UNIVERSITY FOR DEVELOPMENT STUDIES

# MARKOV CHAIN MODELING OF PNEUMONIA CASES IN THE BUILSA NORTH MUNICIPALITY OF THE UPPER EAST REGION OF GHANA

ABDULAI JAFAR

2021

UNIVERSITY FOR DEVELOPMENT STUDIES

# MARKOV CHAIN MODELING OF PNEUMONIA CASES IN THE BUILSA NORTH MUNICIPALITY OF THE UPPER EAST REGION OF GHANA

**ABDULAI JAFAR**

**(UDS/MAS/0001/19)**

**THESIS SUBMITTED TO THE DEPARTMENT OF STATISTICS, FACULTY OF MATHEMATICAL SCIENCES, UNIVERSITY FOR DEVELOPMENT STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER OF PHILOSOPHY DEGREE IN APPLIED STATISTICS**

**SEPTEMBER, 2021**

## DECLARATION

**Student**

I hereby declare that this thesis is the result of my own original work and that no part of it has been presented for another degree in this university or elsewhere:

Candidate's Signature:…… Date:…….

Name:…….

**Supervisors'**

I hereby declare that the preparation and presentation of the thesis was supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies.

Supervisors' Signature:…… Date:……..

Name:…….

## ABSTRACT

Pneumonia is an infection of the lungs. It causes inflammation of the alveoli making it fill with fluid, causing difficulty in breathing and reduces oxygen intake. It is the leading cause of deaths in children worldwide. In this study, secondary data on monthly pneumonia cases collected from the disease control unit from 2015 to 2020 of the Builsa North Municipality was modeled using discrete-time Markov chain modeling. The data was grouped into low, moderate and high states of the pneumonia cases. Chi square test of independence indicated that the data followed the first-order Markov chain assumption. Transition probabilities and relevant disease metrics were estimated for the disease. The model revealed that there is higher chances of moderate pneumonia cases to be recorded in the Builsa North Municipality in the long run than low and high pneumonia cases with a probability of approximately 51%. The expected length of time the pneumonia cases are expected to stay in the low and moderate states were estimated to be approximately 3 months each and 2 months for the high state. It was known from the estimation that it will take the states twenty-eight (28) months to be in equilibrium. The model also revealed that in the long-run the municipality will record on average approximately 116 pneumonia cases.

## ACKNOWLEDGEMENTS

First of all, I am very grateful to my supervisor, Dr. Alhassan Faisal for his counseling and professional guidance during the research.

My intense gratitude goes to Dr. Suleman Nasiru, Head of Department of the Statistics Department for his patience, wisdom and guidance as well as all lecturers of the Department. Also, my profound gratitude goes to Mr. Suitor Kwabena Elvis of the disease control unit of the Builsa North Municipality for his support in providing me with the required data.

Finally, I want to express my profound gratitude to my colleagues and all those who contributed in various ways to the success of this thesis.

## DEDICATION

This work is dedicated to my brother, Mr. Abdullai Tijani.



# TABLE OF CONTENTS

## LIST OF TABLES

## LIST OF FIGURES





## LIST OF ACRONYMS

| | |
|---|---|
| AIDS | Acquired Immunodeficiency Syndrome |
| APA | Aspiration Pneumonia |
| CAP | Community Associated Pneumonia |
| COPD | Chronic Obstructive Pulmonary Disease |
| CV | Coefficient of Variation |
| DTMC | Discrete-Time Markov Chain |
| FPT | First Passage Time |
| HAP | Hospital Acquired Pneumonia |
| HIV | Human Immunodeficiency Virus |
| IRR | Incidence Rate Ratio |
| Max | Maximum |
| Min | Minimum |
| MRT | Mean Recurrent Time |
| NM | Pneumonia |
| PCV | Pneumococcal Conjugate Vaccine |
| PPSV | Pneumococcal Polysaccharide Vaccine |

| | |
|---|---|
| SD | Standard Deviation |
| SE | Standard Error |
| SSP | Steady State Probability |
| ST | Sojourn Time |
| UNICEF | United Nations International Children's Emergency Fund |
| VAP | Ventilator Acquired Pneumonia |
| WHO | World Health Organization |



# CHAPTER ONE

# INTRODUCTION

## 1.1 Background of Study

Pneumonia is the major cause of child death in every region (Wardlaw et al, 2006). It kills children more than malaria, measles and AIDS. Pneumonia is an infection of the lungs that causes inflammation in the alveoli making it fill with fluid resulting in difficulty in breathing and limits oxygen intake. It affects one or both lungs (WHO, 2019). Symptoms may include a cough with or without mucus, chills, difficult breathing and fever and can vary from mild to severe. It can range from mild to severe depending on the type of causative germ, overall health and age. Its development involves bacteria, viruses, fungi and in some rare cases parasites. However, it is most commonly caused by bacteria (American Lung Association, 2020). Most common cause of pneumonia in young children are viruses while adults with weaker immune systems are more frequently infected by bacteria (Crum and Nancy, 2008).

There are three types of pneumonia based on the causative pathogen which include; fungal pneumonia, bacterial pneumonia and viral pneumonia (Jill, 2018). Pneumonia is spread through the airborne droplets that are dispersed by coughing or sneezing from an infected person. It can also be spread through contamination of the blood (Susan, 2019). It is transmitted when germs from infected person spread to another person either through Inhaling infected particles from coughs and sneezes of an infected person, when a person touches an infected surface or when a person with an infection coughs into his or her hand and then shakes an

uninfected person who touches the mouth or eyes without washing (Zawn, 2017). The viruses and bacteria that are commonly found in a child's nose or throat can infect the lungs when inhaled. They may also spread through droplets from a cough or sneeze and through blood, especially during and shortly after birth (WHO, 2019).

Community Acquired Pneumonia (CAP) causing bacteria are divided into typical and atypical bacteria. Mycoplasma pneumoniae, Chlamydophila pneumoniae and Legionella pneumophila are the atypical agents whilst Streptococcus pneumoniae is the main typical agent causing pneumonia (Eliakim et al., 2012). About one third of pneumonia cases is as a result of viruses which often affects children less than 5 years of age which include respiratory syncytial virus (RSV), human rhinovirus, and herpes simplex virus (HSV) (Eliakim et al., 2012).

Pneumonia causing fungi include coccidioidomycosis, histoplasmosis and Cryptococcus. When one immune system becomes weak the fungi present in the body multiply, spread and start attacking the system which then leads to infection (Eliakim et al., 2012).

Anyone can get pneumonia, however those with high risk of getting infected by pneumonia include; children less than 5 years of age, people 65 and above years of age and individuals with other medical conditions such as diabetes, respiratory diseases, malnourishment and immune deficiency disorders. Smoking and alcoholism which reduces amount of white blood cells count in the bloodstream are also factors (Backhaus et al., 2016).

While most healthy children can fight the infection with their natural defenses, children with compromised immune systems as a result of malnutrition, especially due to insufficient breastfeeding and pre-existing illness such as measles increases a child's risk of contracting pneumonia (WHO, 2019). Also, parents smoking, living in crowded homes and indoor air pollutions increase a child's susceptibility to pneumonia (WHO, 2019). However, adequate nutrition, zinc intake and community-based case management are the major strategies for childhood pneumonia prevention (Manivel et al., 2015).

### 1.2 Problem Statement

Contagious diseases have shaped the world and are still with us with most epidemics caused by these diseases. Their presence can lead to a wide spread of morbidity and mortality as well as social, political and economic disruption (Mirza et al., 2020).

Pneumonia is a contagious disease which is the leading cause of death in the world among children below five years of age (Le et al, 2017). There are about 120 million cases of pneumonia in children below five years per year and about 14 million progresses to severe cases (Lassi et al, 2017). Older people have higher risk of getting pneumonia and are more likely to die from it. In the United States, pneumonia is the most common cause of hospitalization in indults. About one million adults seek care in hospitals every year and about 50,000 die from this disease (Ventola, 2016).

In Ghana, pneumonia is rated as the leading cause of morbidity and mortality with an annual death record of 16,200 children, accounting for 20% of deaths per year (Iddrisu et al, 2019).

Pneumonia not only the leading cause of morbidity and mortality in the country and the world at large, it also affects other aspects especially the economic and population growth. This study seeks to model monthly pneumonia cases in the Builsa North Municipality using discrete time Markov chain modeling to estimate relevant metrics such as long-run probabilities of pneumonia cases, mean recurrence time, sojourn time, mean first passage time as well as long-run expected cases of the disease.

### 1.3 Objectives of Study

#### 1.3.1 General Objective

The main objective of the study is to model monthly Pneumonia cases in the Builsa North Municipality using discrete-time Markov chain analysis.

### 1.3.2 Specific Objectives

Specifically, the study intends to;

i. develop three-state discrete-time Markov chain model for the pneumonia cases;
ii. estimate the probabilities of the steady state, sojourn time and mean recurrent time for the various states of the pneumonia cases;
iii. compute the first passage time probabilities for the various states of the pneumonia cases.

## 1.4 Significance of the Study

Findings from this study could be used by policy makers to put more effective measures to minimize the prevalence of pneumonia in the Builsa North Municipality and the country as a whole. Also, the study could provide bases for further researches on pneumonia in Ghana.

## 1.5 Scope of the Study

Markov chain modeling was employed to estimate transition probabilities, steady state probabilities, mean recurrence time, sojourn time, the mean first passage time and the long-run expected value of the pneumonia cases. A major limitation of the study is that medical history of individuals was not considered as well as sex and age. Thus the model was not able to distinguish between males and females and age in terms of the estimated metrics of the disease. Also, Due to lack of time, the study only focused on one municipal hospital in Ghana.

## 1.6 Organization of Thesis

The study was structured into five chapters. Chapter one consists of the introduction of the research which entails the background, problem statement, objectives, significance and organization of the thesis. Chapter two entails the literature review on Markov chain analysis and pneumonia. Chapter three comprises of the methodology employed in the study. Chapter four contains analysis and discussion of results and chapter five outlined the summary, conclusions and recommendations.



# CHAPTER TWO

# LITERATURE REVIEW

## 2.1 Introduction

The aim of this chapter is to present a review of literature on the application of Markov chains to diverse fields as well as literature review on pneumonia.

## 2.2 Application of Markov Modeling

Liu et al. (2009), Markov chain has been widely applied in the disciplines of natural science, engineering, economics and management. In 2000, Markov chain was applied to forecast the prevalence of AIDS among homosexuals in England and Wales (Twumasi, 2018).

Yan et al. (2018), Markov chain model is widely used in forecasting market demand of customers in service demand. In solving problems using Markov chain models, a vector of initial probabilities of the system is constructed. Using Markov chains to determine the product evolution on market requires the present state and transition probabilities only. Recurrent and transient states are the two known states of Markov processes. The certainty that a process will return at a certain known state in the future determines the recurrent state. Probabilistic systems are often modeled using Markov chains when the probability of a future event is dependent on only the present and not on the past. The Markov chain is a relevant method in a sense that its use makes easy getting significant information that may contribute to the improvement of marketing and strategic management to improve the activity through the development of strategies based on the product life cycle.

Forecasting an event's future state involves knowledge of its likelihood of transitioning from one state to another state. The transition of an event from one state to another depends on the immediate preceding state with constant probability forming the basis of the Markov chain (Kassa et al., 2017).

In determining the equilibrium of market share of products in the present period as a bases for forecasting market shares in the future by Datong (2011) in Nigeria the first-order Markov modeling was employed. In the process of modeling these underlining dynamics the method of Unweighted Restricted Ordinary Least Squares was employed in the estimation of transition probability matrix. The switching probabilities of subscribers among service providers and scope of subscriber's loyalty were computed so as to determine the likely marketing mix on equilibrium market share.

Markov chain modeling was also applied by Ansah et al. (2013) to predict subscriber's brand switching behaviour and ergodic market share of the major service providers in Ghana. Eight marketing mixes including Trust, customer loyalty, Price, Brand Image, switching cost, Service Quality, Value Offered, Satisfaction and Switching Cost were considered. To estimate the ergodic market share, a transition probability matrix was derived by relating the marketing mixes to one another. It was concluded from their study that, ergodic market share can be attained and stabilized if no provider takes an act that modifies the matrix of transition probabilities.

In Ethiopia, Kassa et al. (2017) employed Markov chain modeling to predict monthly market share of restaurants. Four restaurant categories were considered which constitute the states of the model to examine customer loyalty, retention and lost. A transition matrix was formed from the result. It was concluded that customers preference strengthens on the forth restaurant.

Elavarasan et al. (2018) emphasized that, the method of Markov chain analysis is useful in the comprehension of stochastic traits of rainfall and droughts through the examination of probabilities for each severity class, residence times in each drought category, times for reaching any drought state from the non-drought category and residence times in each drought category. They also asserted that the methodology can be adequately used as an analytical tool for forecasting transitions among categories of drought severity up to 3 months ahead. Also, the Markov chain modeling was used to develop an early warning system for drought management in two climatic areas of Virginia, USA (Paulo et al., 2005).

Also, a three-state Markov chain model of Plasmodium, Falciparium and Parasitemia transmission in Ghana was fitted by Pobbi (2012). He used the Markov model to describe the transition dynamics of malaria. The states of the model were defined as susceptible, infected and dead over discrete intervals of time. The model was based on the time homogeneity such that individuals were assumed to move from one state to another at constant rate which was independent of time. Also, only one transition was permissible between two consecutive surveys. But the model was used to determine a steady state probability distribution which was unwarranted and questionable. This is because;

the steady state or limiting distribution from the proposed Markov model was not feasible since it must be based on the assumption of the ergodicity of the chain. However, the chain was not ergodic (aperiodic and irreducible) and that computing the limiting or steady state distribution was debatable. Nevertheless, he discovered in his study that the expected time to first infection of the disease was 11 days with total length of time of non-severe malaria being 17 days. Moreover, he found out that the life expectancy of both susceptible and infected individuals were both 55 years.

Adigun et al. (2019) modeled infectious diseases using Markov chain by considering five states which include: exposed, infected, immune, recovered and dead. Their study revealed that the past history of infectious diseases will affect the future only through the present state of infections in diseases which is a strong property of the Markov chain.



**2.3 Pneumonia**

Pneumonia was first described by Hippocrates (460–370 BC). The first descriptions of its clinical and pathological features were made 22 centuries later in 1819 by Laennec while Rokitansky in 1842 was the first to differentiate lobar and bronchopneumonia. Pneumonia is defined as an infection of the pulmonary parenchyma which is caused by different organisms. It is a group of specific infections, each with different pathogenesis, epidemiology, presentation and clinical route. The classification of pneumonia is based on the causative microorganisms. It is common in children, though the specific microbial etiology remains unknown in more than a third of patients. Blood culture is the only test

performed to provide a specific diagnosis, which may only be positive in 5–10 % of patients and up to 20 % in the most severely ill patients (Mackenzie, 2016).

Pneumonia is a form of acute respiratory infection that affects one or both lungs. It is an infection of the tiny air sacs of the lungs called alveoli. In persons with pneumonia, the alveoli are filled with pus and fluid which makes breathing painful and reduces oxygen intake. Pneumonia is more common in elderly people and young children. However, it can occur at any age. Pneumonia is the leading cause of deaths for children under five years accounting for 15% of all child deaths though the number of children dying from pneumonia has decreased substantially (Bernadeta, 2018). It kills an approximated number of 1.4 million children below the age of five years, accounting for about 18% of all children's deaths under five years old worldwide (Mhandu, 2019). It causes death of children and families everywhere, but it is most prevalent in sub-Saharan Africa and South Asia (Pant et al, 2020). In 2017, Ethiopia, India, Pakistan, Democratic Republic of Congo and Nigeria accounted for more than half of all deaths from childhood pneumonia (Howie et al, 2019). Children younger than five years of age especially infants who are from zero to five months of age and not exclusively breastfed are more likely to die 15 times due to pneumonia than children who are exclusively breastfed, Tong (2013).

Despite progress against pneumonia, over 800,000 children still die from it every year. However, by 2017, this number decreased by almost two-thirds due to enhancements in the major risk factors such as childhood wasting, falling global poverty, poor sanitation, air pollution and a better availability of health

technology such as antibiotics and pneumococcal vaccines. In 2017, 2.56 million people died from pneumonia with almost a third of the victims being children under five years. Death rates from pneumonia are highest in Sub-Saharan Africa and Southeast-Asia. The difference between richer and poorer countries is also large with European countries suffered a rate of 10 deaths per 100,000 whilst poorer countries had a rate of more than 100 deaths per 100,000 in 2017 (Swedberg et al., 2020).

Pneumonia is classified based on the clinical context in which a patient develops symptoms of infection. These categories include community-acquired pneumonia (CAP), hospital acquired pneumonia and ventilator-associated event. (Sharma et al., 2017).

Legionnaire's disease, a severe form of pneumonia which results from the infection with Legionella can manifest as a multisystem disease. Usually, it involves the lungs and the gastrointestinal tract. It is also associated with substantial mortality. Legionella commonly affects people older than fifty years of age though cases have been reported in infants and neonates. Pneumonia as a result of Legionella is commonly found in clusters that are not related to person-to-person transmissions. Most of the Legionella infections are gotten through infested water or soil. Risk factors for acquiring Legionella disease include high humidity, work in gardens with compost and rainfall. Pneumonia caused by other pathogens is hard to be distinguished from Legionnaires disease because it shows similar clinical symptoms. However, infection by Legionella can be distinguished

by the presence of raised creatinine kinase levels and diarrhea (Sharma et al., 2017).

### 2.4 Causes and Symptoms of Pneumonia

The pathogens involved in the development of pneumonia are viruses, bacteria, fungus and in rare cases parasites. In babies and children pneumonia is commonly caused by viruses and adults with weaker immune systems are more frequently infected by bacteria. It can occur at any age but children under two years of age and adults above sixty-five years are at higher risk of being infected. It can be spread through contact, sneezing or coughing without covering the mouth or nose, touching surfaces and in hospitals (Kim et al., 2018).

Typically, Pneumonia is caused by bacteria or virus one has been subjected to in the environment. Infection can be passed between people from direct contact or inhaling droplets in the air from coughing or sneezing. Pneumonia can be more rarely caused by a parasite or fungus (Bosh et al., 2013).

People with poor nutrition, preexisting lung diseases, smokers, those around tobacco smoke and other chronic health problems are at higher risk of contracting pneumonia. Also, those who have not been immunized for Streptococcus pneumonia bacteria are at higher risk for lung infections (Bernadeta, 2018).

Pneumonia can be diagnosed in myriad modes. It can be diagnosed by healthcare providers by a physical examination, symptoms or by ordering diagnostics. For pathogenic bacteria in the infected part of the body to be researched, laboratory test can include cell cultures and chest X-rays. There should be a combination of

radiological, clinical findings and laboratory findings to intensify the probability of appropriate diagnosis. Laboratory tests in addition to chest X-rays can aid in the confirmation of the diagnosis of pneumonia by presence of specific findings, such as infiltration in the lung, which would still require qualified assessment in concurrence with clinical picture (Tong and Ba, 2013).

Malnourished children, especially those with inadequate zinc intake, those that are not exclusively breastfed and those with compromised immune system are at high risk of developing pneumonia as well as those with other illness such as measles or HIV. Environmental factors such as living in crowded homes, being exposed to parental smoking and other indoor air pollutions also contribute to children's susceptibility to pneumonia and its consequences (Wardlaw et al, 2018).

The transmission of Pneumonia is generally restricted to local communities and not easily spread across borders. It can be controlled if basic health measures are available (Bernadeta, 2018).

Symptoms of pneumonia may come on quickly or may worsen slowly over time. Pneumonia patients often have a cough, difficulty breathing, fever or chills, low energy and poor appetite. Diarrhea, nausea and chest pain are sometimes exhibited in persons with pneumonia, (Barbara et al., 2020). According to Wardlaw et al. (2018), a wide range of symptoms may be exhibited by children with pneumonia contingent to age and cause of the infection. Common symptoms include difficult rapid breathing, headaches, fever, cough, chills, loss of appetite

and wheezing. Severe cases of pneumonia can cause hypothermia, convulsions, lethargy and feeding problems in young infants.

Pneumonia is caused by a blend of a diversity of factors, including the environment, pathogens, health systems and health-seeking behaviours. Therefore, there is no single intercession that can effectively control, treat or prevent pneumonia. As such, a confluence of key interventions to control pneumonia would include improvements in nutrition, environmental living conditions, early diagnosis, immunization against specific pathogens and treatment of the disease. The practice of increased breastfeeding will also help reduce childhood mortality due to pneumonia (Tong and Ba, 2013).

**2.5 Preventive Measures of Pneumonia**

Pneumonia is the major cause of child death in every region. It kills more children than malaria, measles and AIDS combined. The key preventive measures for children are adequate nutrition, indoor air pollution reduction and increased immunization rates with vaccines that help prevent development of infections which directly cause pneumonia in children, such as Haemophilus influenzae type b (Hib) and those immunizations which prevent infections that can lead to pneumonia as a complication (Wardlaw et al., 2018).

In a study conducted by Obu (2017) on pharmacotherapy of pneumonia in children under five years in two hospitals in the Ashanti Region of Ghana revealed that the antibiotics used commonly were oral amoxicillin and cefuroxime as first-line antibiotics for the out-patient treatment of community-

acquired pneumonia with Oral co-amoxiclav and erythromycin being the second-line antibiotics used for out-patient management of community acquired pneumonia (CAP). He also found that co-amoxiclav was used for severe pneumonia cases resistant to cefuroxime or amoxicillin and erythromycin was used either for suspected cases of atypical bacterial pneumonia or as a substitute in the case of penicillin allergy. He revealed that for in-patients, iv ampicillin or cefuroxime alone or in combination with gentamicin were first line antibiotic therapies and intravenous co-amoxiclav and iv ceftriazone were the second-line antibiotics used for patients not responding to the first-line therapy. However, the main adjunctive therapies were paracetamol and ibuprofen and non-pharmacological methods such as tepid sponging for fever management, cough medicines for cough in a few cases and oxygen for hypoxaemia for the management of pneumonia.

Though antibiotics can be effective for many of the bacterial causing pneumonia, their resistance is growing among the pneumonia causing bacteria. However, antibiotics are ineffective for viral causes of pneumonia (Jansen et al., 2018).

According to Bree (2019) there are two kinds of pneumonia vaccines: pneumococcal conjugate vaccine (PCV13 or Prevnar13) and pneumococcal polysaccharide vaccine (PPSV23 OR Pneumovax23). These vaccines reduce but do not prevent ones risk of getting pneumonia.

The PCV13 prevents against 13 kinds of bacteria that cause serious infection in children and adults. It is part of the standard vaccination protocol which is given

as a three or four dose series for babies from 2 months old and the last dose given by 15 months and it is given as a one-time injection for adults aged 65 and older.

PPSV23 is a one-time dose that protects against 23kinds of bacteria and it is recommended for adults over age 65 who has already received the PCV13 vaccine.

Carol (2020), Pneumococcal conjugate vaccine (PVC13/Prevar13) and Pneumococcal polysaccharide vaccine (PPSV23/Pneumovax) are the two kinds of preventive measures for pneumonia.

The Centers for Disease Control and Prevention (retrieved, 12/5/2021) recommends PCV13 for all children younger than 2 years old and people 2 years and older with certain medical conditions as well as adults 65 years and older. It also recommended PPSV23 for adults 65 years or older, people 2 years through 64 years with certain medical conditions and adults 19 through 64 years old who smoke cigarettes. Other preventive measures include; frequent washing of hands, avoidance of smoking and maintaining good health habits. Other preventive measures include; frequent washing of hands, avoidance of smoking and maintaining good health habits.

# CHAPTER THREE

# METHODOLOGY

## 3.1 Introduction

According to Giebeler et al. (2013), methodology is the general research strategy that outlines the way in which research is to be undertaken and among other things identifies the methods to be used in it. The source of data, Markov chain analysis and the theoretical framework of the analysis are presented on this chapter.

## 3.2 Source of Data

Secondary data on monthly pneumonia cases were collected from Ghana Health Service in order to realize the objectives of the study. The data were taken from the Disease Control Unit of the Sandema hospital for the period of January 2015 to December 2020.

## 3.3 Markov Chain

A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends on the state of the previous event. It was named after Russian mathematician, Andrey Andreyevich Markov (Paul, 2017). According to Sundberg, J. and Klacksell, G. (2012) Markov chain is a mathematical model that describes a process with probabilities, states and transitions where each state has its own probability for going into another state or remaining in the current state. It is a mathematical system that experiences transitions from one state to another according to a given set of probabilistic rules.

A sequence of random variables $X_n, n = 1,2,3,....$ is a Markov chain if it follows the Markov property which states that the probability of the present state $X_n$ of a process is dependent on only its immediate previous state $X_{n-1}$. Thus

$$P[X_n = i_n \mid X_0 = i_0, X_1 = i_1, X_2 = i_2, ..., X_{n-1} = i_{n-1}] = P[X_n = i_n \mid X_{n-1} = i_{n-1}] \quad (3.1)$$

Where $P(i / j)$ is the probability of *i* given *j*.

Markov processes can be classified in to four based on the nature of time parameter and state space. These include;

i. Discrete-time Discrete-space Markov process,
ii. Discrete-time Continues-space Markov process,
iii. Continues-time Discrete-space Markov process,
iv. Continues-time Continues-space Markov process.

Discrete-state Markov process is known as Markov chain.

Markov chain can be of first order, second order or higher orders. However, first-order homogenous discrete-time Markov chain will be employed in this study.

The first order discrete time Markov chain (DTMC) possesses the memory-less property, which states that the probability of the next state of a system is only dependent on the present state of the system and not on any prior states. Hence the first order Markov chain, since the involvement of the past states might lack accurate information in the determination of the probability of the next state.

A random process $[X_t, t \in T]$ is a first order Markov chain if for any value $t_0, t_1, ..., t_n$ the conditional cumulative distribution function of $X(t_n)$ for given values $X(t_0), X(t_1), ..., X(t_{n-1})$ depends only on $X(t_{n-1})$. Thus

$$p[X(t_n) \leq x_n \mid X(t_{n-1}) \leq x_{n-1}, X(t_{n-2}) \leq x_{n-2}, X(t_{n-3}) = x_{n-3}, ..., X(t_0) \leq x_0]$$

$$= p[X(t_n) \leq x_n \mid X(t_{n-1}) \leq x_{n-1}] \qquad (3.2)$$

This implies that the present state of a process is only dependent on only the immediate past state and this property is referred to as Markov property.

### 3.3.1 Discrete-time Markov Chain

A discrete-time Markov chain is a sequence of random variables $X_0, X_1, X_2, ...$ which follows the Markov property namely, the probability of moving to the next state of a process depends only on the present state of the process and not on its previous states. The possible outcomes of $X_i$ constitutes countable state space, S of the chain (Grimmett and Stirzaker, 2020).

The monthly pneumonia cases were modeled using the discrete-time Markov chain analysis.

A discrete-time process $X_t$, $t = 0, 1, 2, 3, ...$ is said to be a Markov chain if for all $i, j, t, ..., n,$ the following holds:

$$P[X_t = j \mid X_{t-1} = i, X_{t-2} = \alpha ..., X_0 = \gamma] = P[X_t = j \mid X_{t-1} = i] = P_{ij}(t) \qquad (3.3)$$

Where $P_{ij}(t)$ is known as state transition probability, which is a conditional probability that the process will be in state *j* at time *t* given that the process is in state i at time *t-1*. Thus, the current state is sufficient to determine the next state.

Markov chain can also be homogenous or non-homogenous.

A homogenous Markov chain is one in which the transition probabilities are independent of time. Thus;

$$P[X_t = j / X_{t-1} = i, X_{t-2} = \alpha, ..., X_0 = \gamma] = P[X_t = j / X_{t-1} = i] = P_{ij} \qquad (3.4)$$

and $p_{ij}$ holds for all times.

Where the homogeneous transition probability satisfies the following conditions;

i. $0 \leq p_{ij} \leq 1$

ii. $\sum_j p_{ij} = 1$

This follows that the states are mutually exclusive and exhaustive.

From (3.4) the Markov chain rule can be obtain as follow;

$$p[X_t = j, X_{t-1} = i_1, X_{k-2} = i_2 ..., X_0 = l_t]$$

$$= P[X_t = j \mid X_{t-1} = i_1, X_{t-2} = i_2, ..., X_0 = i_t] P[X_{t-1} = i_1, X_{t-2} = i_2, ..., X_0 = i_t]$$

$$= p[X_t = j \mid X_{t-1} = i_1] p[X_{t-1} = i_1 \mid X_{t-2} = i_2] [X_{t-2} = i_2 \mid X_{t-3} = i_3, X_4 = i_4, ..., X_0 = i_t] p[X_{t-3} = i_3, X_{t-4} = i_4, ..., X_0 = i_t]$$

$$= p[X_t = j \mid X_{t-1} = i_1]p[X_{t-1} = i_1 \mid X_{t-2} = i_2]p[X_{t-2} = i_2 \mid X_{t-3} = i_3][X_{t-3} = i_3 \mid X_{t-4} = i_4,..., X_0 = i_t]p[X_{t-4} = i_4, X_{t-5} = i_5,..., X_0 = i_t]$$

$$= p[X_t = j \mid X_{t-1} = i_1]p[X_{t-1} = i_1 \mid X_{t-2} = i_2]p[X_{t-2} = i_2 \mid X_{t-3} = i_3]p[X_{t-3} = i_3 \mid X_{t-4} = i_4]...p[X_1 = i_{t-1} \mid X_0 = i_t]$$

$$= p_{i_1 j}\, p_{i_2 i_1} ... p_{i_t i_{t-1}}\, p[X_0 = i_t]$$

This implies that when the initial state $X_0$ probability is known then the joint probability $p[X_t, X_{t-1}, X_{t-2},..., X_0]$ can be computed.

### 3.3.2 Assumptions of the Markov Chain

i. The future state of pneumonia cases depends only on its current state

ii. The transition probabilities are independent of time

### 3.3.3 State-transition Probability

The conditional probability that a process will be in state *j* given that it is in state *i* is called transition probability denoted by $p(i, j)$ or $p_{ij}$.

For a given initial state *i* and a number of trials $n_j$, the sample of transition counts, $n_{i1}, n_{i2}, n_{i3}$ can be considered as a sample of size $n_j$ from a trinomial distribution with probabilities $p_{i1}, p_{i2}$ and $p_{i3}$ such that $\sum_{j=1}^{3} p_{ij} = 1$ (Teodorescu, 2009).

The probability of this outcome can therefore be given as;

$$f(n_{ij} \mid p_{ij}) = \frac{n_i!}{n_{i1}!n_{i2}!n_{i3}!}\, p_{i1}^{n_{i1}}\, p_{i2}^{n_{i2}}\, p_{i3}^{n_{i3}} = n_i! \prod_{i=1}^{3} \frac{p_{ij}^{n_{ij}}}{n_{ij}}, \quad i, j = 1, 2, 3.$$

The likelihood of the transition probabilities follows a trinomial distribution given by;

$$L(n_{ij} \mid p_{ij}) = n_i! \prod_{i=1}^{3} \frac{p_{ij}^{n_{ij}}}{n_{ij}}, \;\; i, j = 1, 2, 3.$$

Taking logarithm of the likelihood,

$$\log[L(p_{ij} \mid n_{ij}) = n_i! \prod_{i=1}^{3} \frac{p_{ij}^{n_{ij}}}{n_{ij}!}]$$

$$\Rightarrow l(p_{ij}) = \log n_i! + \log \prod_{i=1}^{3} \frac{p_{ij}^{n_{ij}}}{n_{ij}!}$$

$$= \log n_i! + \sum_{j=1}^{3} \log \frac{p_{ij}^{n_{ij}}}{n_{ij}!}$$

$$= \log n_i! + \sum_{j=1}^{3} n_{ij} \log p_{ij} - \sum_{j=1}^{3} \log n_{ij}!$$

Maximizing $l(p_{ij})$ using Lagrange multiplier with the constraint $\sum p_i = 1$ then,

$$L(p_{ij}, \lambda) = l(p_{ij}) + \lambda \left(1 - \sum_{j=1}^{3} p_i\right)$$

$$\Rightarrow L(p_{ij}, \lambda) = \log n_i! + \sum_{j=1}^{3} \log \frac{p_{ij}^{n_{ij}}}{n_{ij}!} + \lambda \left(1 - \sum_{j=1}^{3} p_i\right)$$

$$\Rightarrow L(p_{ij}, \lambda) = \log n_i! + \sum_{j=1}^{3} n_{ij} \log p_{ij} - \sum_{j=1}^{3} \log n_{ij}! + \lambda \left(1 - \sum_{j=1}^{3} p_i\right)$$

Taking partial derivative with respect to $p_{ij}$ and equating to zero,

$$\frac{\partial}{\partial p_{ij}} [L(p_{ij}, \lambda) = \log n_i! + \sum_{j=1}^{3} n_{ij} \log p_{ij} - \sum_{j=1}^{3} \log n_{ij}! + \lambda \left(1 - \sum_{j=1}^{3} p_i\right)] = 0$$

$$\Rightarrow \frac{\partial}{\partial p_{ij}} \log n_i! + \frac{\partial}{\partial p_{ij}} \sum_{j=1}^{3} n_{ij} \log p_{ij} - \frac{\partial}{\partial p_{ij}} \sum_{j=1}^{3} \log n_{ij}! + \frac{\partial}{\partial p_{ij}} \lambda - \frac{\partial}{\partial p_{ij}} - \lambda \sum_{j=1}^{3} p_i = 0$$

$$\Rightarrow \frac{n_{ij}}{p_{ij}} - \lambda = 0$$

$$\Rightarrow \frac{n_{ij}}{p_{ij}} = \lambda$$

$$\Rightarrow p_{ij} = \frac{n_{ij}}{\lambda}$$

$$\Rightarrow p_{ij} = \frac{n_{ij}}{n_i}$$

### 3.3.4 State-transition Probability Matrix

Transition probability matrix is a matrix whose entries constitute transition probabilities. It is an $n \times n$ matrix $P$ with entries $p_{ij}$, where $p_{ij}$ is the $i^{th}$ entry in the $j^{th}$ column of the matrix.

Considering a Markov chain with $n$ states and transition probabilities $p_{ij}$ then the transition probability matrix $P$ of the chain can be presented as;

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \qquad (3.5)$$

The transition probability matrix must satisfy the properties below (Ibe, 2014);

i) $p_{ij} \geq 0, \ i, j = 1, 2, ..., n$

ii) $0 \le p_{ij} \le 1, \;\; i, j = 1, 2, ..., n$

iii) $\sum_{j} p_{ij} = 1$

### 3.3.5 State-transition Diagram

Transition diagram is a graphical representation of a Markov chain. It presents the states and probabilities of transition of the Markov chain. The states are represented by circles and the transitions by arrows. For a Markov chain of $n, \;\; n = 1, 2, 3$ states with transition probabilities $P_{ij}, \;\; i, j = 1, 2, 3$ the transition probability diagram is given as shown by Figure 3.1.

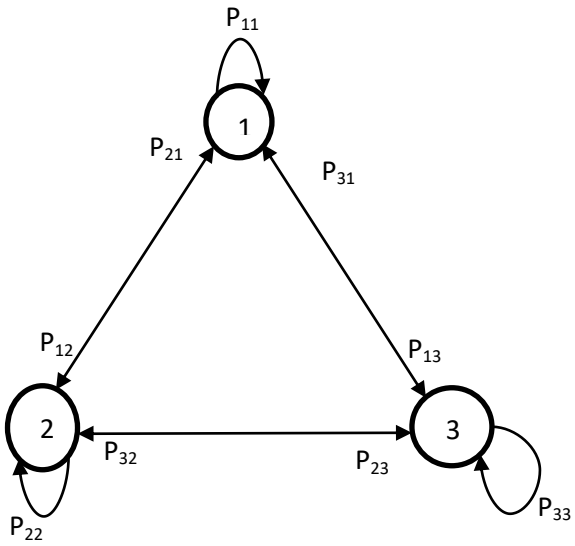


**Figure 3.1: Three-state Transition Diagram**

The corresponding transition probability matrix of the transition diagram is given by,

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

### 3.3.6 *n*-Step Transition Probability

An n-step transition probability is the conditional probability that a system will be in a particular state after *n* transitions given that it is currently in a particular state.

If $P_{ij}^{(n)}$ denotes the conditional probability of a system being in state *j* after *n* transitions given that it is currently in state *i* then,

$$P_{ij}^{(n)} = p(X_{m+n} = j \mid X_m = i) \qquad (3.6)$$

And

$$P_{ij}(0) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$\Rightarrow p_{ij}^{(1)} = p_{ij}(1) = p_{ij}$$

For 2-step transitions, the transition probability denoted by $p_{ij}^{(2)}$ is defined by;

$$P_{ij}^{(2)} = p[X_{m+2} = j \mid X_m = i] \qquad (3.7)$$

Given that $m = 0$,

$$P_{ij}^{(2)} = P[X_2 = j \mid X_0 = i]$$

$$= \sum_k P[X_2 = j, X_1 = k \mid X_0 = i]$$

From the Markov Property,

$$\sum_k p[X_2 = j, X_1 = k \mid X_0 = i] = \sum_k p[X_2 = j \mid X_1 = k, X_0 = i]p[X_1 = k \mid X_0 = i]$$

$$= \sum_k p[X_2 = j \mid X_1 = k]p[X_1 = k \mid X_0 = i]$$

$$= \sum_k p_{ik} p_{kj} \qquad (3.8)$$

$\sum_k p_{ik} p_{kj}$ implies that the probability of being in state j at the end of the second transition, beginning in state *i* is the probability of first going from state *i* to an intermediary state *k* and immediately from state *k* to state *j*. The summation is done over all possible intermediary states k.

A generalization of (3.6) for *n*-transitions is provided by Chapman-Kolmogorov equations

That is for all $0 < m < n$,

$$p_{ij}^{(n)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n-m)} \text{ for any } m = 1, 2, ..., n \text{ and } k = 1, 2, ..., n-1 \qquad (3.9)$$

This implies that the probability of going to state *j* from state *i* after the $n^{th}$ transition is the product of the probability of first going from state *i* to an intermediary state *k* after *m* transitions and from state *k* to state *j* after (*n*-*m*) transitions. The summing is done over all possible *k* states.

For *n*-transitions,

$$p_{ij}^{(n)} = p[X_n = j \mid X_0 = i] = \sum_k p[X_n = j, X_m = k \mid X_0 = i]$$

From the principle of total probability,

$$= \sum_k p[X_n = j / X_m = k] p[X_m = k / X_0 = i]$$

$$= \sum\nolimits_k p_{kj}^{(n-m)} p_{ik}^{m}$$

$$= \sum\nolimits_k p_{ik}^{(m)} p_{kj}^{(n-m)}$$

### 3.3.7 *n*-step Transition Probability Matrix

Let P be the transition probability matrix of a Markov chain, $X_n$, $n = 1, 2, 3, \ldots$ defined as;

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots \\ P_{21} & P_{22} & P_{23} & \cdots \\ P_{31} & P_{32} & P_{33} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

Then, the $n^{th}$ transition probability matrix, is defined as,

$$P^{(n)} = P^n \tag{3.10}$$

And it is given by;

$$P^n = \begin{bmatrix} P_{11}^n & P_{12}^n & P_{13}^n & \cdots \\ P_{21}^n & P_{22}^n & P_{23}^n & \cdots \\ P_{31}^n & P_{32}^n & P_{33}^n & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

However, if *n* is large then it is convenient to use eigenvalue decomposition to compute $P^{(n)}$ from $P = \mathrm{U}\Lambda\mathrm{U}^{-1}$, where $\Lambda$ is a diagonal matrix of eigen values and U is a matrix whose columns are the corresponding eigenvectors. Thus,

$$P^{(n)} = \mathrm{U}\Lambda^{n}\mathrm{U}^{-1} \tag{3.11}$$

The entries of $P^{n}$ will be the probabilities after *n*-steps (transitions).

### 3.3.8 Limiting-state Probability

The following conditions must be satisfied for the existence of limiting-state probabilities (Ibe, 2014);

i. In any irreducible, aperiodic Markov chain the limit, $\lim_{n\to\infty} P_{ij}(n) = \pi_j$ exists and is independent of the initial distribution.

ii. In any irreducible, periodic Markov chain the limit $\lim_{n\to\infty} P_{ij}(n) = \pi_j$ exists and is independent of the initial distribution.

However, these must be interpreted as the long-run probabilities that the process is in state *j*.

Let *P* be a transition probability matrix of a finite irreducible, aperiodic homogeneous Markov chain with *n* representing the number of transitions. When $n \to \infty$, the Markov chain becomes stationary where the state probabilities do not change with *n* any longer. If the limit of the Markov chain exists, then the limiting state probability is given by;

$$\lim_{n\to\infty} P[X_n = j] = \pi_j, \quad j = 1,2,3,...,N \tag{3.12}$$

Since the *n*-step transition probabilities can be written as;

$$P_{ij}^{(n)} = \sum_k P_{ik}(n) P_{kj},$$

if the limiting state probabilities exists without depending on the initial state then,

$$\lim_{n\to\infty} P_{ij}^{(n)} = \lim_{n\to\infty} \sum_k P_{ik}(n) P_{kj} = \lim_{n\to\infty} \sum_k \pi_k P_{kj} = \pi_j \tag{3.13}$$

If $\pi = [\pi_1, \pi_2, \pi_3, ..., \pi_N]$ denotes the limiting-state probability vector, then the following holds;

$$\pi_j = \pi_k P_{kj} \tag{3.14}$$

$$\pi = \pi P \tag{3.15}$$

$$\sum_j \pi_j = 1 \tag{3.16}$$

### 3.3.9 Sojourn Time

The expected time of stay of a process in a given state is referred to as sojourn time. Considering a state *i* for which $P_{ii} > 0$. The interest here is the probability of the process remaining in the state for *d* time units.

Let the number of time units the process remains in state *i* before leaving it, given that the process enters the state be the random variable $D_i$, then the probability mass function (PMF) of $D_i$ if defined as,

$$P_{D_i}^{(d)} = P[D_i = d] = P[X_0 = i, X_1 = i, X_2 = i, X_3 = i, ..., X_{d-1} = i], \quad X_{d \neq 1}$$

$$= P_{ii}^{(d-1)}(1 - P_{ii}) \tag{3.17}$$

If the states of the process represent the members of an observed sequence, then $P_{D_i}(d)$ represents the probability that the sequence remains unchanged *d-1* times before changing.

Since the random variable $D_i$ is geometrically distributed, then the expected (mean) sojourn time of the process in state *i* is defined as,

$$E[D_i] = \frac{1}{1 - P_{ii}} \tag{3.18}$$

For $P_{ii} \neq 1$ the $E[D_i]$ exists.

However, if the state *i* is an absorbing state, then $P_{ii} = 1$ and $E[D_i] = \infty$. This is true since the process remains indefinite in the state.

### 3.3.10 First Passage Time

Let $f_{ij}(n)$ be the conditional probability that, the first time a process enters state *j* occurs in just *n* transitions, given that the process is currently in state *i*. that is

$f_{ij}(n)$ is known as probability of first passage time from state *i* to state *j* in *n* transitions. The probability of first passage from state *i* to state *j* is defined as,

$$f_{ij} = \sum_{n=1}^{\infty} f_{ij}(n) \tag{3.19}$$

Which is the conditional probability that the process will ever enter state *j* given that it was initially in state *i* (Ibe, 2014).

### 3.3.11 Recurring Time

Let the random variable $T_i$ be the recurrence time of state *i*. That is $T_i$ is the time taken until the process returns to state *i*, given that it was initially in state *i* and it is given by;

$$T_i = \min\{n \geq 1 : X_n = i \mid X_0 = i\} \tag{3.20}$$

### 3.3.12 Long-run Expectation

Consider a Markov chain with *n*, $n = 1, 2, 3, \ldots$ states. If the limiting state probability vector and the mean values of the various states of the chain are;

$$\pi = [\pi_1, \pi_2, \pi_3, \ldots]$$

and

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \end{bmatrix}$$

and if the long-run expected value of the chain is $L_r$ ,then $L_r$ is defined as,

$$E\left[L_r\right]=\left[\pi_1,\pi_2,\pi_3,...\right]\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\\ \vdots\end{bmatrix} \tag{3.21}$$

### 3.4 Empirical Framework of the Study

The monthly data collected for this study on pneumonia cases were classified into three states as low, moderate and high pneumonia cases.

#### 3.4.1 Verification of the First-order Markov Chain Assumption

Chi square test of independence was used to investigate whether the data on monthly pneumonia cases satisfied the first-order Markov chain assumption for the three states of the pneumonia cases.

Table 3.1 presents the data layout of the monthly pneumonia cases and the entries $n_{ij}$ representing the number of pneumonia cases in each month belonging to each state of the pneumonia cases.

**Table 3.1: Frequency of Monthly Pneumonia Cases in Each State**

| | State of Pneumonia Cases | | | |
|---|---|---|---|---|
| **Month** | **Low (L)** | **Moderate (M)** | **High (H)** | **Row Total** |
| **1** | $n_{1l}$ | $n_{1m}$ | $n_{1h}$ | $R_1$ |
| **2** | $n_{2l}$ | $n_{2m}$ | $n_{2h}$ | $R_2$ |
| **3** | $n_{3l}$ | $n_{3m}$ | $n_{3h}$ | $R_3$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| **12** | $n_{12l}$ | $n_{12m}$ | $n_{12h}$ | $R_{12}$ |
| **Column Total** | $C_l$ | $C_m$ | $C_h$ | T |



The chi square statistic was used to test the hypotheses;

**Null hypothesis** $H_0$: the first-order Markov chain assumption holds for the data on monthly pneumonia cases at alpha significance level.

**Alternative hypothesis,** $H_a$: the first-order Markov chain assumption does not hold for the data on monthly pneumonia cases at alpha significance level.

The chi square test statistic is given by,

$$\chi^2 = \sum \frac{(0-E)^2}{E} \tag{3.22}$$

where, **O** is the observed frequency in each of the pneumonia classes (states) in each month and

**E** is the expected frequency in each of the classes (states) in each month.

A critical value in the table of probabilities for the chi square distribution and the corresponding p-value were estimated with degree of freedom, $df = (r-1)(c-1)$

r = number of rows and c = number of columns.

### 3.4.2 Maximum Likelihood Estimation of State-transition Probabilities

The transition probabilities, $p_{ij}$ for the various states of the pneumonia cases were estimated using the method of maximum likelihood estimation (MLE) from transition frequencies. The maximum likelihood estimation (MLE) is a method which estimates the parameters of a statistical model given observations, by finding the parameter values that maximize the likelihood of making the observations given the parameters (Rossi, 2018).

The data collected on monthly pneumonia cases were modeled into three-state Markov chain with state space, S = low(L), moderate(M) and high(H). The state of pneumonia cases for the next month was expected to depend only on the state of the pneumonia cases in the current month. The observed pneumonia cases $n_{ij}, i, j = L, M, H$ from one pneumonia state *i* to another state *j* and $\sum_j n_{ij} = n_i$ are presented in Table 3.2.

The definition of the notations $n_{ij}$ in Table 3.2 are;

$n_{ll}$ : Number of pneumonia cases in the low state given that it was in the low state.

$n_{lm}$ : Number of pneumonia cases in the moderate state given that it was in the low state.

$n_{lh}$ : Number of pneumonia cases in the high state given that it was in the low state.

$n_{ml}$ : Number of pneumonia cases in the low state given that it was in the moderate state.

$n_{mm}$ : Number of pneumonia cases in the moderate state given that it was in the moderate state.

$n_{mh}$ : Number of pneumonia cases in the high state given that it was in the moderate state.

$n_{hl}$ : Number of pneumonia cases in the low state given that it was in the high state.

$n_{hm}$ : Number of pneumonia cases in the moderate state given that it was in the high state.

$n_{hh}$ : Number of high pneumonia cases preceded by high pneumonia cases.

$n_{l}$ : Total number of low pneumonia cases.

$n_{m}$ : Total number of moderate pneumonia cases.

$n_{h}$ : Total number of high pneumonia cases.

**Table 3.2: Frequency of Pneumonia Cases from One State (*i*) to Another State (*j*)**

| | | Current Month | | | |
|---|---|---|---|---|---|
| | **State** | **Low (L)** | **Moderate (M)** | **High (H)** | **Total** |
| **Previous Month** | **Low (L)** | $n_{ll}$ | $n_{lm}$ | $n_{lh}$ | $n_l$ |
| | **Moderate (M)** | $n_{ml}$ | $n_{mm}$ | $n_{mh}$ | $n_m$ |
| | **High (H)** | $n_{hl}$ | $n_{hm}$ | $n_{hh}$ | $n_h$ |

The maximum likelihood estimates of the transition probabilities $p_{ij}$ is given by;

$$P_{ij} = \frac{n_{ij}}{\sum n_{ij}} = \frac{n_{ij}}{n_i} \tag{3.23}$$

The transition probabilities of the pneumonia cases from one state to another defined by $p(j \mid i) = p_{ij}$ for $i, j \in S$ are given in Table 3.3.

**Table 3.3: Transition Probabilities of pneumonia cases being in state *j* given that it was in state *i***

| | | Current Month | | |
|---|---|---|---|---|
| | **State** | **Low (L)** | **Moderate (M)** | **High (H)** |
| **Previous Month** | **Low (L)** | $p_{ll}$ | $p_{lm}$ | $p_{lh}$ |
| | **Moderate (M)** | $p_{ml}$ | $p_{mm}$ | $p_{mh}$ |
| | **High (H)** | $p_{hl}$ | $p_{hm}$ | $p_{hh}$ |

The transition probability matrix $P = P(j \mid i) = P_{ij}$ is given below

$$P = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix} \end{array}$$

Where $\sum_j p_{ij} = 1, \quad j = l, m, h$. That is

$$p_{ll} + p_{lm} + p_{lh} = p_{ml} + p_{mm} + p_{mh} = p_{hl} + p_{hm} + p_{hh} = 1$$

The transition probabilities $p_{ij} = p(j/i)$ in the transition matrix are defined as follows;

$p_{ll} = p(l|l)$: Probability of low monthly pneumonia cases remaining low.

$p_{lm} = p(m|l)$: Probability of transitioning of low monthly pneumonia cases to moderate pneumonia cases.

$p_{lh} = p(h|l)$: Probability of monthly pneumonia cases being high given that it was low.

$p_{ml} = p(l|m)$: Probability of monthly pneumonia cases being low given that it was moderate.

$p_{mm} = p(m|m)$: Probability of moderate monthly pneumonia cases remaining moderate.

$p_{mh} = p(h|m)$: Probability of moderate monthly pneumonia cases being high.

$p_{hl} = p(l|h)$: Probability of high monthly pneumonia cases being low.

$p_{hm} = p(m \mid h)$ : Probability of high monthly pneumonia cases being moderate.

$p_{hh} = p(h \mid h)$ : Probability of high monthly pneumonia cases being high.

### 3.4.3 Estimation of Long-run (Equilibrium) Probabilities

If the long-run (equilibrium) probabilities of low pneumonia cases, moderate pneumonia cases and high pneumonia cases are $p_l, p_m$ and $p_h$ respectively, then,

$$[P_l \ P_m \ P_h] \times \begin{bmatrix} P_{ll} & P_{lm} & P_{lh} \\ P_{ml} & P_{mm} & P_{mh} \\ P_{hl} & P_{hm} & P_{hh} \end{bmatrix} = [P_l \ P_m \ P_h]$$

Let $\begin{bmatrix} \pi_l \\ \pi_m \\ \pi_h \end{bmatrix}$ represent the long-run (equilibrium) probabilities,

$$\Rightarrow \begin{bmatrix} \pi_l \\ \pi_m \\ \pi_h \end{bmatrix} = [P_l \ P_m \ P_h] \times \begin{bmatrix} P_{ll} & P_{lm} & P_{lh} \\ P_{ml} & P_{mm} & P_{mh} \\ P_{hl} & P_{hm} & P_{hh} \end{bmatrix} \qquad (3.24)$$

(3.24) gives the estimates of the long-run (equilibrium) probabilities for each of the states. Thus,

For low pneumonia cases, $\pi_l = P_l P_{ll} + P_m P_{ml} + P_h P_{hl}$

For moderate pneumonia cases, $\pi_m = P_l P_{lm} + P_m P_{mm} + P_h P_{hm}$

For high pneumonia cases, $\pi_m = P_l P_{lh} + P_m P_{mh} + P_h P_{hh}$

Where

$\sum \pi_i = 1, \ \ i = l, m, h$ (Raheem et al, 2015).

### 3.4.4 Estimation of Stationary Matrix

Let $P$ be the transition probability matrix of the low, moderate and high states of the pneumonia cases defined as;

$$P = \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix}$$

Then the matrix after the $n^{th}$ transition $P^n$ can be obtain by multiplication as,

$$P^2 = P \times P = \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix}^2$$

$$P^3 = P \times P^2 = \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix}^3$$

$$P^4 = P \times P^3 = \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix}^4$$

.
.
.

$$P^n = P \times P^{n-1} = \begin{bmatrix} p_{ll} & p_{lm} & p_{lh} \\ p_{ml} & p_{mm} & p_{mh} \\ p_{hl} & p_{hm} & p_{hh} \end{bmatrix}^n$$

If after the $n^{th}$ transition the rows of the matrix become the same and does not change with $n$ anymore, then $P^n$ is the stationary (equilibrium) transition probability matrix.

### 3.4.5 Expected Length of Time of the States of the Pneumonia Cases

#### 3.4.5.1 The low state

A low state is defined as a state of months of low pneumonia cases. Let $l$ be the length of the low state of pneumonia cases. $l$ is defined as a sequence of consecutive months of low pneumonia cases preceded and followed by months of moderate or high pneumonia cases. The probability of months of low pneumonia cases is defined as;

$$P(L) = (P_{ll})^{(l-1)}(1-P_{ll})$$

And the expected length of low state pneumonia cases is given by;

$$E(L) = \frac{1}{(1-P_{ll})} \tag{3.25}$$

$(1-P_{ll})$ is the probability of a month being of moderate or high pneumonia cases.

#### 3.4.5.2 The Moderate State

A moderate state is a state of months of moderate pneumonia cases. A sequence of consecutive months of moderate pneumonia cases preceded and followed by months of low or high pneumonia cases is referred to as the length of moderate

state of pneumonia cases. The probability of length of state of moderate pneumonia cases m is defined as;

$$P(M) = (P_{mm})^{(m-1)}(1-P_{mm})$$

The expected length of moderate state of pneumonia cases is given as,

$$E(L) = \frac{1}{(1-P_{mm})} \tag{3.26}$$

Where $(1-P_{mm})$ is the probability of a month not being of moderate pneumonia cases. That is the probability of a month being of low or high pneumonia cases.

### 3.4.5.3 The High State

A high state is a state of months of high pneumonia cases. The length of state of high pneumonia cases *h* is defined as a sequence of consecutive months of high pneumonia cases preceded and followed by months of low and moderate pneumonia cases. The probability of length h of high pneumonia cases is defined as;

$$P(H) = (P_{hh})^{(h-1)}(1-P_{hh})$$

The expected length *h* of high state of pneumonia cases is also defined as;

$$E(L) = \frac{1}{(1-P_{hh})} \tag{3.27}$$

Where $(1-P_{hh})$ is the probability of not being of high pneumonia cases or the probability of a month being of low or moderate pneumonia cases.

### 3.4.6 Estimation of Long-run Expected Value

Let $\pi=[\pi_l,\pi_m,\pi_h]$ and $\mu=\begin{bmatrix}\mu_l\\ \mu_m\\ \mu_h\end{bmatrix}$ represent the steady state probabilities and the expected values of the low, moderate ad high states of the pneumonia cases.

The average pneumonia cases in the long-run, $E[L_r]$ will be estimated as;

$$E[L_r]=[\pi_l,\pi_m,\pi_h]\begin{bmatrix}\mu_l\\ \mu_m\\ \mu_h\end{bmatrix} \tag{3.28}$$

# CHAPTER FOUR

# RESULTS AND ANALYSIS

## 4.1 Introduction

Under this chapter, analysis, discussion and interpretation of the results from data on pneumonia cases are presented. These involve preliminary analysis and further analysis of the pneumonia cases.

## 4.2 Preliminary Analysis

It is seen in Table 4.1 that the minimum and maximum of pneumonia cases in the Builsa North Municipality are 9 and 370 respectively with an average pneumonia cases of about 121 during the period of study from 2015 to 2020. Also, Table 4.1 revealed that the variance is higher than the mean which suggests the presence of over-dispersion.

**Table 4.1: Descriptive Statistics of Pneumonia cases**

| Statistic | Value |
|---|---|
| Mean | 120.540 |
| Variance | 5675.860 |
| Min | 9.000 |
| Max | 370.000 |
| SD | 75.350 |
| CV | 65.500 |

An investigation of pneumonia cases on Monthly bases for the period considered for the study revealed that the least average pneumonia cases was recorded in the Month of February. Interestingly, March and November had the highest mean

pneumonia cases in the municipality with reported cases of approximately 165 as seen in Table 4.2. Further exploration to unveil which of the Months had the highest pneumonia cases recorded as well as the least cases during the period of consideration revealed that August had the least pneumonia cases and March had the highest pneumonia cases recorded with reported values of 9 and 370 respectively. Also, March had the highest variability followed by August and the Month of November had the lowest variability of pneumonia cases as shown by their coefficient of variations in Table 4.2.

**Table 4.2: Descriptive Statistics of Monthly Pneumonia Cases**

| Month | Mean | CV | Min | Max |
|---|---|---|---|---|
| January | 109.000 | 42.950 | 61.000 | 223.000 |
| February | 74.300 | 60.880 | 12.000 | 130.000 |
| March | 146.500 | 83.510 | 39.000 | 370.000 |
| April | 90.700 | 45.240 | 35.000 | 151.000 |
| May | 77.300 | 57.050 | 24.000 | 152.000 |
| June | 107.200 | 42.200 | 63.000 | 192.000 |
| July | 118.700 | 67.850 | 45.000 | 270.000 |
| August | 132.000 | 80.580 | 9.000 | 305.000 |
| September | 144.700 | 69.590 | 76.000 | 346.000 |
| October | 179.500 | 49.490 | 81.000 | 318.000 |
| November | 146.800 | 40.230 | 88.000 | 246.000 |
| December | 119.800 | 49.270 | 61.000 | 223.000 |

The correlation between the low, moderate and high states of the pneumonia cases was investigated using correlation coefficients as shown in Table 4.3. It can be seen from the table that there exists positive correlation between the low,

moderate and high states of the monthly pneumonia cases. The correlation between low pneumonia cases and moderate pneumonia cases is approximately 0.1. The correlation coefficient between low and high pneumonia cases is approximately 0.2 and the correlation coefficient between moderate and high pneumonia cases is also approximately 0.3. However, this linear relationship is insignificant since the correlation coefficients between the various states ranges from 0.1 to 0.3 approximately which indicates weak correlation between the variables. That is an increase in pneumonia cases in a state does not necessarily leads to an increment of pneumonia cases in another state.

**Table 4.3: Correlation Between the Three States of the Pneumonia Cases**

| State | Low | Moderate | High |
|---|---|---|---|
| Low | 1.0000 | | |
| Moderate | 0.0913 | 1.0000 | |
| High | 0.2059 | 0.2599 | 1.0000 |

The pattern of correlation between the low, moderate and high states of the monthly pneumonia cases are shown by Figure 4.1. It can be seen from the figure that an increment or decrement in any of the states shows no significant change in the other states.

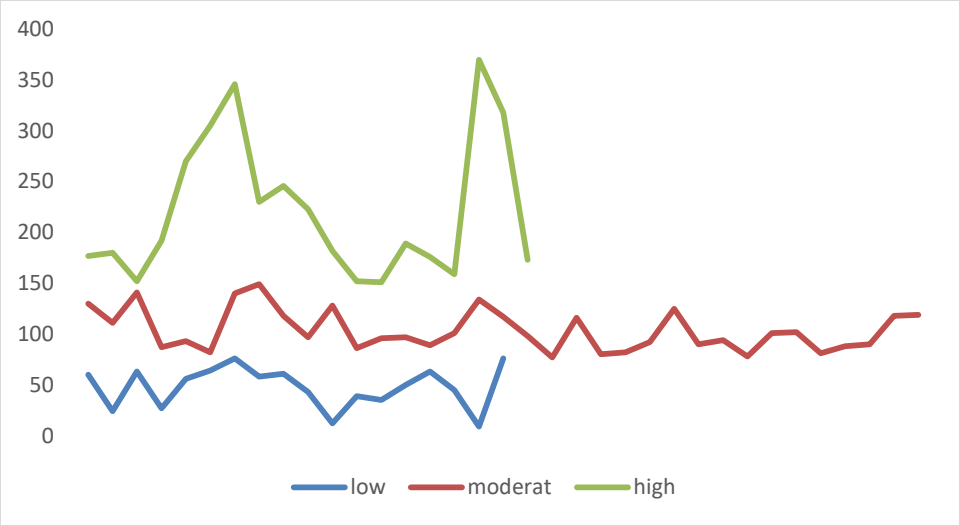


**Figure 4.1: Correlation Between the three States of the Pneumonia Cases**

The trend of pneumonia cases was visualized using the time series plot in Figure 4.2 below. The graph indicates a decreasing linear trend of the pneumonia cases over the period of study from 2015 to 2020.

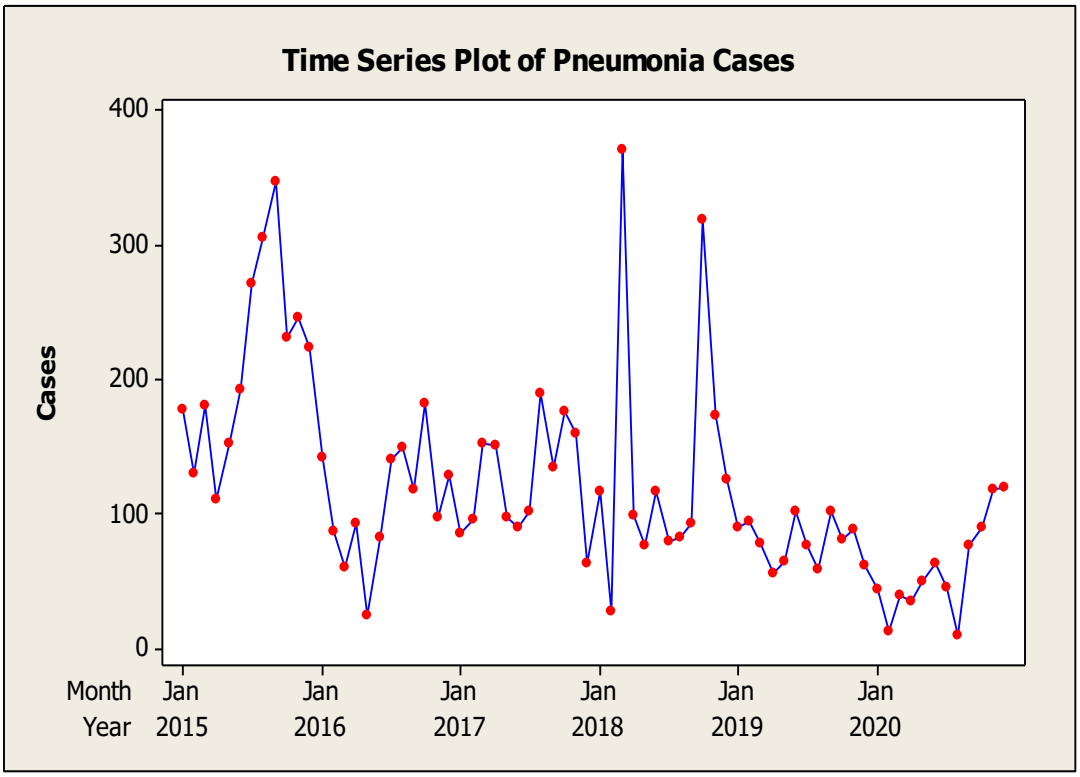


**Figure 4.2: Time Series Plot of the Pneumonia Cases**

Since the data on the monthly pneumonia cases is an over dispersed count data, the data was fitted using the negative binomial regression to cater for the over dispersion in order to explore the effect of the months on the pneumonia cases.

The estimates of the effect of the months on the incidence rate ratios (IRR) of pneumonia cases with reference to the month of April and their standard errors (SE), p-values and z-values are presented in Table 4.4. From Table 4.4, the given p-values revealed that only the month of October was significant with reference to the month of April. Also, whilst other months are held constant, the pneumonia cases incidence rates were expected to increase for all the Months with the exception of February and May as compared to the month of April. October had

the highest expected incidence rate of pneumonia cases since it was expected to have 1.98 times greater pneumonia incidence rate compared to that of April. Also, the pneumonia incidence rate was expected to decrease by 0.82 and 0.85 times for the months of February and May respectively as compared to April.

**Table 4.4: Monthly Effect on Pneumonia Incidence Rate**

| Month | IRR | SE | Z-value | P-value |
|---|---|---|---|---|
| January | 1.202 | 0.387 | 0.570 | 0.567 |
| February | 0.820 | 0.265 | -0.620 | 0.538 |
| March | 1.616 | 0.519 | 1.490 | 0.135 |
| May | 0.853 | 0.275 | -0.490 | 0.622 |
| June | 1.182 | 0.380 | 0.520 | 0.603 |
| July | 1.309 | 0.421 | 0.840 | 0.402 |
| August | 1.456 | 0.468 | 1.170 | 0.242 |
| September | 1.596 | 0.512 | 1.460 | 0.146 |
| October | 1.980 | 0.635 | 2.130 | 0.033 |
| November | 1.619 | 0.520 | 1.500 | 0.133 |
| December | 1.322 | 0.425 | 0.870 | 0.386 |
| Constant | 90.667 | 20.651 | 19.790 | 0.000 |
| Chi Square | | 2176.950 | | |

### 4.3 Further Analysis

The transition matrix which was used to estimate the probability of transitioning from a level of pneumonia cases to another required three states as pneumonia cases can assume three levels of low, moderate or high. The pneumonia cases were then classified into three states as Low (L), Moderate (M) and High (H) states. The classification of the pneumonia cases was achieved using the first quartile (Q1), second quartile (median) and upper quartile (Q3) of the pneumonia cases. The pneumonia cases less than or equal to the first quartile of approximately 76 were considered low. The cases between the first quartile and the upper quartile of approximately 76 and 152 respectively were classified as moderate and any pneumonia case greater than or equal to the upper quartile of 152 pneumonia cases were classed as high.

From the data collected on pneumonia cases for a period of seventy-two (72) months, the transitioning from one state to another of pneumonia cases was observed and compiled.

Table 4.5 presents the transition frequencies between the three states of the pneumonia cases which denote the number of pneumonia cases that are in a particular state in the current month given that the pneumonia cases were in a particular state in the previous month. As shown in Table 4.5, given that the pneumonia cases were in the low state in the previous month the transition frequencies with which the cases will make transitions to the current month in the low, moderate and high states were 11, 6 and 1 respectively. This indicates that when pneumonia cases were in the low state in the previous month, minimum

transitions were made to the high state and maximum transitions were made to the low state in the current month. Also, given that previously, the pneumonia cases were in the moderate state, it would make maximum transitions to the moderate state in the current month with transition frequency of 22, transition frequency of 7 to the high state in the current month and would make the least transitions to the low state in the current month with transition frequency of 6. Moreover, it is depicted from Table 4.5 that, given the pneumonia cases being in the high state in the previous month, it will make transitions to the low, moderate and high states in the current month with transition frequencies of 1, 8 and 9 respectively. This means that, if the pneumonia cases were in the high state in the previous month, it will make fewer transitions to the low state and more transitions to the high state than the moderate state in the current month.

It can be deduced from above that when low pneumonia cases are recorded in the previous month then, there is high possibility that low pneumonia cases will be recorded in the current month. Also, when moderate pneumonia cases are recorded in the previous month then the likelihood that moderate pneumonia cases will be recorded in the current month is high and when high pneumonia cases are recorded in the previous month then, there is high likelihood that high pneumonia cases will be recorded in the current month. Therefore, when measures are put in place to reduce pneumonia cases to the low state then, there is high probability of recording low pneumonia case.



**Table 4.5: Transition Frequencies of Monthly Pneumonia Cases**

| | | Current | | |
|---|---|---|---|---|
| | **State** | **L** | **M** | **H** |
| | **L** | 11 | 6 | 1 |
| **Previous** | **M** | 6 | 22 | 7 |
| | **H** | 1 | 8 | 9 |

To estimate the transition probabilities from Table 4.5 from which the transition probability matrix was formulated, the data on pneumonia cases was tested to investigate whether the first-order Markov chain assumption holds for the three states by performing chi square test of independence. The chi square test statistic of 22.895 with p-value of 0.0000 indicated that the first-order Markov chain assumption holds for the states of the pneumonia cases.

Table 4.6 presents the transition probabilities of the pneumonia cases from one state to another. It can be seen from the Table 4.6 that there is 0.6110 chances of pneumonia cases being in the low state in the current month given that it was in the low state in the previous month. Also, from the Table 4.6, given that the pneumonia cases were in the low state in the previous month the probability of being in the moderate state in the current month was 0.3330. It can also be seen that there is 0.0560 likelihood that the pneumonia cases will be in the low state in the current month given that it was in the low state in the previous month.

Also, given that in the previous month the pneumonia cases were in the moderate state, then the probability that it will be in the low state in the current month is 0.1710. The chances of being in the moderate state in the current month given that

the pneumonia cases were in the moderate state in the previous month is 0.6290 and given that the pneumonia cases were in the moderate state in the previous month, then the probability of the pneumonia cases being in the high state in the current month is 0.2000.

Furthermore, given that the pneumonia cases were in the high state in the previous month, the probability of transitioning to the low state in the current month is 0.0560 as seen in the transition matrix. There is 0.4440 chances of the pneumonia cases being in the moderate state in the current month given that the cases were in the high state in the previous month and given that the pneumonia cases were in the high state in the previous month, then there is 0.5000 likelihood of the cases being in the high state in the current state.

It can be inferred that when low pneumonia cases are recorded in the previous month in the municipality, there is a higher likelihood that low pneumonia cases will be observed in the current month in the municipality with a probability of 61.1% as compare to probabilities of 33.3% and 5.6% in moving to the moderate and high state respectively in the current month. This implies that when measures are put in place to reduce pneumonia cases in the municipality, there is the tendency that the pneumonia cases to be recorded will remain low. Therefore, there is the need for stake holders to put in place preventive measures of pneumonia as well as ensure their adherence to improve upon the reduction of pneumonia cases in the municipality.

**Table 4.6: Transition Probabilities of the Pneumonia Cases**

| | | Current Month | | |
|---|---|---|---|---|
| | **State** | **Low (L)** | **Moderate (M)** | **High (H)** |
| **Previous Month** | **Low (L)** | 0.6110 | 0.3330 | 0.0560 |
| | **Moderate (M)** | 0.1710 | 0.6290 | 0.2000 |
| | **High (H)** | 0.0560 | 0.4440 | 0.5000 |

The transition probability matrix P is

$$P = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.6110 & 0.3330 & 0.0560 \\ 0.1710 & 0.6290 & 0.2000 \\ 0.0560 & 0.4440 & 0.5000 \end{bmatrix} \end{array}$$

The communication among the three states of the pneumonia cases was visualized by plotting a transition diagram as shown in Figure 4.3 using the transition probability matrix.

From the figure, the transition probability of remaining in the law, moderate and high state of pneumonia cases were 61.10%, 62.9% and 50% respectively. The probability of transitioning from the low state of pneumonia cases to the moderate and high states of the pneumonia cases were 33.30% and 5.60% respectively and the probabilities of moving from the moderate state to the low state and from the high state to the low state were 17.10% and 5.60%. Also, the probability of transitioning from the moderate state of the pneumonia cases to the high state was 20.00% and 44.40% vice versa. It can be inferred that there is high probability of remaining in a particular state of the pneumonia cases than moving to a different state. This means that if preventive measures of pneumonia are adhered to reduce the prevalence of pneumonia cases then the likelihood of pneumonia cases

remaining low would be high. It is therefore prudent for stakeholder to create awareness of the lethal impact of pneumonia as well put in place appropriate preventive measures so as to reduce pneumonia cases in the Builsa North municipality.

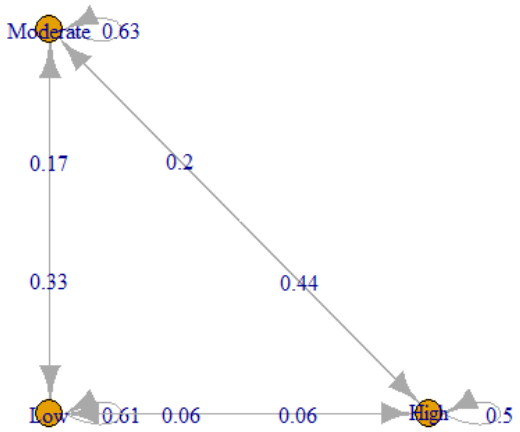


**Figure 4.3: Transition Diagram for the Three States of Pneumonia Cases**

The steady state probabilities (SSP), the mean recurrence time (MRT) and the sojourn time (ST) for each of the states of the pneumonia cases were estimated from the transition matrix of the pneumonia cases. Table 4.7 presents the estimates of the SSP, MRT and the ST. From Table 4.7, the steady state probabilities of the pneumonia cases for the low, moderate and high states were calculated as $\pi_1 = 0.258, \pi_2 = 0.510$ *and* $\pi_3 = 0.233$ respectively. The steady state probabilities indicate that in the long-run the probabilities of pneumonia cases being in the low, moderate and high states will be approximately 26%, 51% and 23% respectively. It can be deduced from the steady state probabilities of the states that in the long-run there are higher chances of pneumonia cases to be in the

moderate state than the low and high states. That is the long-run the probability of the pneumonia cases being in the moderate state is 51%, approximately 26% of the pneumonia cases will be in the low state and 23% of the pneumonia cases will be in the high state in the Builsa North Municipality.

The time taken for the pneumonia cases to return to each of the states for the pneumonia cases were obtained as seen in Table 4.7 The recurrent time for the low state was estimated as 3.883 months, the moderate state was estimated as 1.962 months and a recurrent time of 4.297 months was estimated for the high state. This implies that it takes approximately 4 months, 2 months and 4 months for pneumonia cases to return to the low, moderate and high states of the pneumonia cases after leaving those states. It is seen that it takes the pneumonia cases shorter time to return to the moderate state than to the low and high states.

Also, the expected time it takes the pneumonia cases to stay in a particular state of the pneumonia cases in the municipality was estimated as indicated in Table 4.7. The expected length of time the pneumonia cases are expected to stay in the low and moderate states were estimated to be approximately 3 months each and 2 months for the high state. Hence the complete cycle of the pneumonia cases in the Builsa North municipality was estimated to be 8 months.

**Table 4.7: The SSP, MRT and ST of Pneumonia Cases for the Builsa North Municipality**

| State | SSP | MRT | ST |
|---|---|---|---|
| Low | 0.258 | 3.883 | 3 |
| Moderate | 0.510 | 1.962 | 3 |
| High | 0.233 | 4.297 | 2 |

To investigate the number of months it takes for the various states of the pneumonia cases to be in equilibrium, the *n*-step transition probabilities were estimated from the transition probability matrix of the pneumonia cases. It was known from the estimation that it will take the states after twenty-eight (28) months which is two years four months to be in equilibrium. The 28-step transition probability matrices are given below. From the steady state matrix, the probability of the pneumonia cases being in the low, moderate and high states after twenty-eight months given that the pneumonia cases were in the low, moderate and high states were 0.2575645, 0.5097059 and 0.2327296 respectively.

$$P = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.6110 & 0.3330 & 0.0560 \\ 0.1710 & 0.6290 & 0.2000 \\ 0.0560 & 0.4440 & 0.5000 \end{bmatrix} \end{array}$$

$$P^2 = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.43340 & 0.437784 & 0.128816 \\ 0.22324 & 0.541384 & 0.235376 \\ 0.13814 & 0.519924 & 0.341936 \end{bmatrix} \end{array}$$

$$P^3 = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.3468822 & 0.4768826 & 0.1762352 \\ 0.2421574 & 0.5193764 & 0.2384662 \\ 0.1924590 & 0.5248524 & 0.2826886 \end{bmatrix} \end{array}$$

$$P^4 = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.3033611 & 0.4937194 & 0.2029195 \\ 0.2501256 & 0.5132052 & 0.2366692 \\ 0.2231727 & 0.5197347 & 0.2570925 \end{bmatrix} \end{array}$$

.
.
.

$$P^{27} = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.2575645 & 0.5097059 & 0.2327296 \\ 0.2575645 & 0.5097059 & 0.2327296 \\ 0.2575644 & 0.5097059 & 0.2327296 \end{bmatrix} \end{array}$$

$$P^{28} = \begin{array}{c} \\ L \\ M \\ H \end{array} \begin{array}{c} \begin{array}{ccc} L & M & H \end{array} \\ \begin{bmatrix} 0.2575645 & 0.5097059 & 0.2327296 \\ 0.2575645 & 0.5097059 & 0.2327296 \\ 0.2575645 & 0.5097059 & 0.2327296 \end{bmatrix} \end{array}$$

Table 4.8 shows the first passage times (FPT) probabilities of pneumonia cases assuming they are in the low state. From Table 4.8, the probability that low pneumonia cases will be observed in the next month is 0.6110 assuming that in the current month pneumonia cases are in the low state. Assuming the pneumonia cases in the current month is low, the probability that there will be moderate pneumonia cases in the municipality in the next month is 0.3330. There is 0.0560 chances of witnessing high pneumonia cases in the municipality in the next month given that pneumonia cases are low in the current month. This implies that assuming low pneumonia cases are observed in the current month then, there is higher probability of low pneumonia cases being observed in the next month. That is, if measures are put in place to decrease pneumonia cases, there will be high likelihood of having low pneumonia cases in the Builsa North municipality.

Table 4.8 also indicates that, if low pneumonia cases are observed in the fourth month as the current month then, there is 0.0377 likelihood of observing low pneumonia cases in the next month. Given that in the fourth month the pneumonia cases are in the low state, the probability of observing moderate pneumonia cases in the next month is 0.1009. Also, for the high pneumonia cases to be observed in the next month, given that the forth month has low pneumonia cases, then the probability will be 0.0993.

In the eight months, if the current pneumonia cases are low, the probability of observing low pneumonia case in the next month is 0.0211. There will be 0.0176 likelihood of having moderate pneumonia cases in the next month given that pneumonia cases in the eight month is low and the probability that there will be high pneumonia case in the municipality in the next month given that there are low pneumonia cases is 0.0572. It can be inferred that, when low pneumonia cases are observed in the eighth month, there will be higher probability of having high pneumonia cases in the next month than observing low and moderate pneumonia cases.

Moreover, if low pneumonia cases are observed currently in the twelfth month then, there will be 0.0120 chance of observing low pneumonia cases in the next month. Also, for moderate pneumonia to be observed in the municipality in the next month given that the twelfth month has low pneumonia cases will be 0.0029 and the probability we will observe high pneumonia cases in the next month is 0.0312. This means that when low pneumonia cases are observed in the twelfth

month then the municipality will have the lowest probability of experiencing moderate pneumonia cases.

It is also seen in Table 4.8 that from the first to the forth months when low pneumonia cases are observed then there is higher probability of observing moderate pneumonia cases in the next month than observing high pneumonia cases and from the fifth to the twelfth month there is high probability of observing high pneumonia cases than moderate pneumonia cases.

**Table 4.8: FPT Probabilities Assuming the current state of Pneumonia Cases is Low**

| | States | | |
|---|---|---|---|
| **Month** | **Low** | **Moderate** | **High** |
| 1 | 0.6110 | 0.3330 | 0.0560 |
| 2 | 0.0601 | 0.2280 | 0.1008 |
| 3 | 0.0454 | 0.1530 | 0.1067 |
| 4 | 0.0377 | 0.1009 | 0.0993 |
| 5 | 0.0323 | 0.0659 | 0.0882 |
| 6 | 0.0279 | 0.0427 | 0.0768 |
| 7 | 0.0243 | 0.0275 | 0.0664 |
| 8 | 0.0211 | 0.0176 | 0.0572 |
| 9 | 0.0183 | 0.0113 | 0.0492 |
| 10 | 0.0160 | 0.0072 | 0.0423 |
| 11 | 0.0139 | 0.0046 | 0.0363 |
| 12 | 0.0120 | 0.0029 | 0.0312 |

The first passage time (FPT) probabilities given that the current pneumonia cases are in the moderate state are presented in Table 4.9. From the table, in the first month, given that the current pneumonia cases are in the moderate state, the probability of observing low pneumonia cases in the next month is 0.1710, 0.6290 likelihood of getting moderate pneumonia cases in the next month and 0.2000 probability of observing high pneumonia cases in the next month. It can be depicted from Table 4.9 that in the first month, when current pneumonia cases are moderate, then there is higher probability of having moderate pneumonia cases in the next month than low and high pneumonia cases in the municipality.

In the fourth month, the probability of experiencing low pneumonia cases in the next month if the current pneumonia cases are in the moderate state is 0.0810. Also, given that the current pneumonia cases are in the moderate state, then the likelihood of observing moderate pneumonia cases in the next month is 0.0528 while, the likelihood of observing high pneumonia cases in the next month is 0.0826.

Also, in the eighth month, given that current pneumonia cases are moderate, the probability of observing low pneumonia cases in the next month is 0.0458, 0.0077 likelihood of observing moderate pneumonia cases in the next month and 0.0427 probability of observing high pneumonia cases in the next month. This can be inferred that when current pneumonia cases in the eighth month are in the moderate state, then the municipality has high probability of recording low pneumonia cases in the next month than moderate and high cases.

Table 4.9 also indicated that, assuming the current pneumonia cases in the twelfth month is moderate, the probability of pneumonia cases being low in the next month is 0.0262. There is 0.0012 chance of getting moderate pneumonia cases in the next month given that the current pneumonia cases are moderate whilst the probability of having high pneumonia cases in the next month is 0.0232. This implies that whenever current pneumonia cases in the twelfth month are moderate, then there is high probability of recording low pneumonia cases in the next moth than moderate and high cases.

Table 4.9 revealed that when current pneumonia cases are moderate, there is high probability of observing high pneumonia cases than low pneumonia cases in the next month from the first to the forth months, whilst from the fifth month to the twelfth month the likelihood of observing low pneumonia cases is higher than observing high pneumonia cases.



**Table 4.9: FPT Probabilities Assuming the current state of Pneumonia Cases is Moderate**

| | State | | |
|---|---|---|---|
| **Month** | **Low** | **Moderate** | **High** |
| 1 | 0.1710 | 0.6290 | 0.2000 |
| 2 | 0.1188 | 0.1457 | 0.1354 |
| 3 | 0.0955 | 0.0872 | 0.1024 |
| 4 | 0.0810 | 0.0528 | 0.0826 |
| 5 | 0.0699 | 0.0323 | 0.0690 |
| 6 | 0.0606 | 0.0199 | 0.0585 |
| 7 | 0.0527 | 0.0124 | 0.0499 |
| 8 | 0.0458 | 0.0077 | 0.0427 |
| 9 | 0.0398 | 0.0048 | 0.0367 |
| 10 | 0.0346 | 0.0030 | 0.0315 |
| 11 | 0.0301 | 0.0019 | 0.0270 |
| 12 | 0.0262 | 0.0012 | 0.0232 |



The first passage time (FPT) probabilities for the Builsa North Municipality given that the pneumonia cases are currently high are shown in Table 4.10. Given that the current pneumonia cases in the Builsa North Municipality is high, the probability of observing low pneumonia cases in the municipality in the next month is 0.0560. Similarly, the probability of observing moderate pneumonia cases in the municipality in the next month given that the current pneumonia cases in the municipality is high is 0.4440 as well as 0.5000 chances of observing high pneumonia cases in the next month given that there are currently high pneumonia cases in the municipality. This means that in the first month there is

higher probability of recording high pneumonia cases in the municipality in the next month given that the municipal currently have high pneumonia cases.

In the forth month assuming the current pneumonia cases are in the high state, the probability of having moderate pneumonia cases in the next month is 0.751, 0.0947 chances of observing low pneumonia cases in the month and 0.0514 probability of observing high pneumonia cases in the next month. This shows that in the fourth month, when current pneumonia cases are high, there is higher likelihood of having low pneumonia cases in the Builsa North Municipal in the next month than having moderate and high pneumonia cases.

In addition, it is seen in Table 4.10 that in the eighth month, when the current pneumonia cases are high, then the probability of having low pneumonia cases in the next month is 0.0550 whilst the likelihood of observing moderate pneumonia cases in the next month is 0.0091. Also, the probability of observing high pneumonia cases in the next month given that the pneumonia cases are currently in the high state is 0.0259. That is in the eighth month when pneumonia cases are currently high, there is high probability of having low pneumonia cases in the next month in the municipality than observing moderate and high pneumonia cases.

Moreover, in the twelfth month, if the current pneumonia cases are high, the probability of having low pneumonia cases in the next month is 0.0314. There is 0.0013 probability of observing moderate pneumonia cases in the next month given that the pneumonia cases are currently in the high state. However, if the

current pneumonia cases are in the high state then, the probability of having high pneumonia cases in the next month is 0.0140. This shows that in the twelfth month when the current pneumonia cases in the municipal is high, then there is lower probability of observing moderate pneumonia cases in the next month.

Furthermore, it can be seen in Table 4.10 that, for the first three months, when the current pneumonia cases are in the high state, there is high probability of the cases being in the moderate state than being in the low state in the next month whilst from the forth to the twelfth months, there is high probability of observing low pneumonia cases than moderate pneumonia cases in the next month.

**Table 4.10: FPT Probabilities Assuming the current state of Pneumonia cases is High**

| | State | | |
|---|---|---|---|
| **Month** | **Low** | **Moderate** | **High** |
| 1 | 0.0560 | 0.0444 | 0.5000 |
| 2 | 0.1039 | 0.2406 | 0.0919 |
| 3 | 0.1047 | 0.1331 | 0.0658 |
| 4 | 0.0947 | 0.0751 | 0.0514 |
| 5 | 0.0833 | 0.0432 | 0.0423 |
| 6 | 0.0727 | 0.0253 | 0.0356 |
| 7 | 0.0633 | 0.0150 | 0.0303 |
| 8 | 0.0550 | 0.0091 | 0.0258 |
| 9 | 0.0478 | 0.0055 | 0.0222 |
| 10 | 0.0416 | 0.0034 | 0.0190 |
| 11 | 0.0361 | 0.0021 | 0.0163 |
| 12 | 0.0314 | 0.0013 | 0.0140 |

The long-run expected number of pneumonia cases was estimated from the long-run probabilities, $\pi$ and mean vector, $\mu$ of the various states of the pneumonia cases.

The long-run probabilities were estimated as,

$$\pi = [0.258 \quad 0.510 \quad 0.233]$$

Mean of the three states of the pneumonia cases were estimated as,

$$\mu = \begin{bmatrix} 47.833 \\ 103.629 \\ 220.579 \end{bmatrix}$$

The expected pneumonia cases in the municipality in the long-run was estimated as,

$$\text{Expected pneumonia cases} = [0.258 \quad 0.510 \quad 0.233] \begin{bmatrix} 47.833 \\ 103.629 \\ 220.579 \end{bmatrix}$$

$$= 116.458$$

This implies that the Builsa North Municipality in the long-run would record an average pneumonia cases of approximately 116.

# CHAPTER FIVE

# SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## 5.1 Introduction

In this chapter, the summary of the findings of the research, conclusion and recommendations of the study are presented.

## 5.2 Summary

This research was primarily to model monthly Pneumonia cases in the Builsa North Municipality for the period of seventy-two months (six years) using Markov chain modeling to estimate significant metrics of the disease such as steady state probabilities, the long-run expected cases, recurrent times, sojourn times, mean recurrent times and the first passage times of the various states of the pneumonia cases.

Using time series plot, the trend of the pneumonia cases was visualized which revealed a decreasing non-seasonal trend over the period of study from 2015 to 2020. Also, estimated correlation coefficients indicated that there exists positively weak correlation between the low, moderate and high states of the monthly pneumonia cases.

Findings from the research revealed that when pneumonia cases are in a particular state in the current month then, there is higher probability of the pneumonia cases being recorded in the same state in the next month compare to transitioning to the other states. The study also discovered that in the long-run there will be higher chances of recording moderate pneumonia cases in the municipality with a

probability of approximately 51% as compared to low and high pneumonia cases with probabilities of 26% and 23% respectively and the complete cycle of the pneumonia cases was estimated to be 8 months. It was also revealed that in the long-run the municipality will record on average approximately 116 pneumonia cases.



### 5.3 Conclusions

The major aim of the study was to model monthly pneumonia cases in the Builsa North Municipality from January, 2015 to December, 2020 using the first-order Markov chain modeling. The data on the monthly pneumonia cases was tested to confirm its conformity with the first-order Markov chain assumption before fitting the model. The data was modeled into three states, low, moderate and high states. The relationship between the states was explored which revealed a positively weak correlation between the states. It also revealed a non-seasonal trend of the pneumonia cases for the period of study with the aid of time series plot.

The model revealed that, there is a high long-run probability of recording moderate pneumonia cases than that of low and high pneumonia cases. It was discovered from the study that for a given current state of pneumonia cases there is higher probability of observing pneumonia cases in the same state. Hence if measures are put in place to be observed in other to put pneumonia cases to the low state in the municipality, there would be a higher possibility of observing low pneumonia cases in the municipality than moderate and high cases.

### 5.4 Recommendations

From the findings of the study, the following recommendations were made;

i. Since there is high probability of pneumonia cases being in the same state as its current state, it is recommended that adequate attention be given to pneumonia by the government and other agencies by putting in place more efficacious measures and ensuring their adherence to minimize pneumonia cases in the municipality and the country at large.

ii. It is also recommended that public health education be organized to create awareness of the lethal impact of pneumonia especially on children so as to intensity compliance to preventive measures and remedies in order to minimize the prevalence of pneumonia infections.

iii. It is also recommended that future studies can consider other states such as susceptibility, infection and recovery states of pneumonia as well as sex in estimating the relevant metrics of pneumonia

iv. It is also recommended that future studies should consider other models to estimate relevant epidemiological quantities of pneumonia to see which one would be most appropriate.

## REFERENCES

Adigun, K., Adeleke, A., Adewusi, O., Olubiyi, O., Halid, O., & Babalola, B. (2019). Modelling Infectious Diseases Using Markov Chain. *American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)*, *61*(1), 280-288.

Ait-Khaled, N., Enarson, D. A., & Stop TB Initiative. (2003). *Tuberculosis: a manual for medical students* (No. WHO/CDS/TB/99.272). World Health Organization.

American Lung Association (2020). Preventing pneumonia. https://www.lung.org/lung-health-diseases/lung-disease-lookup/pneumonia/preventing-pneumonia.Accessed, 11/5/2021

American Lung Association. (2020). Learn about pneumonia. *https://www.lung.org/lung-health-diseases/lung-disease-lookup/pneumonia/learn-aboutpneumonia.* Retrieved, 25/12/2020

American Lung Association. (2020). What causes pneumonia?. *https://www.lung.org/lung-health-diseases/lung-disease-lookup/pneumonia/what-causespneumonia#:~:text=the%20proper%20treatment.-,Bacteria,in%20the%20upper%20respiratory%20tract*. Accessed, 22/12/2020

Ansah-Narh, T., Nortey, E. N. N., & Amponsah, K. D. (2013). Prediction of Subscribers'brand Switching Behaviour and Ergodic Market Share of Network Service Providers In Ghana Using Markov Chain Model, 2(4), 298-303

Backhaus, E., Berg, S., Andersson, R., Ockborn, G., Malmström, P., Dahl, M., ... & Trollfors, B. (2016). Epidemiology of invasive pneumococcal infections: manifestations, incidence and case fatality rate correlated to age, gender and risk factors. *BMC infectious diseases*, *16*(1), 1-12.

Barbara Jones, M.D., Nathan Dean, M.D., Richard Wunderink, M.D. & Marianna Sockrider, M.D. (2020). What is pneumonia? *American Thoracic Society*. https://www.thoracic.org/patients/patient-resources/resources/what-is-pneumonia.pdf. Retrieved, 25/12/2020.

Bernadeta D. (2018). "Pneumonia". *Our World of Data.* https://ourworldindata.org/pneumonia. *Retrieved, 13/01/2021.*

Bosch, A. A., Biesbroek, G., Trzcinski, K., Sanders, E. A., & Bogaert, D. (2013). Viral and bacterial interactions in the upper respiratory tract. *PLoS pathogens*, *9*(1), e1003057.

Bree N. (2019). Types of Pneumonia. *(*https://www.healthline.com/health/pneumonia#types. Accessed, 23/12/2020)

Bree, N.(2019). Everything you need to know about pneumonia.https://www.healthline.com/health/pneumonia. Retrieved, 27/12/2020.

Carol, D. (2020). What is bacterial pneumonia?

Centers of Disease Control and Pneumococcal Vaccination. *https://www.cdc.gov/vaccines/vpd/pneumo/index.html#:~:text=CDC%20recommends%20PCV13%20for%20all,their%20clinician%2C%20to%20get%20PCV13.* Accessed, 2021, 5, 12

Centers of Disease Control and prevention (2020).https://www.cdc.gov/pneumococcal/about/risktransmission.html#:~:text=Adults%20at%20Ris%20for%20Pneumococcal%20Disease&text=With%20chronic%20illnesses%20(chronic%20heart,%2C%20or%20damaged%2Fabsent%20spleen.Accessed, 15/12/2020.

Corey Whelan, (2018). How to Prevent Pneumonia: Vaccine, Other Tips, and More. *(healthline.com)*. Accessed, 3/28/2021.

Crum-Cianflone, N. F. (2008). Bacterial, fungal, parasitic, and viral myositis. *Clinical microbiology reviews*, *21*(3), 473-494.

Datong, G. M. (2011). A Markov Chain Model Analysis of GSM Network Service Providers of Marketing Mix. Ijens.*Org, II (4), 38-43. Retrieved from* http://www.ijens.org, *10/11/2020*

Elavarasan, D., Vincent, D. R., Sharma, V., Zomaya, A. Y., & Srinivasan, K. (2018). Forecasting yield by integrating agrarian factors and machine learning models: A survey. *Computers and Electronics in Agriculture*, *155*, 257-282.

Eliakim-Raz, N., Robenshtok, E., Shefet, D., Gafter-Gvili, A., Vidal, L., Paul, M., & Leibovici, L. (2012). Empiric antibiotic coverage of atypical pathogens for community-acquired pneumonia in hospitalized adults. *Cochrane Database of Systematic Reviews*, (9).

Giebeler, A., Newhauser, W. D., Amos, R. A., Mahajan, A., Homann, K., & Howell, R. M. (2013). Standardized treatment planning methodology for passively scattered proton craniospinal irradiation. *Radiation oncology*, *8*(1), 1-17.

Grimmett, G., & Stirzaker, D. (2020). Probability and random processes. Oxford university press.

Howie, S. R., & Murdoch, D. R. (2019). Global childhood pneumonia: the good news, the bad news, and the way ahead. *The Lancet Global Health*, *7*(1), e4-e5.

https://www.cdc.gov/pneumococcal/about/risk-transmission.html#:~:text=Adults%20at%20Risk%20for%20Pneumococcal%20Disease&text=With%20chronic%20illnesses%20(chronic%20heart,%2C%20or%20damaged%2Fabsent%20spleen. Accessed, 15/12/2020.

Ibe, O. (2014). Fundamentals of applied probability and random processes. *Academic Press.*

Iddrisu, A. K., Alhassan, A., & Amidu, N. (2019). Survival analysis of birth defect infants and children with pneumonia mortality in Ghana. *Advances in Public Health, 2019.*

Jansen, K. U., Knirsch, C., & Anderson, A. S. (2018). The role of vaccines in preventing bacterial antimicrobial resistance. *Nature medicine*, *24*(1), 10-19.

Jill, S.S. (2018). Why pneumonia can be deadly for some people. *https://www.healthline.com/health/pneumonia/can-you-die-from-pneumonia#risk.* Accessed, 15/12/2020.

Kaplan, W., Wirtz, V. J., & Mantel-Teeuwisse, A. (2018). Priority medicines for Europe and the World 2013 update.

Kassa, A. M., Abrham, E., & Seid, T. (2017). Application of Markov Chain Analysis Model for Predicting Monthly Market Share of Restaurants. *International Journal of Recent Engineering Research and Development*, *2*(3), 48-55.

Kermack, W. O., & McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of*

*london. Series A, Containing papers of a mathematical and physical character*, *115*(772), 700-721.

Kim, G. L., Seon, S. H., & Rhee, D. K. (2017). Pneumonia and Streptococcus pneumoniae vaccine. *Archives of pharmacal research*, *40*(8), 885-893.

Kim, G. L., Seon, S. H., & Rhee, D. K. (2017). Pneumonia and Streptococcus pneumoniae vaccine. *Archives of pharmacal research*, *40*(8), 885-893.

Koivula, I., Sten, M., & Makela, P. H. (1994). Risk factors for pneumonia in the elderly. *The American journal of medicine*, *96*(4), 313-320.

Lassi, Z. S., Imdad, A., & Bhutta, Z. A. (2017). Short-course versus long-course intravenous therapy with the same antibiotic for severe community-acquired pneumonia in children aged two months to 59 months. *Cochrane Database of Systematic Reviews*, (10).

Le Roux, D. M., & Zar, H. J. (2017). Community-acquired pneumonia in children—a changing spectrum of disease. *Pediatric radiology*, *47*(11), 1392-1398.

Liu, X., Ren, L., Yuan, F., Yang, B. & Nanjing, P.R. (2009). Meteorological drought Forecasting using Markov Chain model. *In: 11th International Conference on Environmental Science and Information Application Technology, Chania, Crete, Greece, pp 23–26* http://toc.proceedings.com/18229webtoc.pdf. *Retrieved, 10/11/2020*

Lohani, V. K., Loganathan, G. V., & Mostaghimi, S. (1998). Long-term analysis and short-term forecasting of dry spells by Palmer Drought Severity Index. *Hydrology Research*, *29*(1), 21-40.

Mackenzie, G. (2016). The definition and classification of pneumonia. *Pneumonia*, *8*(1), 1-5.

Manivel, A., Sudarwati, S., & Herdiningrat, R. S. (2015). Clinical Profile and Mortality in Children with Pneumonia. *Althea Medical Journal*, *2*(2), 235-240.

McHugh, M. L. (2013). The chi-square test of independence. *Biochemia medica*, *23*(2), 143-149.

Mhandu, E. (2019). Building a low-cost biomedical device to improve accuracy in pneumonia diagnosis in under five children.

Mirza, A. Z., Shamshad, H., Osra, F. A., Habeebullah, T. M., & Morad, M. (2020). An overview of viruses discovered over the last decades and drug development for the current pandemic. *European Journal of Pharmacology*, 173746.

Obu, E. (2017). *Pharmacotherapy of pneumonia in children under five years in two hospitals in the Ashanti Region of Ghana* (Doctoral dissertation).

Pant, A., Jain, A., Nayak, K. C., Gandhi, D., & Prasad, B. G. (2020, July). Pneumonia detection: An efficient approach using deep learning. In *2020*

*11th International Conference on Computing, Communication and Networking Technologies (ICCCNT)* (pp. 1-6). IEEE.

Paulo, A. A., & Pereira, L. S. (2007). Prediction of SPI drought class transitions using Markov chains. *Water resources management*, *21*(10), 1813-1827.

Paulo, A. A., Ferreira, E., Coelho, C., & Pereira, L. S. (2005). Drought class transition analysis through Markov and Loglinear models, an approach to early warning. *Agricultural water management*, *77*(1-3), 59-81.

Peter, C. (2017). What you should know about pneumonia. https://www.medicalnewstoday.com/articles/151632. Accessed, 20/12/2020.

Pobbi, M. A. (2012). *A three-state Markov Chain Model of plasmodium falciparum parasitemia transmission in Ghana* (Doctoral dissertation).

Samuel, M. (2011). Pneumonia in Pre-School Children.

Sharma, L., Losier, A., Tolbert, T., Cruz, C. S. D., & Marion, C. R. (2017). Pneumonia updates on Legionella, Chlamydophila, and Mycoplasma pneumonia. *Clinics in chest medicine*, *38*(1), 45.

Sundberg, J., and Klacksell, G. (2012). Can you describe the stock index with a Markov chain?.

Susan, C. (2019). Pneumonia symptoms & risk factors. (https://www.news-medical.net/health/Pneumonia-Symptoms-Risk-Factors.aspx. Accessed, *23/12/2020*)

Swedberg, E., Shah, R., Sadruddin, S., & Soeripto, J. (2020). Saving young children from forgotten killer: pneumonia.

Teodorescu, I. (2009). Maximum likelihood estimation for Markov Chains. *arXiv preprint arXiv:0905.4131*.

The Lancet Global Health Editorial (2018). The disgraceful neglect of childhood pneumonia. *The Lancet Global Health, 6(12), e1253.*

Tong, N., & BA, M. (2013). Background paper 6.22 pneumonia. *A Public Health Approach to Innovation*, *1*(1), 7-8.

Twumasi, CL. (2018). *Markov Chain Modeling of HIV, Tuberculosis and Hepatitis-B Transmission: A Study of a Regional Hospital in Ghana* (Doctoral dissertation, University of Ghana).

UNICEF, W. (2012). Pneumonia and diarrhoea: tackling the deadliest diseases for the world's poorest children. *New York: UNICEF*.

US Department of Health and Human Services. (2018). AIDS Info Glossary of HIV/AIDS Related Terms.

Ventola, C. L. (2016). Immunization in the United States: recommendations, barriers, and measures to improve compliance: part 1: childhood vaccinations. *Pharmacy and Therapeutics*, *41*(7), 426.

Wardlaw, T. M., Johansson, E. W., & Hodge, M. J. (2006). *Pneumonia: the forgotten killer of children*. Unicef.

WHO, (2020). Pneumonia is the leading cause of death in children. https://www.who.int/maternal_child_adolescent/news_events/news/2011/pneumonia/en/. *Retrieved, 13/01/2021*.

World Health Organization, (2019). Pneumonia.https://www.who.int/news-room/fact-sheets/detail/pneumonia. *Accessed, 6/12/2020.*

World Health Organization. (2019). *WHO guidelines on tuberculosis infection prevention and control: 2019 update* (No. WHO/CDS/TB/2019.1). World Health Organization.

Yan, Q., Qin, C., Nie, M., & Yang, L. (2018). Forecasting the electricity demand and market shares in retail electricity market based on system dynamics and Markov chain. *Mathematical Problems in Engineering*, *2018*.

Zawn, V. (2017). "Is Pneumonia contagious? *Is pneumonia contagious? Causes and transmission | MedicalRecords.com*. Accessed, 25/11/2020.