right) E\left[\eta_{k,t}^2 I\left\{\eta_{k,t} < 0\right\}\right] < 1$$

(Francq and Zakoian, 2011).

3.6 MS-GARCH Model

Financial stock returns tend to possess sharp peaks and thick tails and showcasing asymmetric effects and variability in their price volatilities. These characteristics can be subjected to the conditional volatility of the returns which are time-varying in nature. Also, studies have maintained the high persistence of the GARCH model attributable to regime shifts in the parameters over a period (Diebold, 1986; Lamoureux and Lastrapes, 1990; Mikosch and Starica, 2004).

This high persistence in GARCH model parameters motivates this study to apply a MS-GARCH or RS-GARCH model which allows regime-switches in the latter model parameters. Consider that y_t is a variable of interest at say time t having a mean of zero (0) and also serially uncorrelated, then the moment conditions hold: $E(y_t) = 0$ and $E(y_t y_{t-l}) = 0$ for $l \neq 0$ and $\forall\ t > 0$. Also, consider I_{t-1} being a set of information observed/witnessed up to time $t-1$. This implies that $I_{t-1} = \{y_{t-i}, i > 0\}$. Hence an expression for the specification of the general MS-GARCH can be stated as:

$$y_t \mid (s_t = k,\ I_{t-1}) \sim D(0,\ h_{k,t},\ \xi_k), \tag{3.12}$$

with standardised innovations $\eta_{k,t} = y_t / h_{k,t}^{1/2}\ \underset{\sim}{iid}\ D(0,1,\xi_k)$.

From equation (3.12), $D(0,\ h_{k,t},\ \xi_k)$ is a distribution that is continuous with zero means, $h_{k,t}$ is a varying time variance and additional shape parameters (for instance, asymmetry) gathered in the vector ξ_k. Moreover, assume unobservable (latent) variable s_t, defined on the discrete space $\{1, 2, ..., K\}$, that evolves according to an unobserved first-order homogenous Markov chain with matrix transition probability $\mathrm{P} = \{p_{i,j}\}_{i,j=1}^{K}$ or in matrix form for the transition probability matrix P (probabilities associated with making a switch from one regime to the other) as given below:

$$\mathrm{P}=\begin{bmatrix} p_{1,1} & \cdots & p_{1,K} \\ \vdots & \ddots & \vdots \\ p_{K,1} & \cdots & p_{K,K} \end{bmatrix}. \tag{3.13}$$

From the above matrix, the probability of transition from state $s_{t-1}=i$ to state $s_t=j$ can be determined from the relation $p_{i,j}=\mathrm{P}\left[s_t=j \mid s_{t-1}=i\right]$. The following constraint hold for the transition probability matrix: $0<p_{i,j}<1\ \forall\ i, j\in\{1,2,...,K\}$ and $\sum_{j=1}^{K} p_{i,j}=1, \forall\ i\in\{1,2,...,K\}$. Given the parametrisation of $D(\bullet)$, the variance of y_t conditional on the realisation of $s_t=k$ and the information set I_{t-1} is determined by squaring and blowing expectation through for equation (3.12) yielding;

$$E\left[y_t^2 \mid s_t=k,\ I_{t-1}\right]=h_{k,t}. \tag{3.14}$$

Haas et al. (2004) noted in their study that the variance conditioned on y_t is assumed to follow a model of a GARCH-type. Specifically, conditional on the state (regime) $s_t=k$, then $h_{k,t}$ is with a specification expressible as a function of past returns that is y_{t-1}, past variance which is $h_{k,t-1}$ and the additional regime-dependent vector of parameters θ_k:

$$h_{k,t}=h\left(y_{t-1}, h_{k,t-1}, \theta_k\right). \tag{3.15}$$

From equation (3.15), $h(\bullet)$ is a $I_{t-1}-$measurable function defining the filter for the conditional variance as well as also ensuring its positivity. Basing on the assumption that $h_{k,1}=\bar{h}_k\ (k=1,2,...,K)$, where $\bar{h}_k$ is defined as the fixed initial variance level for the regime (state) k, hence setting the fixed initial variance level of the regime (state) k equal to the unconditional variance in the regime k, then different scedastic specifications can be obtained depending on the form of $h(\bullet)$. For instance, consider that;

$$h_{k,t} = \omega_k + \alpha_k y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{3.16}$$

where $\omega_k > 0,\ \alpha_k > 0,\ \beta_k \geq 0$ and $\alpha_k + \beta_k < 1\ (k = 1, 2, ..., K)$, then the MS-GARCH (1, 1) model presented by Haas et al. (2004) is obtained. In this light let the parameter $\theta_k = (\omega_k,\ \alpha_k,\ \beta_k)'$. On this note, the MS-GARCH model can be deduced to mean a Markov chain with a transition kernel which is a mixture of distributions.

3.6.1 Conditional Distributions

The specification associated with a model becomes complete through the definition of the conditional distribution of the standardised innovations in each regime of the MC. The Normal distribution is limited in line with its application due to the inability of the distribution to address stylised facts in financial time series. In this respect, the most common distributions utilised to model financial log returns are the Student-t (ST), GED, Skewed student-t (SST) and Skewed GED (SGED). These distributions are standardised to have their expected value and variance to be zero (0) and unity (1) respectively. The standardised ST distribution has its Probability Density Function (PDF) given by:

$$f_{ST}(\eta;\upsilon)=\frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{(\upsilon-2)\pi}\,\Gamma\left(\frac{\upsilon}{2}\right)}\left(1+\frac{\eta^2}{(\upsilon-2)}\right)^{-\frac{\upsilon+1}{2}}, \quad \eta\in\mathbb{R}, \tag{3.17}$$

where $\Gamma(\bullet)$ is the Gamma function, υ is the degree of freedom (shape) parameter and η is the standardised normal random variable. It is worth noting from equation (3.17) that the second-order moment is existential iff $\upsilon>2$. The kurtosis of the ST distribution is seen to be higher for lower υ. The PDF of ST is symmetric, and the degrees of freedom will help determine distribution at the tails.

Furthermore, the standardised GED has a PDF expressed as:

$$f_{GED}(\eta;\upsilon)=\frac{\upsilon\exp\left\{-\frac{1}{2}\left|\eta/\lambda\right|^{\upsilon}\right\}}{\lambda 2^{\left(1+1/\upsilon\right)}\Gamma\left(1/\upsilon\right)}, \quad \lambda=\left\{\frac{\Gamma\left(1/\upsilon\right)}{4^{1/\upsilon}\Gamma\left(3/\upsilon\right)}\right\}^{1/2}, \quad \eta\in\mathbb{R}. \tag{3.18}$$

From equation (3.18), $\Gamma(\bullet)$ is the Gamma function, η is the standardised normal random variable, λ is a constant and the shape parameter (υ) satisfying the condition of $\upsilon>0$. The GED is noted to be a distribution that is symmetric and defines three parameters which is the mode of the distribution, dispersion of the distribution and the shape parameter ensuring the control of skewness.

In addition, Hansen (1994) defined the SST density and with this approach corresponding to a re-parametrisation that ensured that the distribution exhibit a zero (0) mean and unit (1) variance. The density for the SST is given as:

$$f_{SST}\left(z\,|\,\eta,\xi\right)=\begin{cases} bc\left(1+\dfrac{1}{\eta-2}\left(\dfrac{bz+a}{1-\eta}\right)^{2}\right)^{-\frac{(\eta+1)}{2}}, & z<-\dfrac{a}{b} \\ bc\left(1+\dfrac{1}{\eta-2}\left(\dfrac{bz+a}{1+\eta}\right)^{2}\right)^{-\frac{(\eta+1)}{2}}, & z\geq-\dfrac{a}{b} \end{cases}. \tag{3.19}$$

From the above density, define $z=\left(r_t-\mu_t\right)/\sigma_t$ with a restriction on the parameters η and ξ which sought to control the conditional distributional shape as; $2<\eta<\infty$ and $-1<\xi<1$. Also, consider defining the constants as stated in equation (3.19) to be: $a=4\xi c\left(\eta-2\big/\eta-2\right)$, $b^2=1+3\xi^2-a^2$ and $c=\Gamma\left(\eta+1\big/2\right)\Big/\sqrt{\pi(\eta-2)}\,\Gamma\left(\eta\big/2\right)$. Assuming that $\xi=0$, then the SST is seen to reduce to the ST distribution. The parameters for the degree of freedom (η) and non-centrality (ξ) controls the thickness of the tail and asymmetry of the distribution respectively.

Also, considering the PDF of the GED defined for equation (3.18), then by extension the SGED distribution can be stated as:

$$f_{SGED}\left(z\,|\,\mu,\sigma,\eta,\xi\right)=\frac{C}{\sigma}\exp\left\{-\frac{1}{\left(1-sign\left(z-\mu+\delta\sigma\right)\xi\right)^{\eta}\theta^{\eta}\sigma^{\eta}}\left|z-\mu+\delta\sigma\right|^{\eta}\right\}, \tag{3.20}$$

where defining $C=\eta\big/2\theta\,\Gamma\left(1\big/\eta\right)^{-1}$, $\theta=\Gamma\left(1\big/\eta\right)^{1/2}\Gamma\left(3\big/\eta\right)^{-1/2}S\left(\xi\right)^{-1}$, $\delta=2\xi AS\left(\xi\right)^{-1}$, $\eta>0$ and $-1<\xi<1$.

Consider defining $S\left(\xi\right)=\sqrt{1+3\xi^2-4A^2\xi^2}$ and $A=\Gamma\left(2\big/\eta\right)\Gamma\left(1\big/\eta\right)^{-1/2}\Gamma\left(3\big/\eta\right)^{-1/2}$. Also the expected value $\mu=E\left(z\right)$, σ is found to be the standard deviation associated with the random variable z, ξ is the parameter for skewness, $sign$ represent the sign function and the Gamma function as $\Gamma(a)=\int_0^{+\infty} y^{a-1}e^{y}dy$. The height and tails of the PDF are controlled by the parameter η while the parameter of skewness (ξ) exists to control the descent rate of the PDF around the mode of z.

3.7 Model Estimation

In model estimations of the MS-GARCH and mixture of GARCH models, the techniques of ML or Bayesian MCMC can be employed. The likelihood function can be used to evaluate these approaches. Consider that $\Psi=(\theta_1,\xi_1,\theta_2,\xi_2,\ldots,\theta_k,\xi_k,P)$ is a vector of model parameters then the likelihood function is given as:

$$\ell(\Psi \mid I_T)=\prod_{t=1}^{T} f(y_t \mid \Psi, I_{t-1}). \tag{3.21}$$

From equation (3.21), $f(y_t \mid \Psi, I_{t-1})$ represent the density of y_t given past observations I_{t-1}, and model parameters Ψ. Also, the conditional density y_t for a MS-GARCH is stated as:

$$f(y_t \mid \Psi, \mathrm{I}_{t-1})=\sum_{i=1}^{K}\sum_{j=1}^{K} p_{i,j}\, z_{i,t-1} f_D(y_t \mid s_t=j,\Psi,I_{t-1}). \tag{3.22}$$

From equation (3.22), $z_{i,t-1}=P[s_{t-1}=i \mid \Psi, I_{t-1}]$ denotes the filtered probability of state i at the time $t-1$ obtained via the filter of Hamilton (Hamilton and Susmel, 1994).

From equation (3.22), maximising the logarithm of the likelihood function yield the ML estimator $\hat{\Psi}$.

For the Bayesian perspective of MCMC estimation, this study made use of the approach by Ardia (2008) in building the kernel of the posterior distribution $f(\Psi \mid \mathrm{I}_T)$ through the combination of the likelihood with a diffuse (truncated) prior $f(\Psi)$. Hence building the prior from independent priors is as follows:

$$
\begin{aligned}
& f(\Psi) = f(\theta_1, \xi_1) f(\theta_2, \xi_2) \ldots f(\theta_K, \xi_K) f(P) \\
& f(\theta_k, \xi_k) \propto f(\theta_k) f(\xi_k) I\{(\theta_k, \xi_k) \in CSC_k\} \qquad (k = 1, 2, \ldots, K) \\
& f(\theta_k) \propto f_N\left(\theta_k; \mu_{\theta_k}, diag\left(\sigma^2_{\theta_k}\right)\right) I\{\theta_k \in PC_k\} \qquad (k = 1, 2, \ldots, K). \\
& f(\xi_k) \propto f_N\left(\xi_k; \mu_{\xi_k}, diag\left(\sigma^2_{\xi_k}\right)\right) I\{\xi_{k,1} > 0, \xi_{k,2} > 2\} \quad (k = 1, 2, \ldots, K) \\
& f(P) \propto \prod_{i=1}^{K} \left(\prod_{j=1}^{K} p_{i,j} \right) I\{0 < p_{i,i} < 1\}
\end{aligned}
\tag{3.23}
$$

From equation (3.23), CSC_k is defined to be the covariance-stationary condition and PC_k as the positivity condition in the regime k. $\xi_{k,1}$ is a parameter of asymmetry while $\xi_{k,2}$ is a tailed parameter of the SST distribution in the regime k. Also, define $f_N(\bullet; \mu, \Sigma)$ to be a Multivariate Normal Density with a vector of mean μ and covariance matrix Σ and $\bar{h}_k = \bar{h}_k(\theta_k, \xi_k)$ representing the unconditional variance in the regime k. In ascertaining the prior density for the transition matrix, assume on the basis that K rows are independent and follow a Dirichlet prior with all hyperparameters equal to two (Trottier and Ardia, 2016). In this study, the technique of simulation was utilised since the form of the posterior is not known (the normalising constant is not tractable numerically). Also, the study adopted Vihola (2012) adaptive random-walk Metropolis sampler in generating the MCMC draws for the posterior.

3.8 Model Selection Criteria

There is the likelihood for two or more tentative models to compete with each other and due to this; the need to rely on these criteria's to choose the appropriate model out of the lot is imperative. Because of this, the study made use of the AIC, BIC and DIC to select the best model based on the least value of these criteria's respectively. According to Akaike (1974), AIC is defined by:

$$AIC = 2k - 2\log \ell, \tag{3.24}$$

where k is the number of parameters and ℓ is the value of the likelihood function. Also, Schwarz (1978) proposed BIC and is stated as:

$$BIC = k\log(T) - 2\log(\ell), \tag{3.25}$$

where k, ℓ as defined in equation (3.24) and T is the sample size. Also, Berg et al. (2004) defined DIC in their study as:

$$DIC = \bar{D} + p_D, \tag{3.26}$$

$\bar{D} = E_{\Psi|y}\left[D(\Psi)\right] = E_{\Psi|y}\left[-2\ln f\left(y|\Psi\right)\right]$ is the posterior expectation of the deviance

and

$$p_D = \bar{D} - D(\bar{\Psi}) = E_{\Psi|y}\left[D(\Psi)\right] - D\left[E_{\Psi|y}(\Psi)\right] = E_{\Psi|y}\left[-2\ln f(y|\Psi)\right] + 2\ln f(y|\bar{\Psi})$$

measure the model complexity by the effective number of parameters (that is the difference between the mean of the posterior deviance evaluated at the posterior mean, $\bar{\Psi}$ of the parameters).

3.9 Density and Downside Risk Forecasting

Forecasting is an important tool for risk managers, investors and stakeholders in the financial sector as it helps toward predicting in advance future happenings through the reliance on current sets of stock returns. To this effect, the out-of-sample forecast of stock returns volatility is of very much importance in making sound financial decisions.

In the MS-GARCH models, the generation of a one-step-ahead density and downside risks such as VaR and ES is straightforward. In this study, following Ardia et al. (2018), the mixture of K regime dependent distributions for the one-step-ahead conditional PDF y_{T+1} is given by:

$$f\left(y_{T+1} \mid \Psi, I_T\right) = \sum_{k=1}^{K} \pi_{k,T+1} f_D\left(y_{T+1} \mid s_{T+1} = k, \Psi, I_T\right). \tag{3.27}$$

From equation (3.27), we find the mixing weights to be $\pi_{k,T+1} = \sum_{i=1}^{K} p_{i,k}\, z_{i,T}$ where $z_{i,t-1} = P\left[s_{t-1} = i \middle| \Psi, I_t\right]$ $(i = 1, 2, \ldots, K)$ are the filtered probabilities at time T .

The Cumulative Density Function (CDF) can be obtained by integrating the PDF for equation (3.27) as found below:

$$F\left(y_{T+1} \mid \Psi, I_T\right) = \int_{-\infty}^{y_{T+1}} f\left(z \mid \Psi, I_T\right) dz. \tag{3.28}$$

The predictive PDF and CDF for the classical framework are computed through the replacement of Ψ by the estimator of the ML $\hat{\Psi}$ in equations (3.27) and (3.28). In considering the Bayesian framework, the uncertainty parameter is integrated. For instance; given a sample of posterior represented as $\left\{\Psi^{[m]}, m=1,2,\cdots,M\right\}$, then the predictive PDF can be determined as:

$$f\left(y_{T+1} \mid \mathrm{I}_T\right)=\int_{\Psi} f\left(y_{T+1} \mid \Psi, \mathrm{I}_T\right) f\left(\Psi \mid \mathrm{I}_T\right) d\Psi \\ \approx \frac{1}{M} \sum_{m=1}^{M} f\left(y_{T+1} \mid \Psi^{[m]}, \mathrm{I}_T\right). \tag{3.29}$$

Also, the predictive CDF is given by:

$$F\left(y_{T+1} \mid I_T\right)=\int_{-\infty}^{y_{T+1}} f\left(z \mid I_T\right) dz. \tag{3.30}$$

3.9.1 Value-at-Risk (VaR) and Expected Shortfall (ES)

According to Jorion (2006), the quantile of the log-returns distribution at the desired horizon can be referred to as VaR and ES can be described as the loss expected when the loss exceeds the level of VaR. Ardia et al. (2018) noted that the forecast of VaR considering at time $T+1$ for a risk level α (given the set of information up to time T) then by definition;

$$VaR_{T+1}^{\alpha}=\inf\left\{y_{T+1} \in \mathbb{R} \middle| F\left(y_{T+1} \middle| I_T\right)=\alpha\right\}. \tag{3.31}$$

From equation (3.30), $F\left(y_T \middle| I_T\right)$ is the one-step-ahead CDF with evaluation in y. Also following Ardia et al. (2018), the ES forecast at the time $T+1$ is obtained as:

$$ES_{T+1}^{\alpha}=E\left[y_{T+1} \middle| y_{T+1} \leq VaR_{T+1}^{\alpha}, I_T\right]. \tag{3.32}$$

In this study, we consider risk levels of 1% and 5% for both VaR and ES respectively.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

In this chapter, the data were analysed, results presented, and discussions of the study equally taken into consideration. The analysis of the study was organised into preliminary and further analysis.

4.1 Preliminary Analysis

This section presents the descriptive statistics of the monthly stock returns data for GCB over the sample period considered in respect of the study.

The summary statistics as shown in Table 4.1 indicate that GCB recorded the minimum of -0.2367 and maximum of 0.3221 monthly stock returns for the sample period under review. GCB was also associated with a positive mean of 0.0149 for the period under consideration. This means that investors made gains in the monthly stocks of GCB. The standard deviation can be interpreted to mean that the risk level for investing in the monthly stock returns of GCB was 0.0963. The measure used in assessing the distortions from the normal distribution (skewness) was positive. This tends to confirm that GCB monthly stock return is associated with a distribution of a thicker upper tail as compared to the lower tail. This is also an indication of non-symmetric monthly stock returns hence justifying higher chances of gains than losses for investors of these monthly stocks. For instance, GCB was associated with positive skewness (0.7268) signifying a higher probability of making gains as compared to losses by investors. The excess kurtosis (1.6799) for GCB was positive. This is a confirmation of GCB monthly stock returns exhibiting a leptokurtic distribution and hence indicating more extreme outliers as compared to the Gaussian (Normal) distribution.

Table 4.1: Descriptive Statistics of Monthly Stock Returns

Statistic	**GCB**
Minimum	-0.2367
Maximum	0.3221
Mean	0.0149
Standard deviation	0.0963
Skewness	0.7268
Excess Kurtosis	1.6799

Figure 4.1 indicate that the monthly stock returns of GCB can be generally be described as volatile for the period under review. The monthly stock returns series of GCB tend to showcase some stability in the mean, but the variance suggests some periods exhibiting low or high volatility adducing to the fact of volatility clustering. This evidence of volatility clustering is seen to be characterised by the changes in the variance dynamics from one period to the other and hence suggestive of a model that can best incorporate such switches associated with these variances.

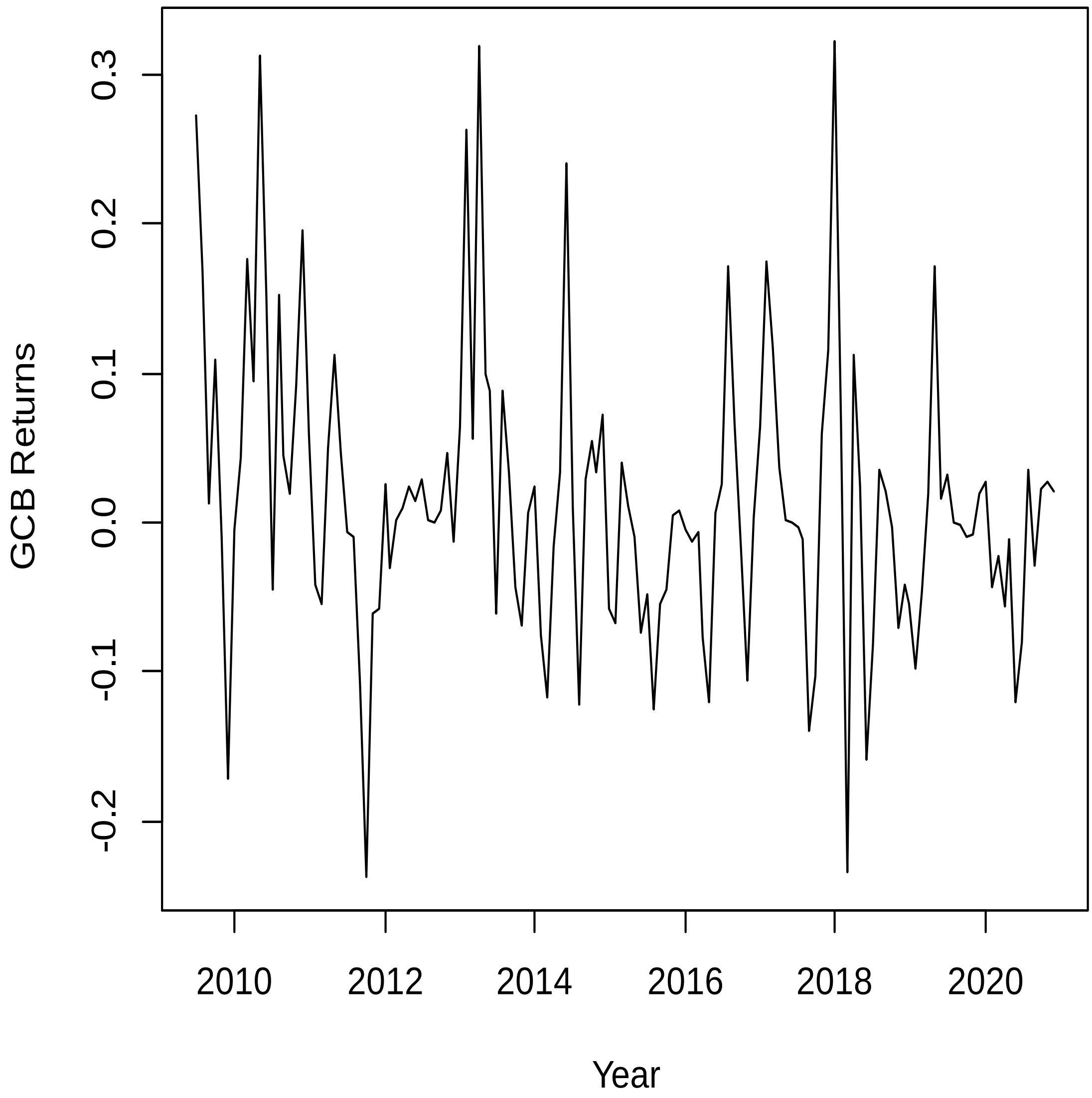

Figure 4.1: Time Series Plot of GCB Monthly Stock Returns

The test of stationarity of the monthly return series of GCB was conducted using ADF and KPSS tests respectively. It was apparent from Table 4.2 that, the p-values of the ADF test for GCB was significant at the level of 5% and hence confirming a rejection of the null hypothesis of non-stationarity. Also, in the case of the KPSS test, the p-values accompanying the GCB monthly stock returns were not significant at the 5% level of significance. This means that we fail to reject the null hypothesis of stationarity for the GCB monthly stock return series. In general, the two tests (ADF and KPSS) suggest the monthly stock return series of GCB were all stationary considering the 5% level of significance (Dickey and Fuller, 1979; Kwiatkowski et al., 1992).

Table 4.2: ADF and KPSS Test of Stationarity

	ADF Test		**KPSS Test**	
Return Series	*Statistic*	*p-value*	*Statistic*	*p-value*
GCB	–4.5099	**0.0100**	0.4315	0.0636

The ACF and PACF plots of GCB monthly stock returns shown in Figure 4.2 tend to display stationarity in the mean but with evidence of few significant spikes at some lags. For instance, the ACF plot of GCB was found to decay rapidly with evidence of the first spike displaying significance. Also, the PACF plot dies out after the first lag but with a significant spike at the first lag. This tends to suggest a tentative model building (ARMA$(0,1)$, ARMA$(1,0)$ and ARMA$(1,1)$) approach can be devised for the mean of the company's monthly stock returns.

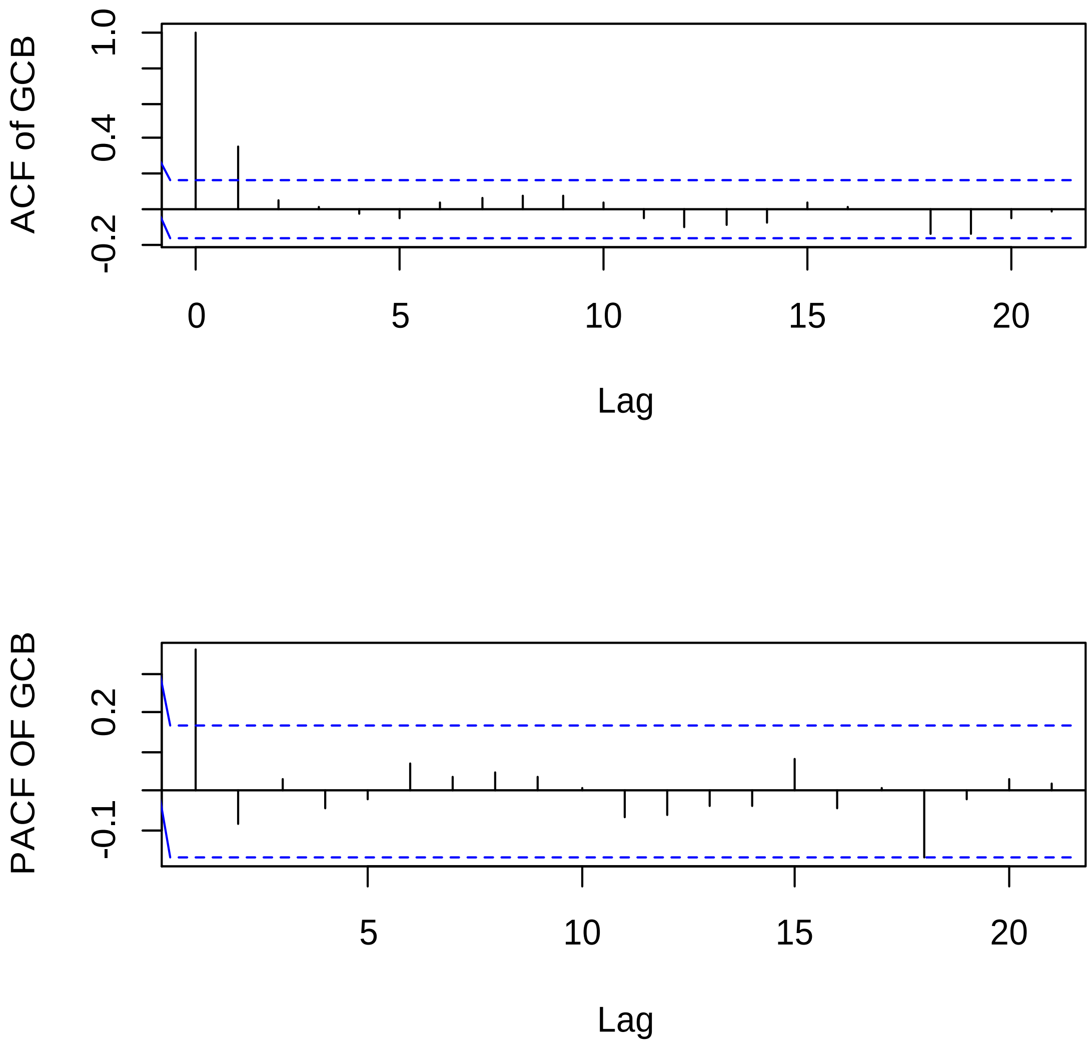

Figure 4.2: Correlograms of GCB Monthly Stock Returns

The ACF plot of GCB squared monthly stock return was associated with a significant spike at lag 2 and other higher lag while the PACF plot was found to have a significant spike at lag 2 as well as other higher lags as displayed in Figure 4.3. The significance of some spikes in the autocorrelation plots is an indication of some dependence in the conditional volatility (squared monthly stock returns) or the existence of time-varying volatility and thus a confirmation for the need to fit GARCH models to the empirical data.

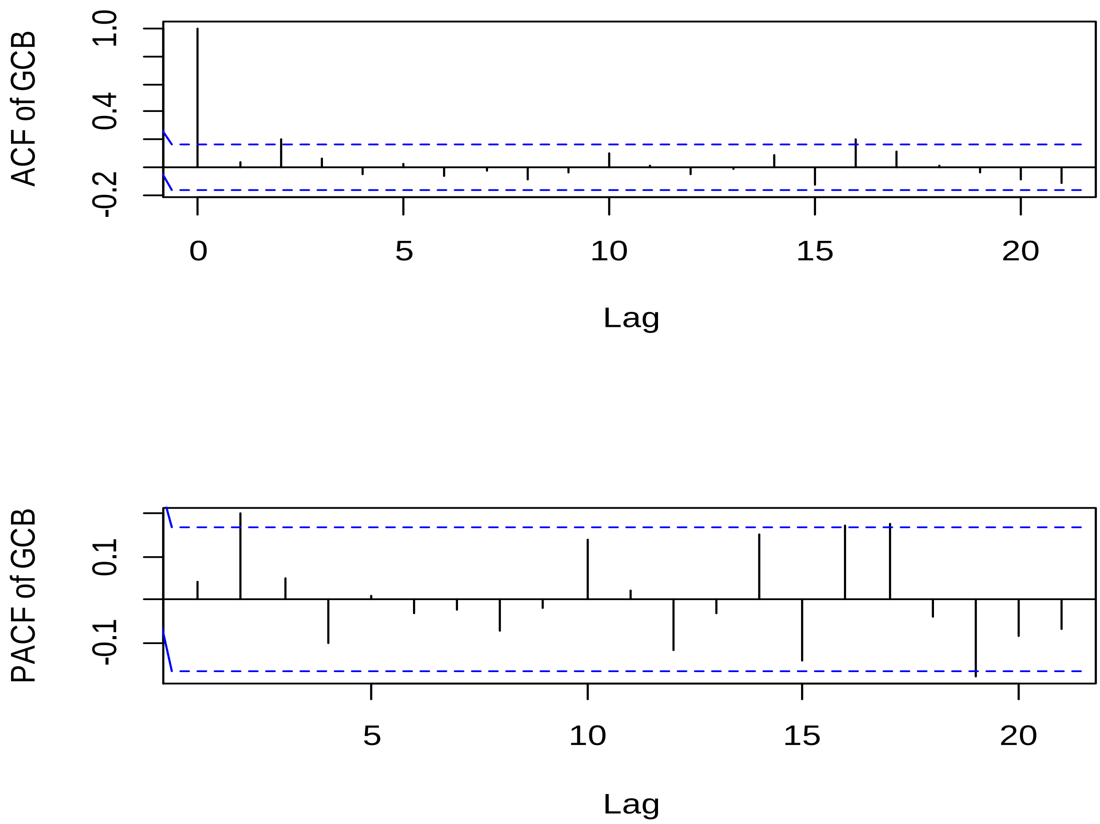

Figure 4.3: Correlograms of GCB Squared Monthly Stock Returns

4.2 Further Analysis

This section builds on the preliminary analysis which hints at the direction to consider for the final analysis. The further analysis considered the development of several MS models for both the classical and Bayesian perspective for which substantive models were selected for the company under study based on using model selection criteria. Also, these selected models were subjected further to risk analysis (VaR and ES).

In this study, several variance specifications of two regimes MS models with varied conditional distributions or innovations (ST, GED, SST and SGED) were assumed towards analysing regime changes in the volatility dynamics of monthly stock returns via the classical and Bayesian approaches.

The study fitted sixteen tentative two regime MS models each for GCB company of which model evaluation through the goodness of fit statistics such as LL, AIC and BIC were used to select a substantive model for the classical estimation while the DIC was used to select a substantive model for the Bayesian estimation.

4.2.1 Comparative Analysis of Tentative Models under the Classical Estimation

Table 4.3 shows estimates of the goodness of fit tests under the classical estimation for the sixteen MS models fitted for the company under study. The goodness of fit statistic (AIC, BIC and LL), the smaller the values of AIC and BIC, and the higher the value of LL, the better the models fits the empirical data such that for all specifications of conditional volatility and conditional distributions are fully satisfied. The LL, AIC and BIC for the two regime specification of E-GARCH with SST conditional distribution for GCB are better as compared to other models (Akaike, 1974; Schwarz, 1978). This means that the monthly stock return series of GCB possesses the characteristics of leverage effects which is in line with financial returns (Black, 1976; Christie, 1982; Masqood et al., 2017) confirming negative past returns of GCB dominates the conditional volatility much more as compared to positive past returns of equal magnitude.

Table 4.3: Classical Tentative MS Models

Models	LL	AIC	BIC
GARCH			
Student-t	151.8591	-283.7183	-254.4457
GED	152.2125	-284.4250	-255.1524
Skewed Student-t	159.3038	-294.6077	-259.4806
Skewed GED	152.4163	-280.8325	-245.7055
E-GARCH			
Student-t	155.6151	-287.2303	-252.1032
GED	156.2992	-288.5984	-253.4713
Skewed Student-t	**164.5412**	**-301.0825**	**-260.1009**
Skewed GED	156.8417	-285.6834	-244.7018
GJR-GARCH			
Student-t	150.0748	-276.1496	-241.0225
GED	144.2425	-264.4850	-229.3580
Skewed Student-t	153.1991	-278.3982	237.4167
Skewed GED	151.0559	-274.1118	-233.1302
T-GARCH			
Student-t	153.1833	-282.3667	247.2396
GED	146.2742	-268.5483	-233.4213
Skewed Student-t	157.9917	-287.9833	-247.0018
Skewed GED	19.7400	-11.4799	29.5016

The parameter estimates for the E-GARCH variance specification with the SST conditional distribution points to the satisfaction of the assumption of covariance stationarity ($|\beta| < 1$) for the monthly stock return series of GCB as given in Table 4.4. Except for the parameter $(\alpha_{1,2})$, all coefficients in the two regimes E-GARCH SST model are statistically significant. The tails can be said to be fatter in the second regime than in the first regime ($\nu_1 = 2.2173$ and $\nu_2 = 83.7193$), the distributional shape of the first regime possessing an almost symmetrical distributional shape ($\xi_1 = 0.1636$) while the second regime can be said to be skewed ($\xi_2 = 14.5157$). The parameters of the GARCH error terms for regime one and regime two can be seen to be reacting differently to past negative returns that are $\alpha_{2,1} = -0.1208$ and $\alpha_{2,2} = 0.5677$ (Ardia et al., 2019). Volatility persistence was also different for both regime one and regime two. The first regime was associated with a volatility persistency of $\alpha_{1,1} + \frac{1}{2}\alpha_{2,1} + \beta_1 \approx 1.6841$ and second regime with volatility persistency of $\alpha_{1,2} + \frac{1}{2}\alpha_{2,2} + \beta_2 \approx 0.6942$. These statistic point to the fact that the first regime is characteristic of a weak volatility reaction to past negative returns (-0.1208) and a high persistence to the process of volatility (1.6841) while the second regime is characteristic of a strong volatility reaction to past negative returns (0.5677) and a low persistence to the process of volatility (0.6942) (Ardia et al., 2019). In general, volatility persistency is dominant in regime one as compared to regime two and thus tend to fall in line with the view Bentes (2014) attesting to the fact that volatility persistency is eminent during periods of market turbulence.

Table 4.4: Classical Parameter Estimates of E-GARCH Skewed Student-t

Parameters	Estimate	Std Error	t value	p-value
$\alpha_{0,1}$	-3.0204	0.0611	-49.4390	**0.0000**
$\alpha_{1,1}$	1.3226	0.0408	32.4478	**0.0000**
$\alpha_{2,1}$	-0.1208	0.0353	-3.4250	**0.0003**
β_1	0.4219	0.0137	30.7637	**0.0000**
ν_1	2.2173	0.0087	254.6337	**0.0000**
ξ_1	0.1636	0.0120	13.6235	**0.0000**
$\alpha_{0,2}$	-2.1738	0.0959	-22.6737	**0.0000**
$\alpha_{1,2}$	-0.0858	0.2246	-0.3820	0.3512
$\alpha_{2,2}$	0.5677	0.0872	6.5130	**0.0000**
β_2	0.4961	0.0087	57.3538	**0.0000**
ν_2	83.7193	1.0670	78.4616	**0.0000**
ξ_2	14.5157	1.0083	14.3967	**0.0000**

The transition probability matrix shows the probabilities associated with transitions from one state (regime) to the other that is;

$$\mathrm{P}_{ij} = \begin{bmatrix} 0.4442 & 0.5558 \\ 0.5263 & 0.4737 \end{bmatrix}.$$

It can be observed that the basic assumptions such as: $0 < p_{i,j} < 1 \ \forall \ i, j \in \{1, 2, ..., K\}$ and $\sum_{j=1}^{K} p_{i,j} = 1, \forall \ i \in \{1, 2, ..., K\}$ have been fully met. The probabilities associated with transitioning simply quantifies the persistency of each regime. The estimate of the state transition probability matrix indicates that monthly stock return switches from low to low volatility regimes $\left(P_{1,1} = 0.4442\right)$ are shorter as compared to switches from high to high volatility regimes $\left(P_{2,2} = 0.4737\right)$. This result tends to suggest that remaining in regime two which is a high volatility regime is rather dominant as compared to regime one which is a low volatility regime by 0.0295. Also, it is evident from the transition probability matrix that, it takes almost close to 56% for monthly stock returns to switch from a low volatility regime to a high volatility regime $\left(P_{1,2} = 0.5558\right)$ while vice versa $\left(P_{2,1} = 0.5263\right)$ is associated with almost close to 53%. This means that monthly stock returns have high persistence on regime two (high volatility regime), with a probability value of 0.5558, indicating that when the process is in regime one (low volatility regime), there is a very high probability that it switches to regime two (high volatility regime) $P_{1,2} = \mathrm{P}\left[s_t = 2 \mid s_{t-1} = 1\right]$ by approximately 56%. The average duration of a low volatility regime is $\left(1 - P_{1,1}\right)^{-1} = 1.7992$ months while that of high volatility regime duration is $\left(1 - P_{2,2}\right)^{-1} = 1.9001$ months. Also, the unconditional probabilities (stable probabilities) of being in state one is 49% and that of state two is 51%.

4.2.2 Comparative Analysis of Tentative Models under the Bayesian Estimation

In the Bayesian estimation, certain parameters were controlled such as setting the thinning factor (***nthin=40***), number of MCMC draws (***nmcmc=10000***) and the number of discarded draws (***nburn=5000***) during the model development stages.

Table 4.5 displays the estimates of the goodness of fit tests under the Bayesian estimation for sixteen MS models fitted for GCB. Spiegelhalter et al. (2002) indicated that minimum DIC associated with a model justifies the choice of selecting such a model. In this vein, following Spiegelhalter et al. (2002), the substantive model to be selected with the minimum DIC (-295.1033) is the E-GARCH variance specification with the SST conditional distribution.

Table 4.5: Bayesian Tentative MS Models

Models	DIC
GARCH	
Student-t	-283.0779
GED	-283.2976
Skewed Student-t	-275.6094
Skewed GED	-276.6863
E-GARCH	
Student-t	-283.7820
GED	-294.5318
Skewed Student-t	**-295.1033**
Skewed GED	-288.0988
GJR-GARCH	
Student-t	-279.8881
GED	-280.4427
Skewed Student-t	-290.8880
Skewed GED	-277.5442
T-GARCH	
Student-t	-281.2652
GED	-281.3729
Skewed Student-t	-288.6879
Skewed GED	-284.4806

The Bayesian diagnostic analysis for the two-regime E-GARCH variance specification with SST conditional distribution was conducted. The trace plots as displayed in Figure 4.4 can be said to be mixing well and hence can be described to be largely stationary. This shows the convergence of the Markov chains to their stationarity.

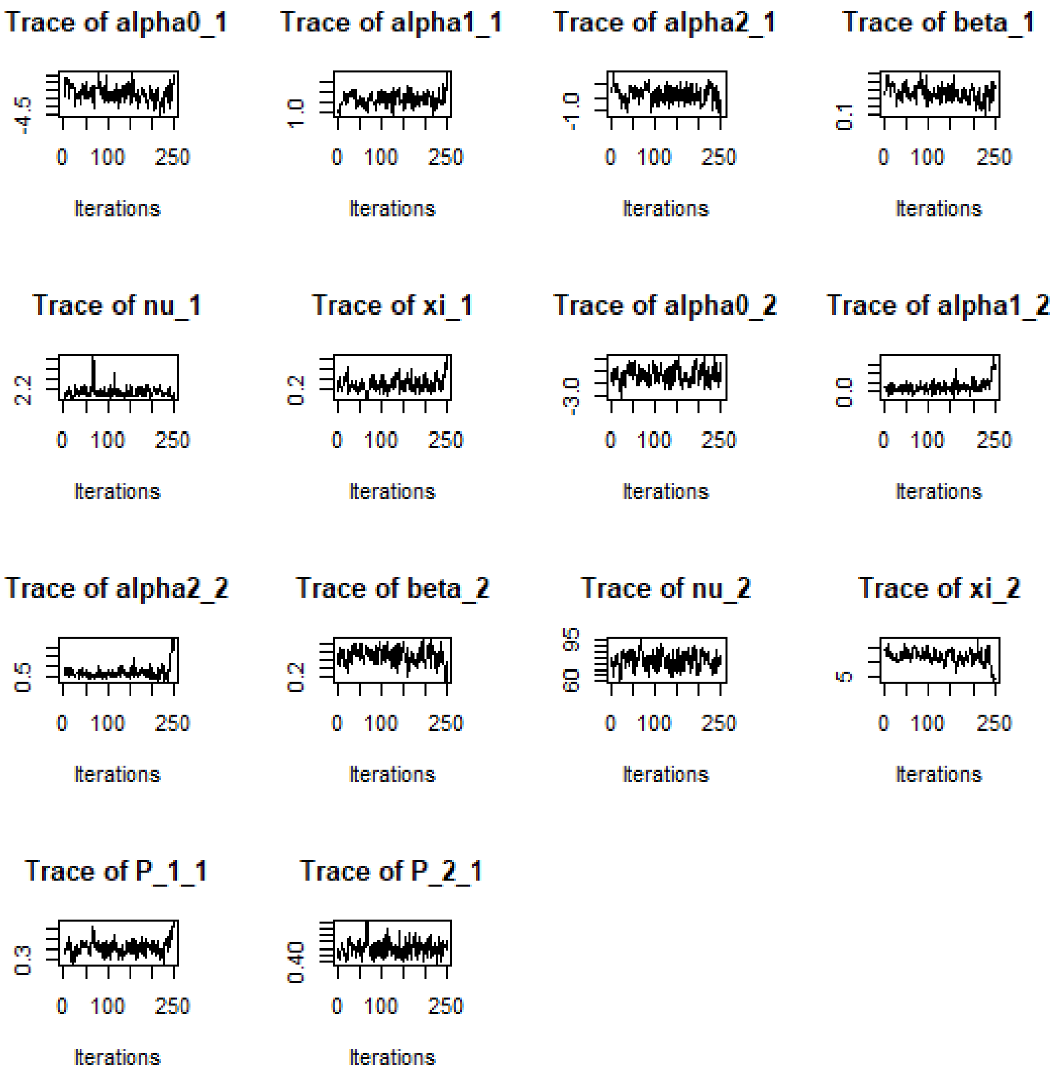

Figure 4.4: Trace Plots of Bayesian E-GARCH Skewed Student-t

Figure 4.5 is the kernel densities of the Bayesian substantive model. It can be seen that the densities display the shape of the normal distribution. This tends to confirm that the posterior distribution depicts that of the normal distribution and hence the satisfaction of the convergence of the Markov chain.

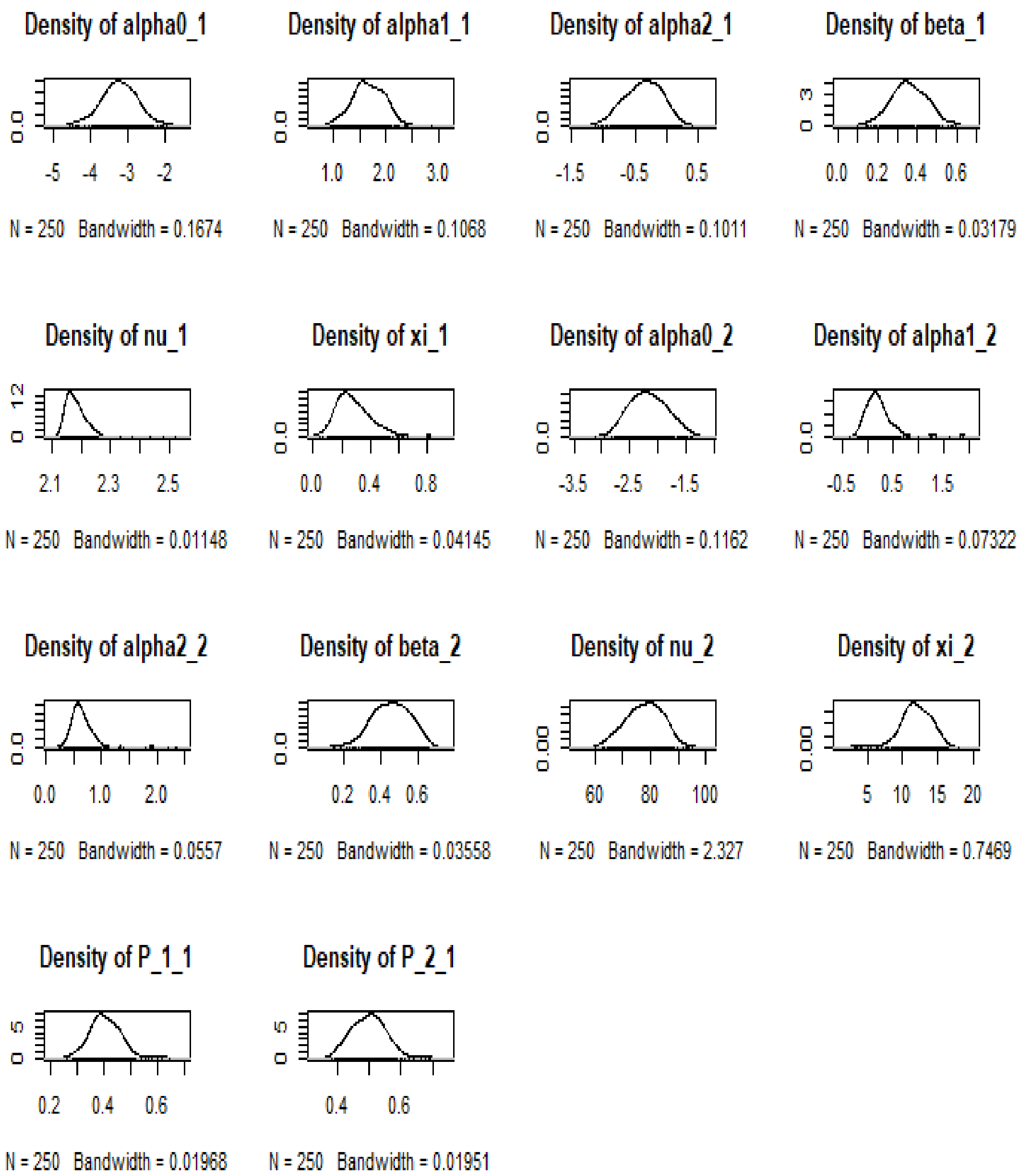

Figure 4.5: Density Plots of Bayesian E-GARCH Skewed Student-t

The ACF plots of the parameters of the model were assessed to examine the behaviour of the Markov chain. It can be established that the autocorrelations decay rapidly for increasing lags as displayed in Figure 4.6. This suggests some convergence in the model parameters.

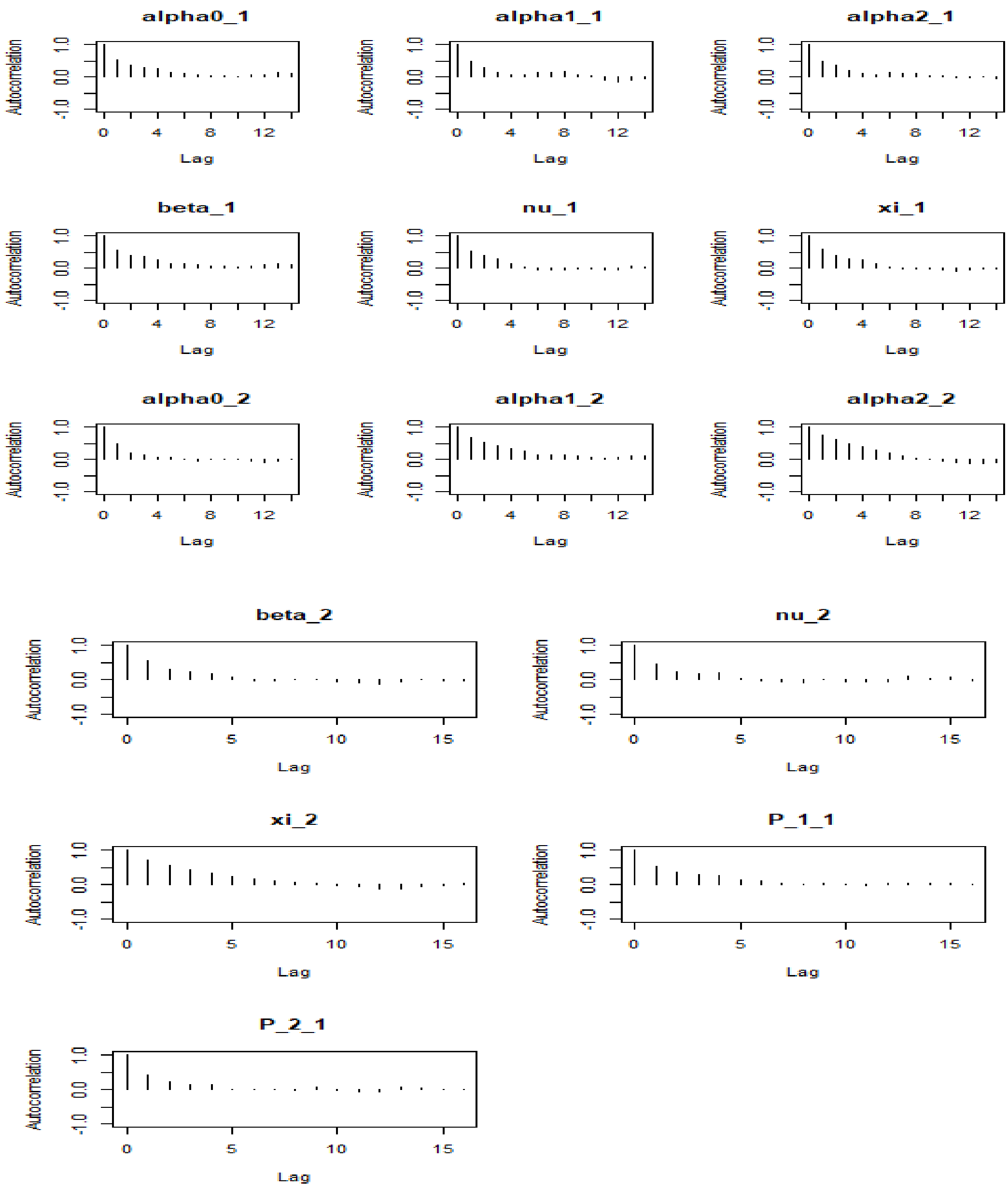

Figure 4.6: ACF Plots of Bayesian E-GARCH Skewed Student-t

Bayesian estimates fitted in this study relied on the starting values of the ML parameter estimates. The posterior estimates as indicated in Table 4.6 tends to assess the efficiency in respect of the method of sampling through resorting to the utilisation of Relative Numerical Efficiency (Geweke, 1989). Literature does not explicitly specify the threshold for achieving an efficient Relative Numerical Efficiency, however, in a study by Korkpoe and Nathaniel (2019), values of Relative Numerical Efficiency near to one can be said to be efficient while those near to zero are an attestation of working with dependent samples. This means that the values of Relative Numerical Efficiency found in Table 4.6 generally can be said to be low but not zero and hence a confirmation that some degree of efficiency is ascertained in our sampling.

The estimates from the variance specification (E-GARCH) with the SST conditional distribution is found to fully meet the covariance stationarity assumption ($|\beta| < 1$). The impact of shocks does not die out soon in the state of high volatility $(\beta_1 < \beta_2)$ for the monthly stock returns of GCB while the densities of the second state (high volatility) do not get flattered $(\nu_1 < \nu_2)$ of which these outcomes are not in line with that of Shiferaw (2018). The symmetry of distribution in the view of Fernandez and Steel (1998) suggest a value of the shape parameter to be equal to zero $(\xi = 0)$. Following Fernandez and Steel (1998), the first and second regime tend to have a dissimilar distribution with regime one found to be characteristic of an almost symmetrical distributional shape $(\xi_1 = 0.3008)$ while the distributional shape of regime two can best be ascribed to be relatively skewed $(\xi_2 = 11.7693)$ which is found to contradict the outcome of Korkpoe and Nathaniel (2019).

The estimated parameters of the GARCH error terms for regime one and regime two can be seen to be reacting differently to past negative returns that are $\left(\alpha_{2,1} = -0.3691\right)$ and $\left(\alpha_{2,2} = 0.6788\right)$. The persistence of volatility can be found to be dissimilar for both regimes (regime one and regime two). The first regime is accompanied by persistent volatility that is: $\alpha_{1,1} + \frac{1}{2}\alpha_{2,1} + \beta_1 \approx 1.8504$ and second regime followed by a persistent volatility that is: $\alpha_{1,2} + \frac{1}{2}\alpha_{2,2} + \beta_2 \approx 1.0325$. These statistic is an attribution based on the fact that regime one tends to possess the features of a weak volatility reaction to past negative returns $\left(\alpha_{2,1} = -0.3691\right)$ and high persistence concerning the volatility process (1.8504) while regime two is typical of a strong volatility reaction to past negative returns $\left(\alpha_{2,2} = 0.6788\right)$ and low persistence concerning the volatility process (1.0325).

Table 4.6: Bayesian Parameter Estimates of E-GARCH Skewed Student-t

Parameters	Mean	SD	SE	TSSE	RNE
$\alpha_{0,1}$	-3.2211	0.4831	0.0306	0.0674	0.2052
$\alpha_{1,1}$	1.6755	0.3040	0.0192	0.0359	0.2874
$\alpha_{2,1}$	-0.3691	0.2876	0.0182	0.0366	0.2469
β_1	0.3594	0.0905	0.0057	0.0137	0.1733
ν_1	2.1840	0.0470	0.0030	0.0059	0.2544
ξ_1	0.2910	0.1297	0.0082	0.0157	0.2722
$\alpha_{0,2}$	-2.1697	0.3309	0.0209	0.0359	0.3402
$\alpha_{1,2}$	0.2401	0.3209	0.0203	0.0519	0.1526
$\alpha_{2,2}$	0.6788	0.2950	0.0187	0.0499	0.1401
β_2	0.4530	0.1013	0.0064	0.0120	0.2864
ν_2	77.9543	6.6233	0.4189	0.7498	0.3121
ξ_2	11.7955	2.3331	0.1476	0.3981	0.1374

Footnote: SD=Standard deviation, SE=Naive standard error of the mean, TSSE=Time series standard error based on an estimate of the spectral density at zero, RNE=Relative Numerical Efficiency

The posterior mean probability transition matrix below gives the transitions from one state or regime to the other which is;

$$P_{ij} = \begin{bmatrix} 0.4098 & 0.5902 \\ 0.5010 & 0.4990 \end{bmatrix}.$$

The transition matrix is found to have satisfied assumptions of a state probability transition matrix such as:

$0 < p_{i,j} < 1 \; \forall \; i, j \in \{1, 2, ..., K\}$ and $\sum_{j=1}^{K} p_{i,j} = 1, \forall \; i \in \{1, 2, ..., K\}$. The posterior estimate of the state transition matrix is found to indicate that monthly stock return switches from high to high volatility regimes $(\mathrm{P}_{2,2} = 0.4990)$ are longer as compared to switches from low to low volatility regimes $(\mathrm{P}_{1,1} = 0.4098)$. These results tend to suggest that remaining in high volatility regimes is found to be dominant as compared to low volatility regimes by 0.0860. Also, the posterior transition probability matrix reveals that switching from a low to a high volatility regime; takes almost close to 59% $(\mathrm{P}_{1,2} = 0.5902)$. This means that monthly stock returns have high persistence on regime two (high volatility regime), with a probability value of 0.5902, indicating that when the process is in regime one (low volatility regime), there is a very high probability that it switches to regime two (high volatility regime), $P_{1,2} = \mathrm{P}[s_t = 2 \mid s_{t-1} = 1]$ by approximately 59%. Also, an approximate 50% $(\mathrm{P}_{2,1} = 0.5010)$ was accompanied by switching from a high to low volatility regime. The average associated with the duration of the low volatility regime is $(1 - \mathrm{P}_{1,1})^{-1} = 1.6943$ months while the duration of the high volatility regime is $(1 - \mathrm{P}_{2,2})^{-1} = 1.9960$ months. The posterior unconditional probability (stable probability) of being in the first (low volatility) regime is 46% and 54% for being in the second (high volatility) regime approximately.

4.2.3 Comparative Risk Analysis of Classical and Bayesian Substantive Model

Under this section, risk analysis (such as VaR and ES) at the horizon of up to three were utilised to quantify the quantile of the future distribution at the 1% and 5%. The substantive model (E-GARCH SST) under the classical and Bayesian estimation were compared respectively for both VaR and ES at the aforementioned thresholds.

In the comparison of VaR and ES, parameters of the classical E-GARCH SST model were considered fixed while that of the Bayesian paradigm, parameter uncertainties were integrated into the aforementioned model.

The results from Table 4.7 indicates that, at the VaR level of 1% and any given horizon, the Bayesian E-GARCH SST produce less expected losses on an asset (monthly stocks) as compared to the classical counterpart. For instance, at horizon one of the Bayesian E-GARCH SST, there is a 1% probability that investors of GCB could lose more than an approximated 17.13% of investment on the monthly stocks as compared to that of 16.98% under the classical approach. In another vein, at horizon one, there is a 99% chance that an investor will lose less than 17.13% of its investment in the monthly stocks of GCB under the Bayesian E-GARCH SST as compared to less than 16.98% under the classical E-GARCH SST. Also, the 5% VaR level tend to favour the Bayesian perspective except for horizon one that the expected losses (0.1503) were the same for the two estimation approaches.

The ES measures the expected loss when the loss is beyond the level of VaR. The results showcase the superiority of the Bayesian model at the 1% and 5% respectively in producing less ES at horizons two and three. However, at the 5% level of ES and horizon one, the classical model is preferred. The statistic reported in Table 4.7 can be interpreted to mean for instance, under the Bayesian approach of horizon three, an investor is 95% confident that the expected loss for an investment in the monthly stock of GCB will be approximately 29% beyond the 5% level of VaR as compared to the expected loss of 28% under the classical approach. This simply means that in the "worst-case scenario", the expected loss on investment of GCB to an investor is 29% under the Bayesian as compared to 28% expected loss under the classical.

Generally, it can be deduced that with less expected losses accompanying the Bayesian model as compared to the classical model, then preference should be given to the Bayesian models when computing VaR and ES.

Table 4.7: VaR and ES Estimates of Substantive Models

	Classical E-GARCH SST				***Bayesian E-GARCH SST***			
	VaR 1%	**VaR 5%**	**ES 1%**	**ES 5%**	**VaR 1%**	**VaR 5%**	**ES 1%**	**ES 5%**
$h=1$	-0.1698	-0.1503	-0.1770	-0.1652	-0.1713	-0.1503	-0.1958	-0.1635
$h=2$	-0.2925	-0.1533	-0.6853	-0.2842	-0.4574	-0.2158	-7.7145	-1.9968
$h=3$	-0.2775	-0.1579	-0.6318	-0.2781	-0.4160	-0.1564	-0.6346	-0.2943

Footnote: h=horizon

4.2.4 In-Sample Classical and Bayesian Conditional Volatility

The conditional volatility of the classical and Bayesian substantive models can be found to exhibit the same pattern across the sample period as shown in Figure 4.7. However, the surges of volatility can be said to be sharper in the Bayesian (blue colour) as compared to the classical (magenta colour), but both found to be reverting fast to the moderate level. In general, the classical approach (magenta colour) tends to underestimate the conditional volatility as the plotted series is found to lie below the Bayesian approach (blue colour) over the entire sample period. This volatility uptick in some cases of the plot can be subjected to investors being nervous about happenings in the financial stock market.

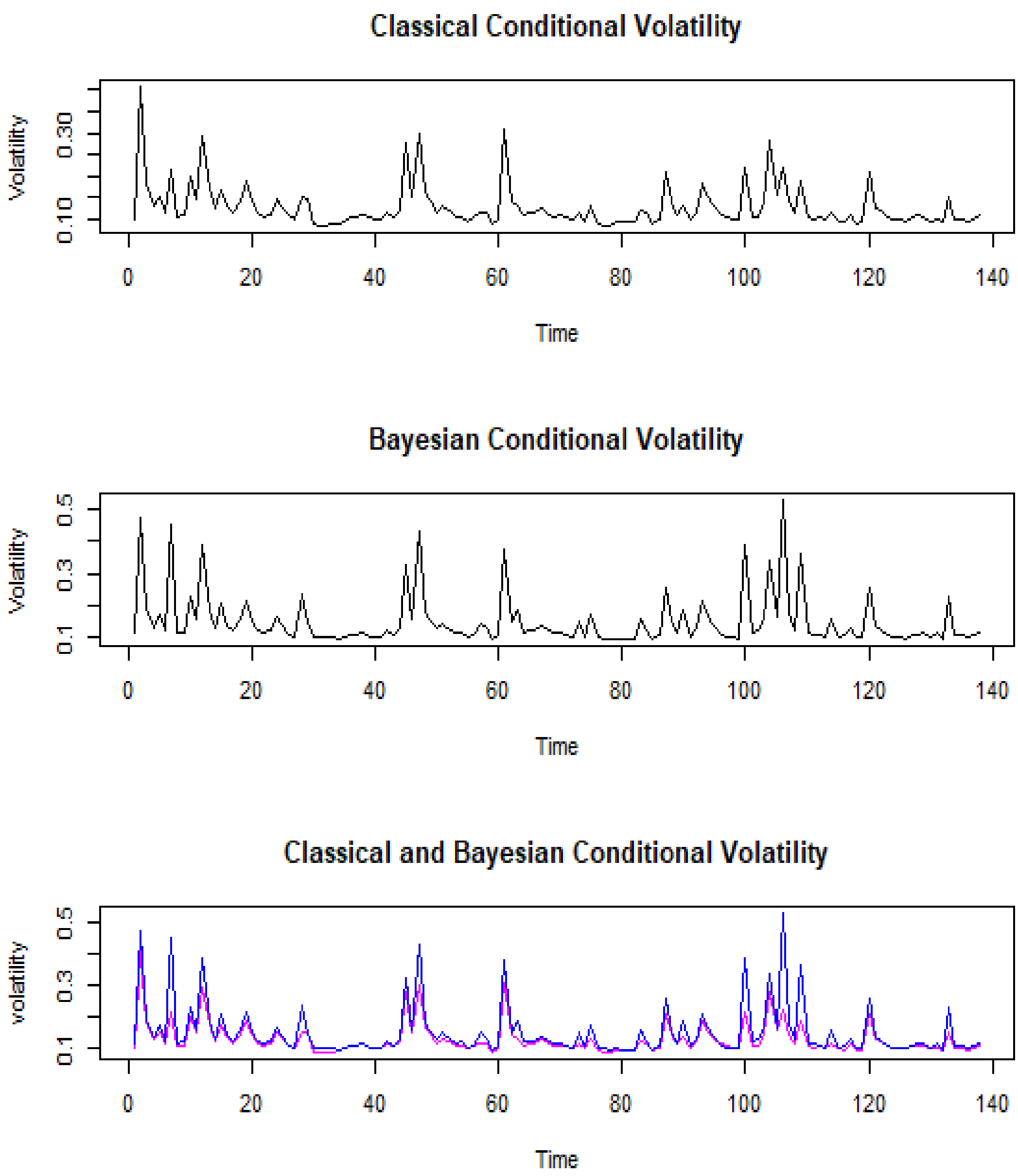

Figure 4.7: Plot of In-Sample Conditional Volatility for Classical and Bayesian

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter presents the summary of the main findings of the study, conclusions and recommendations.

5.1 Summary

The study fundamentally focused on employing classical and Bayesian Switching volatility models in analysing stock returns in the Ghanaian stock market. The empirical data that were used constituted monthly closing stocks of GCB covering 138 months (07/2009 to 12/2020).

The summary statistics indicate that the monthly stock returns of GCB exhibit feature such as non-normality, skewness and leptokurtic (peakedness which is sharp) distributional shape akin to any other financial time series data.

The stock return series showcased fewer and less extreme outliers, and with the period under review showing volatility. The series also exhibits some stability in the mean, but the variance exhibits some form of switches such as low and high volatility giving credence to the presence of stylised facts that is volatility clustering. Further exploration of the data found some dependence in the conditional volatility (squared monthly stock returns) in the correlograms (ACF and PACF) suggesting the need towards fitting GARCH type models to the data.

The sixteen MS models fitted each under the classical and Bayesian approach found the estimate of the E-GARCH variance specification with SST conditional distribution (innovation) to have been associated with the highest LL and least AIC and BIC, and DIC respectively. These substantive models selected under the classical and Bayesian estimation find the monthly stock return series to exhibit leverage effects respectively.

The average (expected) duration of the volatility regime (low and high regimes) under the classical and Bayesian estimation was approximately two months respectively. However, both the classical and Bayesian methods were found to have been associated with switches from a low to a high volatility regime taking a little bit longer as compared to the reverse. Also, the unconditional probability in the second (high volatility) regime takes a bit longer than the first (low volatility) regime in both the classical and Bayesian methods.

The risk analysis (VaR and ES) devised in this study at the level of 1% and 5% found the estimates associated with the Bayesian technique to produce less expected losses as compared to the classical counterparts.

5.2 Conclusions

This study examined the switching behaviour of volatility regimes in the monthly stock returns on the GSE. The results revealed the presence of stylised facts in the return series contributing to the switching behaviour in the variance. To justify this assertion, the fit statistic under the classical and Bayesian perspectives confirmed the existence of regimes in the monthly stock returns of GCB throughout the review.

The classical and Bayesian estimation techniques agree on the E-GARCH variance specification with SST conditional distribution (innovation) as the appropriate model towards handling these stylised facts in the observed returns series based on the model selection criteria. The identified models establish the dominance of leverage effects where negative past return tends to have a bearing on the conditional volatility than past positive returns of similar magnitude.

The estimation techniques reconciled on the same features for the two regimes. The first regime exhibits a weak reaction to past negative returns and a strong persistence towards the volatility process while the second regime demonstrates a strong reaction to past negative returns and a low persistence towards the volatility process. Generally, the first regime can best be ascribed to as "turbulent market conditions" while regime two can best portray "tranquil market conditions" by investors or financial market players.

The forecast horizon of up to three finds the model under the Bayesian realm to generally perform better in estimating VaR and ES at the 1% and 5% as compared to that under the classical. This showcases the importance of integrating parameter uncertainty in a Bayesian framework towards risk analysis.

5.3 Recommendations

Based on the summary and conclusions of this study, the following recommendations are worthy of consideration.

i. Investors in the financial sector should invest in the stocks of GCB. This is because GCB stocks give good returns to investors and where there exist "turbulent market conditions", it takes a shorter period to recover.

ii. Financial sector players should make it a point to utilise Bayesian Switching volatility models in forecasting risk since the study find the estimates of this method far better than the classical counterparts.

iii. Further studies should be devised based on developing models that can handle the preponderance of zeros in log returns since this study finds daily log returns to have zeros and hence an alternative to this study was to transform the data into monthly stock returns.

iv. Further studies must be carried out to investigate the asymmetric features of the financial data of various companies on the GSE by utilising probability conditional distributions.

v. Studies in the area of Copula Extreme Value analysis can be carried out to better appreciate the inter-dependencies of financial market variables among companies listed on the GSE.

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