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Heat and Mass Transfer over a Vertical Surface with Convective Boundary Conditions in the Presence of Viscous Dissipation and nth Order Chemical Reaction

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Abstract

This study investigated the effects of an nth order chemical reaction and viscous dissipation on the heat and mass transfer over a vertical surface with convective boundary conditions. A similarity analysis was used to transform the system of partial differential equations describing the problem into ordinary differential equations. The reduced system was solved using the Newton Raphson shooting method alongside the Forth-order Runge - Kutta algorithm. The results are presented graphically and in tabular form for various controlling parameters.

Keywords: Convective boundary condition; viscous dissipation; chemical reaction; heat and mass transfer; vertical plate, similarity analysis

1.0 Introduction

The subject of coupled heat and mass transfer due to natural convection has gained considerable attention in the scientific community due to its diverse applications in engineering systems. These applications include, cooling of electronic equipment, nuclear reactors, food processing and polymer production among others. Natural or free convection occurs due to the change in density of the fluid medium caused by a heating process. Fluid with lower density is lighter and therefore rises during the process of heating resulting in the movement of the fluid in a process known as convection. This movement of the fluid results to the heat being transferred from one point to another. Natural convection is the most common method used in industrial cooling. Numerous research results on heat and mass transfer have been reported in literature. Makinde (2010) studied similarity solution of MHD flow with heat and mass transfer over a moving vertical plate with magnetic field and thermal convective surface boundary condition. Abdel-Khalek (2009) examined MHD free convection with mass transfer from a moving permeable vertical surface and produced interesting results using the perturbation techniques. Chamkha (2004) investigated unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Ibrahim and Makinde (2010a, b; 2011a, b) and Seini and Makinde (2013a, b) have made significant contributions to the subject of heat and mass transfer in the presence of magnetic field and obtain very interesting results. Seini (2013) also presented the heat and mass transfer problem with unsteady flow conditions. Arthur and Seini (2014) analyzed the MHD thermal stagnation point flow towards a stretching porous surface.

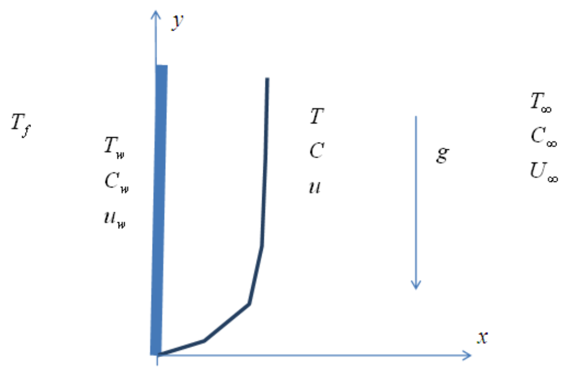
Viscous dissipation plays an important role in natural convection, geological processes, polymer processing and in strong gravitational field processes on large scales and has received remarkable attention from researchers. It also plays a practical role in oil products transportation through ducts. Viscous dissipation changes the temperature distribution of a system by playing a role like an energy source, which affects the heat transfer rate and hence needs to be considered in heat transfer problems.

Gangadhar (2012) presented a similarity solution for natural convection over a moving vertical plate with internal heat generation and viscous dissipation. Ibrahim and Reddy (2013) produced similarity solution of heat and mass transfer for natural convection over a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation, viscous dissipation and chemical reaction by applying shooting iteration technique along with the fourth-order Runge-Kutta method and their results agreed with the results obtained by Gangadhar (2012). Lakshmi et al. (2012) solved numerically MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction and viscous dissipation. They observed among others that an increase in the strength of chemical reacting substances causes an increase in the magnitude of local skin friction, the plate surface temperature, and Sherwood number, but opposite behaviour is seen for local Nusselt number. Singh (2012) examined the viscous dissipation and chemical reaction effects on flow past a stretching porous surface in a porous medium. Singh (2012) considered effects of variable fluid properties and viscous dissipation on mixed convection fluid flow past a vertical plate in porous medium and showed that the velocity initially increases from no slip condition and it acquires a peak value and then gradually decreases and attains the free stream velocity. Kazi et al. (2013) studied the problem of viscous dissipation on MHD natural convection flow along a vertical wavy surface and observed that, as the Prandtl number increases, the velocity, temperature and rate of heat transfer decrease while the skin friction initially decreases and becomes constant near $x = 1.0$ after that position skin friction increases with Prandtl number. Pantokratoras (2004) studied the effect of viscous dissipation in natural convection along a heated vertical plate. Hosain et al. (2007) examined combined effect of conduction and viscous dissipation on MHD free convection flow along a vertical flat plate.

In this present study, our objective is to investigate heat and mass transfer over a vertical surface with convective boundary conditions in the presence of viscous dissipation and n^th order chemical reaction. It is therefore hoped that the results obtained will be useful for industrial application and also serve as reference for future research work.

2 Mathematical Model

Let us consider a steady, laminar, incompressible, convection flow with heat and mass transfer over a vertical plate in a stream of cold fluid at temperature T_∞. We take the left surface of the plate to be heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient, h_f. The cold fluid at the right side of the plate is assumed to be Newtonian, and its property variations due to temperature and chemical species concentration are limited to fluid density. It is also assumed that it is a viscous dissipative fluid .The density variation and the effects of buoyancy are taken into account in the momentum equation (Boussinesq approximation). In addition, there is no applied electric field.



Let the x-axis be taken along the direction of the plate and y-axis normal to it. If *u*, *v*, *T* and *C* are the fluid *x*-component of velocity, *y* component of velocity, temperature and concentration respectively, then under the Boussinesq and boundary-layer approximations, and based on the above assumptions the continuity, momentum, energy and mass transfer (concentration) equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma(C-C_\infty)^n \tag{4}$$

where υ is the kinematic viscosity, T_∞ is the free stream temperature, C_∞ is the free stream concentration, γ is the plate surface rate of chemical reaction , α is the thermal diffusivity and D is the mass diffusivity, β_T is the thermal expansion coefficient, β_C is the solutal expansion coefficient, ρ is the fluid density, g is gravitational acceleration. Subject to the following boundary conditions:

$$u(x,0)=0,\ v(x,0)=0,\ -k\frac{\partial T}{\partial y}=h_f\left[T_f-T(x,0)\right],\ C_w(x,0)=C_w \text{ at y} = 0,$$

$$u(x,\infty)=U,\ T(x,\infty)=T_\infty,\ C(x,\infty)=C_\infty \text{ as } y\to\infty, \tag{5}$$

where h_f is the plate heat transfer coefficient, C_w is the species concentration at the plate surface and k is the thermal conductivity coefficient. The stream function, ψ, satisfies the continuity equation (1) automatically if defined in the usual way as

$$u=\frac{\partial\psi}{\partial y}, \text{ and } v=-\frac{\partial\psi}{\partial x}, \tag{6}$$

The following dimensionless quantities are introduced

$$\psi=(\upsilon xU_\infty)^{1/2} f(\eta), \eta=y\sqrt{\left(\frac{U_\infty}{\upsilon x}\right)},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{7}$$

Substituting equations (7) into the governing equations (2) – (4) and the boundary conditions (5), we obtain the following dimensionless forms:

$$f'''+\frac{1}{2}ff''+Gr_x\theta+Gc_x\phi=0 \tag{8}$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr Ecf''^2=0 \tag{9}$$

$$\phi''+\frac{1}{2}Scf\phi'-Sc\beta x\phi^n=0 \tag{10}$$

The transformed boundary conditions are then;

$$f'(0)=0,\ f(0)=0,\ \theta'(0)=Bi_x[\theta(0)-1],\ \phi(0)=1 \tag{11}$$

$$f'(\infty)=1,\ \theta(\infty)=0,\ \phi(\infty)=0 \tag{12}$$

where the prime symbol denotes differentiation with respect to η and Gr_x = $g\beta_T(T_w-T_\infty)x/U_\infty^2$ represents the local thermal Grashof number, Gc_x = $g\beta_C(C_w-C_\infty)x/U_\infty^2$ represents the local solutal Grashof number, $E_c=U_\infty^2/c_p(T_w-T_\infty)$ represents the Eckert number, $Bi_x=\frac{h_f}{k}(\upsilon x/U_\infty)^{\frac{1}{2}}$ represents the local convective heat transfer parameter. $\Pr=\upsilon/\alpha$ represents the Prandtl number,

$Sc = \upsilon / D$ represents the Schmidt number and $\beta_x = \gamma x (C_w - C_\infty)^{n-1} / U_\infty$ represents the chemical reaction parameter.

We notice that the local parameters Bi_x, Gr_x, Gc_x and β_x in equation (8), (9) and (10) respectively are functions of *x*. However in order to have similarity solution all parameters must be constant and we therefore assume $h_f = ax^{-1/2}$, $\beta_T = bx^{-1}$, $\beta_c = cx^{-1}$ and $\gamma = dx^{-1}$ where a, b, c and d are constants.

3. Numerical Solution

The numerical technique chosen for the solution of the coupled ordinary differential equations (8)- (10) together with the associated transformed boundary conditions (11) - (12) is the standard Newton-Raphson shooting method alongside the fourth-order Runge-Kutta integration algorithm. The numerical procedure was carried out using a Maple 16 software package. From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $\theta(0)$, $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ are also sorted out and their numerical values presented in a tabular form.

4. Results and Discussions

Table 1 shows the comparison of the works of Makinde and Olanrewaju (2010; 2012) with the present study for varying parameters of Biot number (Bi_x), thermal Grashof number (Gr_x) and Prandtl number (*Pr*). It is clear from the results that our present study is consistent with those reported by these authors.

Table 1 Comparison of results with Makinde and Olanrewaju (2010, 2012)

			Makinde and Olanrewaju (2010)		**Makinde and Olanrewaju (2012)**		**Present Study**	
Bi_x	Gr_x	Pr	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0.1	0.72	0.075077	0.249228	0.075077	0.249228	0.075077	0.249228
1.0	0.1	0.72	0.237506	0.762494	0.237506	0.762494	0.237506	0.762494
10	**0.1**	0.72	0.305596	0.969440	0.305596	0.969440	0.305596	0.969440
0.1	**0.5**	0.72	0.076138	0.238623	0.076138	0.238623	0.076138	0.238623
0.1	**1.0**	**0.72**	0.077045	0.2295515	0.077045	0.2295515	0.077045	0.2295515
0.1	0.1	**3.00**	0.0830460	0.169540	0.0830460	0.169540	0.0830460	0.169540
0.1	0.1	**7.10**	0.0867212	0.132788	0.0867212	0.132788	0.0867212	0.132788

Table 2 Numerical results of skin friction coefficient, Nusselt number and the Sherwood number

Pr	Sc	Gr	Gc	Ec	β	n	Bi	$-f''(0)$	$-\theta(0)$	$-\phi(0)$
0.71	0.24	0.1	0.1	0.1	0.1	1	0.1	0.551121	0.065257	0.256443
4.0	0.24	0.1	0.1	0.1	0.1	1	0.1	0.548689	0.057131	0.255745
7.1	**0.24**	0.1	0.1	0.1	0.1	1	0.1	0.551679	0.049986	0.255714
0.71	**1.24**	0.1	0.1	0.1	0.1	1	0.1	0.497511	0.064815	0.500671
0.71	**1.78**	0.1	0.1	0.1	0.1	1	0.1	0.487270	0.064728	0.580345
0.71	**2.64**	**0.1**	0.1	0.1	0.1	1	0.1	0.476812	0.064642	0.682652
0.71	0.24	**0.5**	0.1	0.1	0.1	1	0.1	0.725716	0.066085	0.265455
0.71	0.24	**1.0**	0.1	0.1	0.1	1	0.1	0.913354	0.066774	0.273896
0.71	0.24	**1.5**	**0.1**	0.1	0.1	1	0.1	1.080660	0.067272	0.280612
0.71	0.24	0.1	**0.5**	0.1	0.1	1	0.1	1.082740	0.067337	0.284728
0.71	0.24	0.1	**1.0**	0.1	0.1	1	0.1	1.639527	0.068464	0.307758
0.71	0.24	0.1	**1.5**	**0.1**	0.1	1	0.1	2.137125	0.069072	0.325091
0.71	0.24	0.1	0.1	**0.5**	0.1	1	0.1	0.624443	0.019874	0.260631
0.71	0.24	0.1	0.1	**1.0**	0.1	1	0.1	0.710321	-0.03801	0.265142
0.71	0.24	0.1	0.1	**1.5**	**0.1**	1	0.1	0.791281	-0.09690	0.269070
0.71	0.24	0.1	0.1	0.1	**0.5**	1	0.1	0.589405	0.169866	0.258443
0.71	0.24	0.1	0.1	0.1	**1.0**	**1**	0.1	0.604641	0.213006	0.259223
0.71	0.24	0.1	0.1	0.1	0.1	**2**	0.1	0.554749	0.065284	0.244284
0.71	0.24	0.1	0.1	0.1	0.1	**3**	**0.1**	0.556187	0.065295	0.238093
0.71	0.24	0.1	0.1	0.1	0.1	1	**0.5**	0.529330	0.065089	0.387952
0.71	0.24	0.1	0.1	0.1	0.1	1	**1.0**	0.512332	0.064954	0.513696
0.1	0.1	0.1	0.1	0.1	0.1	1	**1.5**	0.500812	0.064861	0.616576

The result of varying parameter values on the local skin friction coefficient, the local Nusselt number and the local Sherwood number are shown in Table 2. It is observed that increasing the Prandtl number (*Pr*) reduces the local skin friction coefficient together with the heat and mass transfer rate at the surface. The skin friction coefficient and the rate of heat transfer are reduced at the surface whilst the rate of mass transfer increases with increasing Schmidt number (Sc). Increasing the buoyancy forces (*Gr, Gc*) and the reaction rate parameter (β) is observed to increase the local skin friction coefficient and the rate of heat and mass transfers at the surface. The skin friction coefficient and the rate of heat transfer at the surface are reduced whilst the rate of mass transfer increases for increasing values of the convective heat transfer parameter (*Bi*). Furthermore, skin friction coefficient and the rate of heat transfer at the surface are increased whilst the rate of mass transfer reduces for increasing values of the order of chemical reaction (*n*). The Eckert number (Ec) increases the skin friction and the rate of mass transfer whilst the rate of heat transfer at the surface reduces.

Effects of Parameter Variation on Velocity Profiles

The effects of parameter variation on the velocity boundary layer are shown in Figures 1-4. It is observed in Figure 1 that increasing values of the Schmidt number tend to reduce the velocity profile slightly. In Figure 2 and 3, the velocity profiles for increasing the buoyancy forces (Gr, Gc) increase just as that of increasing the Eckert number in Figure 4. We can note here that, increasing buoyancy forces will lead to a better flow kinematics.

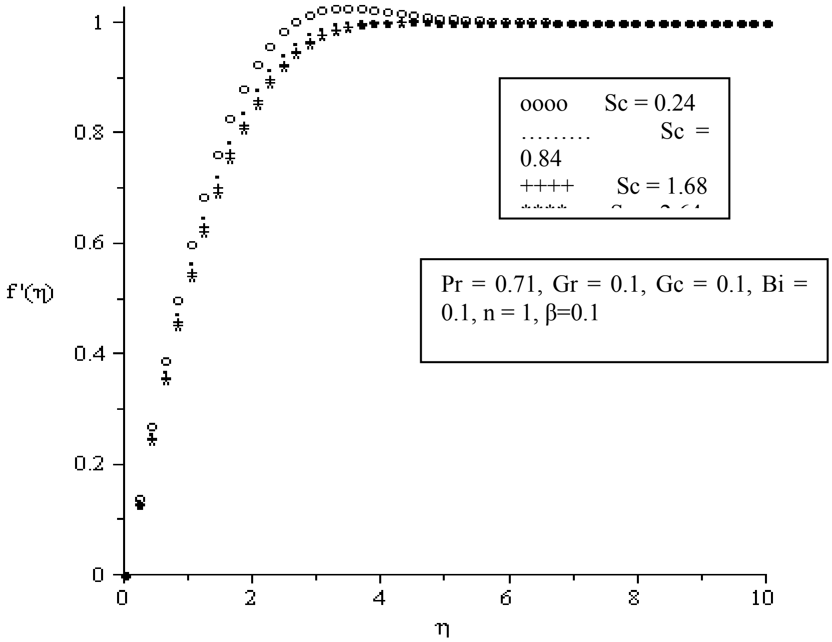


Figure 1: Velocity profiles for varying values of Schmidt number (*Sc*)

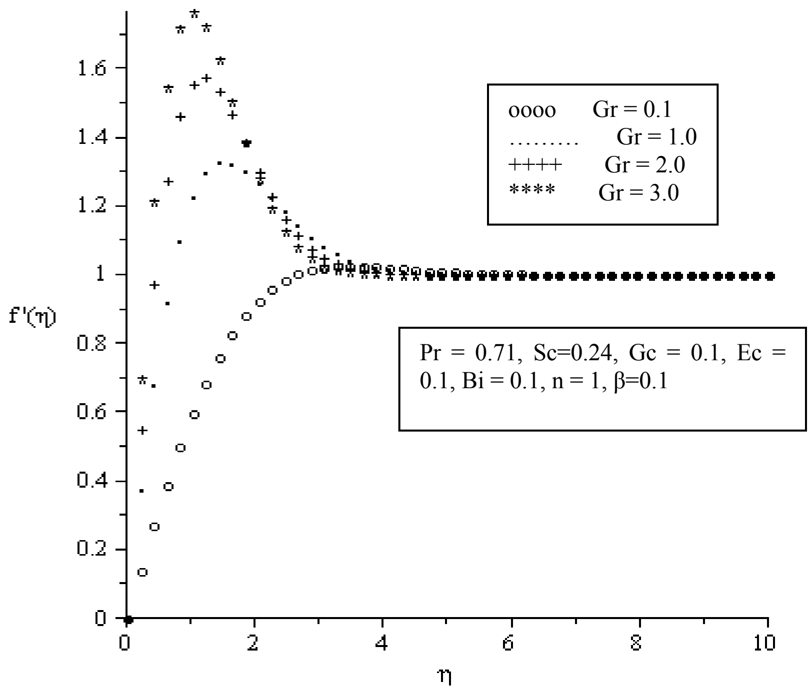


Figure 2: Velocity profiles for varying values of thermal Grashof number (Gr)

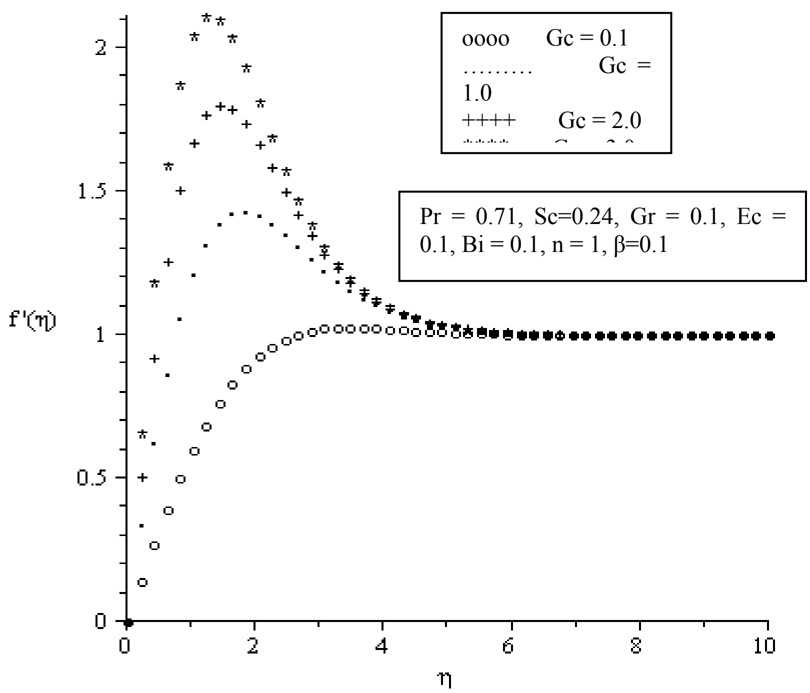


Figure 3: Velocity profiles for varying values of solutal Grashof number (Gc)

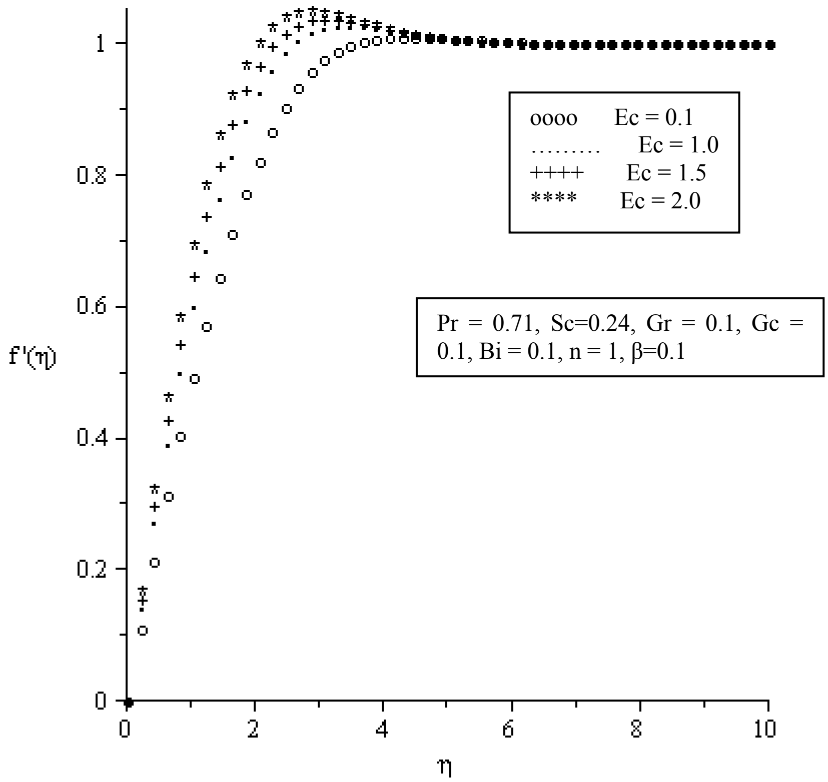


Figure 4: Velocity profiles for varying values of Eckert number (Ec **)**

Effects of Parameter Variation on Temperature Profiles

The effects of parameter variation on temperature profiles are shown in Figures 5-9. In Figure 5, increasing values of the Prandtl number reduces the thermal boundary layer thickness. The temperature profiles of Figure 6 and 7 are reduced as a result of increasing the buoyancy forces. This means that increasing the buoyancy forces enhances the cooling process. In Figure 8 and 9, it is observed that increasing both the Eckert number and the convective heat transfer parameter increase the temperature boundary layer thickness.

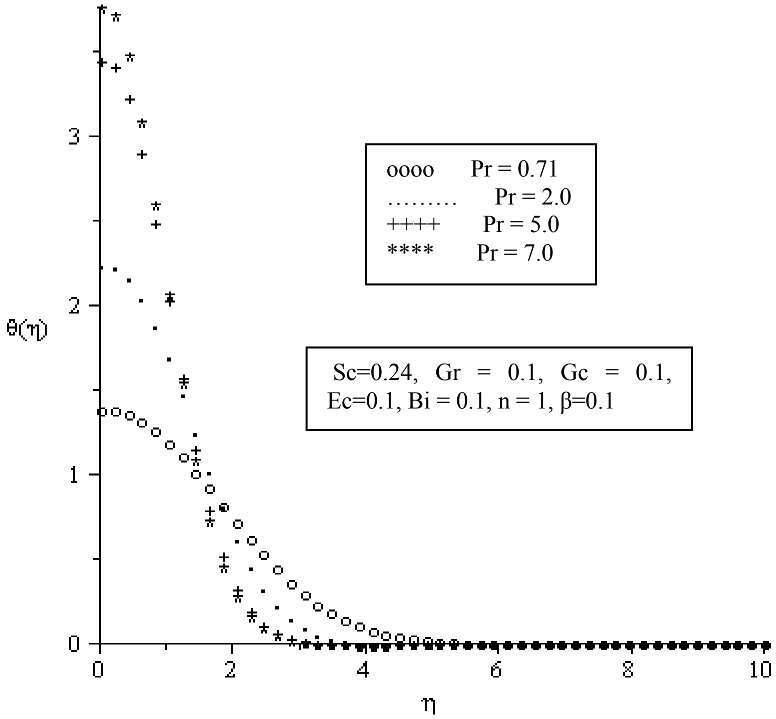


Figure 5: Temperature profiles for varying values of Prandtl number ($\Pr$)

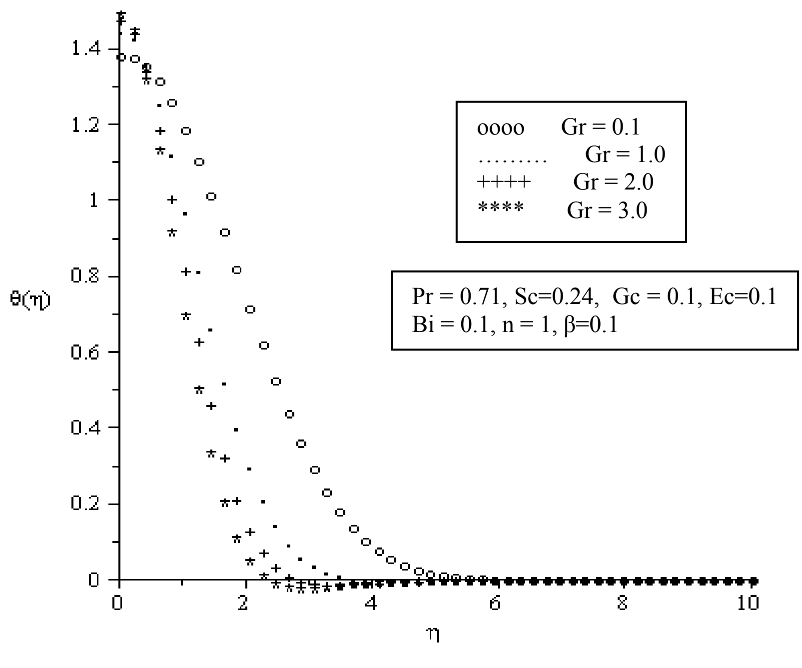


Figure 6: Temperature profiles for varying values of thermal Grashof number (Gr)

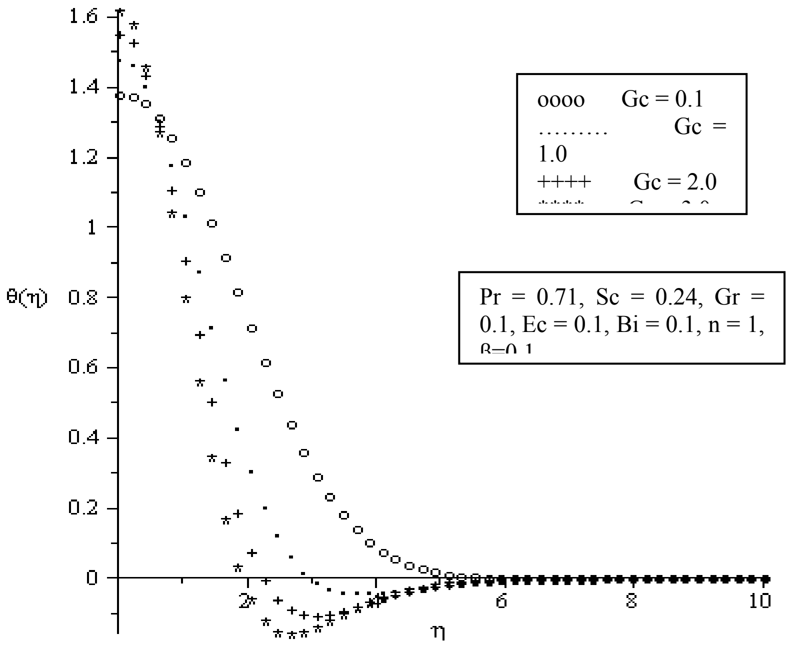


Figure 7: Temperature profiles for varying values of solutal Grashof number (Gc)

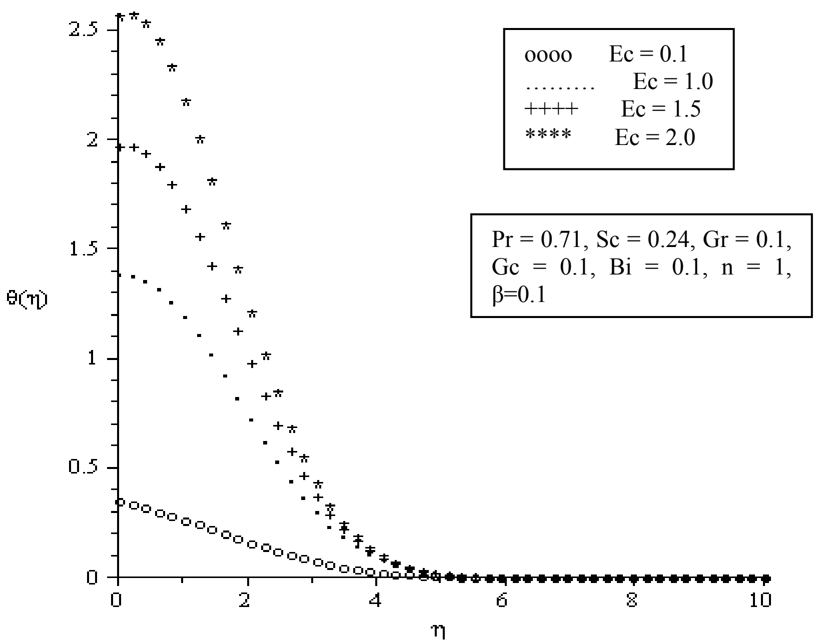


Figure 8: Temperature profiles for varying values of Eckert number (Ec)

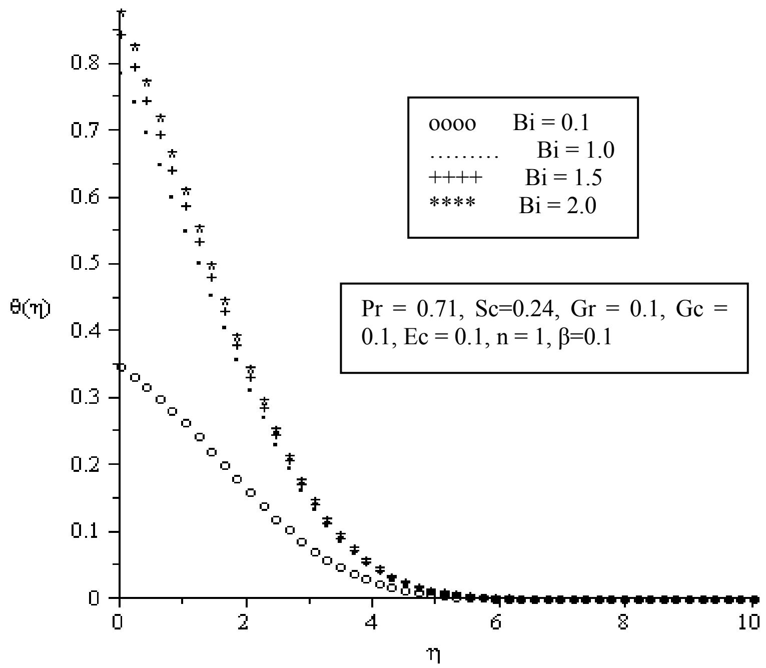


Figure 9: Temperature profiles for varying values of convective heat transfer parameter (Bi)

Effects of Parameter Variation on Concentration Profiles

Figures 10-15 depict the effects of varying parameters on the concentration boundary layer thickness. It is observed in Figure 10-15 that, increasing the Schmidt number, the buoyancy forces, the convective heat transfer parameter and the reaction rate parameter have adverse effect of decaying the concentration boundary layer thickness. Moreover, the boundary layer thickness increases when the order of reaction increases.

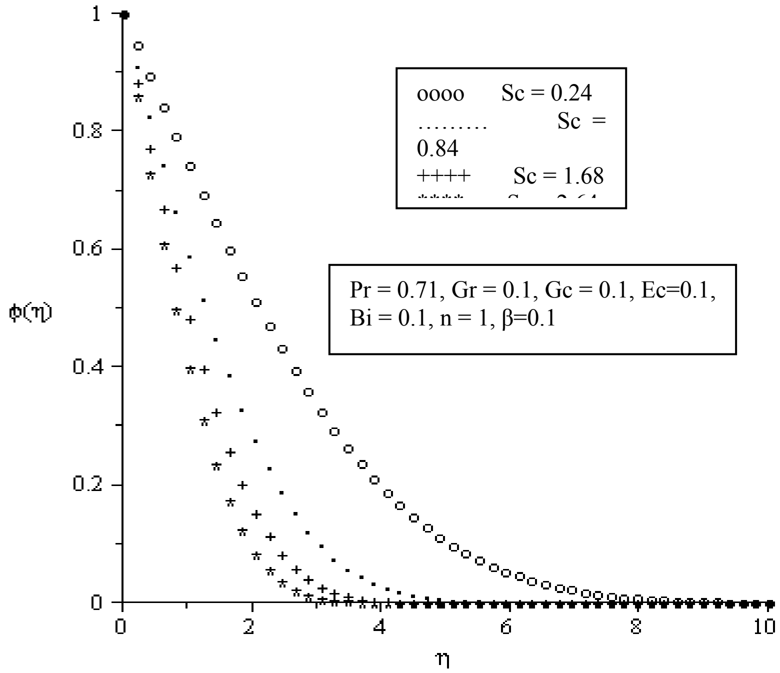


Figure 10: Concentration profiles for varying values of Schmidt number (Sc)

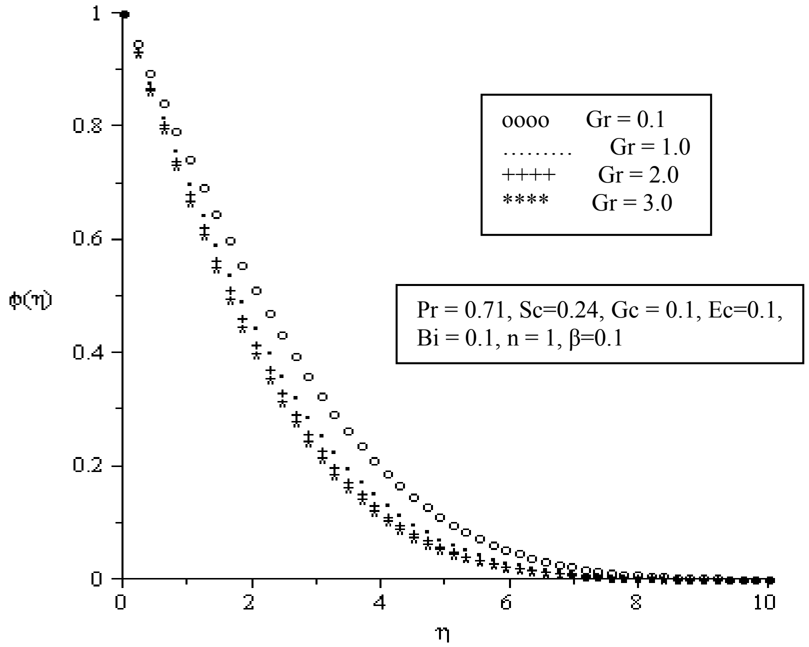


Figure 11: Concentration profiles for varying values of thermal Grashof number (Gr)

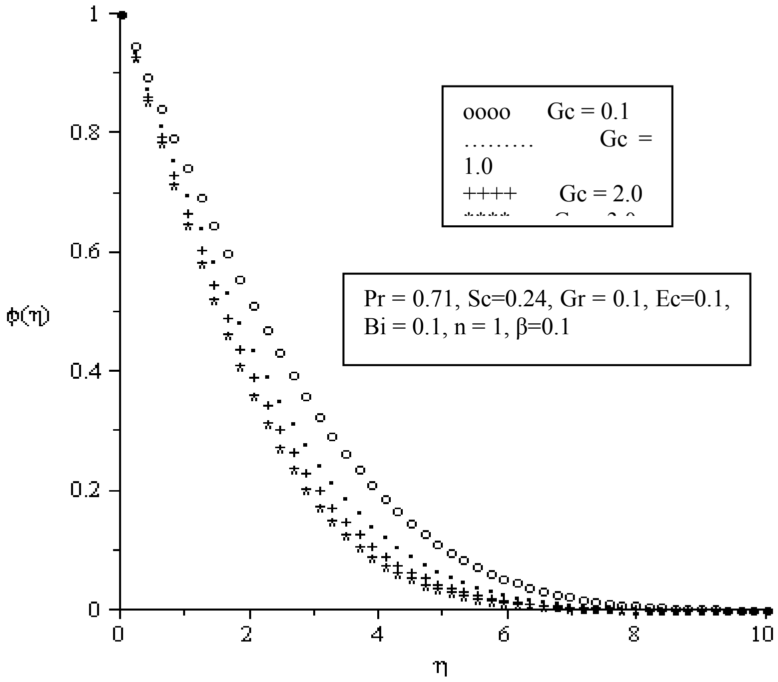


Figure 12: Concentration profiles for varying values of Solutal Grashof number (Gc)

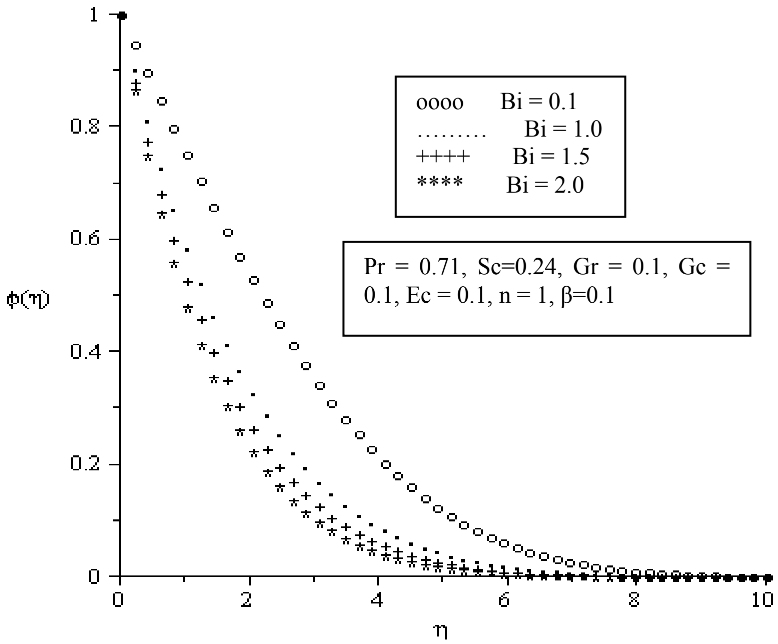


Figure 13: Concentration profiles for varying values of convective heat transfer parameter (Bi)

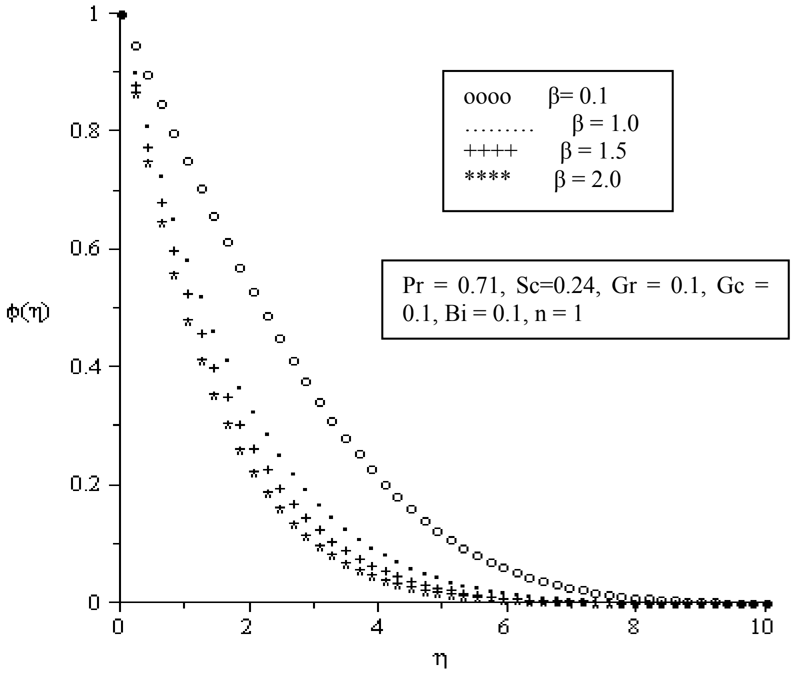


Figure 14: Concentration profiles for varying values of reaction rate parameter (β)

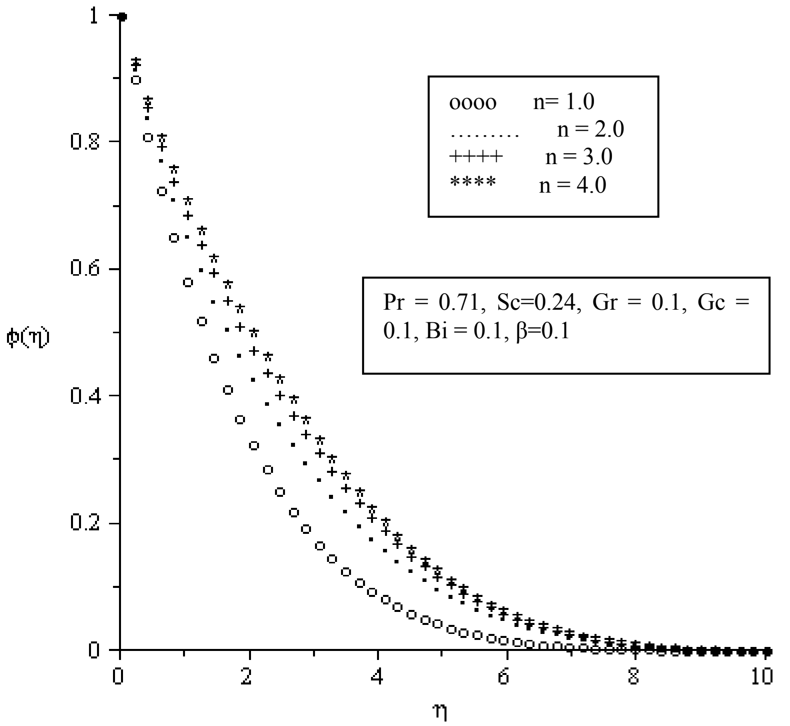


Figure 15: Concentration profiles for varying values of order of chemical reaction (n)

5.0 Conclusion

Heat and mass transfer over a vertical surface with convective boundary conditions in the presence of viscous dissipation and n^{th} order chemical reaction has been studied. Numerical results have been compared to earlier results published in the literature and a perfect agreement was achieved. Among others, our results reveal that:

1. The velocity increases with the increase in Gr, Gc and Ec. It also decreases with the increase in Sc.
2. The temperature reduces with increasing of Gc and Gr; and also rises with increase in $\Pr$, Ec and Bi.
3. The concentration boundary layer decreases with increase in β, Sc, Gr, Gc and Bi; and increases with increasing n .
4. The skin friction at the surface increases for the increase in β, n, Gr, Gc and Ec; and decreases for increasing $\Pr$, Sc and Bi.
5. The rate of heat transfer at the surface increases with the increase of β, n, Gr, Gc and Ec; and decreases with increasing $\Pr$, Sc and Bi.
6. The rate of mass transfer at the surface increases with the increase of β, Sc, Gr, Gc and Bi; and decreases with increasing $\Pr$, Ec and n.

References

[1] A.A. Mamun, N.M. Hosaind. A. Azim and M.A. Maleque (2007),Combined Effect Of Conduction And viscous Dissipation On Magnetohydrodynamic Free Convection Flow Along A Vertical Flat Plate. Journal of Naval Architecture and Marine Engineering 4, 87-98

[2] A. Pantokratoras (2004), Effect of viscous dissipation in natural convection along a heated vertical plate. Applied Mathematical Modelling 29 (2005) 553–564.

[3] A.J. Chamkha (2004), Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption, International. Journal of Engineering Science. 42, 217 - 230.

[4] E.M., Arthur, and Y. I. Seini (2014), MHD Thermal Stagnation Point Flow towards a Stretching Porous Surface. *Mathematical Theory and Modeling*, *4*(5), 163-169.

[5] H. K. Kazi, M.A. Alim and S.L. Andallah (2013), Effects of viscous dissipation on MHD natural convection flow along a vertical wavy surface. J. of Theoretical and Applied Physics 2013, 7:31

[6] J. Singh (2012) Viscous Dissipation and Chemical Reaction Effects on Flow Past a Stretching Porous Surface in a Porous Medium. Adv. Theor. Appl. Mech., Vol. 5, 2012, no. 8, 323 - 331.

[7] K. Gangadhar (2012), Similarity Solution For Natural Convection Over A Moving Vertical Plate With Internal Heat Generation And Viscous Dissipation, *Int. J. of Appl. Math and Mech.* **8** (18): 90-100.

[8] M.P. Lakshmi, N.B. Reddy and T. Poornima (2012), MHD Boundary Layer Flow of Heat and Mass Transfer over a Moving Vertical Plate in a Porous Medium with Suction and Viscous Dissipation. Int. J. Eng. Research and Applications (IJERA) Vol. 2, Issue 5, pp.149-159.
[9] M.M. Abdel-Khalek (2009), Heat And Mass Transfer In MHD Free Convection From A Moving Permeable Vertical Surface By A Perturbation Technique, Comm. Nonlinear Sci. And Num. Sim.14, 2091-2102.
[10] O.D. Makinde (2010), On MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition. Canadian Journal of Chemical Engineering 88: 983–990
[11] P.K. Singh (2012), Effects of Variable Fluid Properties and Viscous Dissipation on Mixed Convection Fluid Flow past a Vertical Plate in Porous Medium. Int. J. Scientific & Engineering Research Volume 3, Issue 7, ISSN 2229-5518.
[12] S.M. Ibrahim and N.B. Reddy (2013), Similarity Solution of Heat and Mass Transfer for Natural Convection over a Moving Vertical Plate with Internal Heat Generation and a Convective Boundary Condition in the Presence of Thermal Radiation, Viscous Dissipation, and Chemical Reaction. Hindawi Publishing Corporation ISRN Thermodynamics Volume 2013, Article ID 790604, 10 pages
[13] S.Y. Ibrahim and O.D. Makinde (2010a), On MHD Boundary Layer Flow of Chemically Reacting Fluid with Heat and Mass Transfer Past a Stretching Sheet, International Journal of Fluid Mech. 2(2) pp. 123-132
[14] S.Y. Ibrahim and O.D. Makinde (2010b), Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction, Scientific Research and Essays Vol. 5(19), pp. 2875-2882.
[15] S.Y. Ibrahim and O.D. Makinde (2011a), Chemically reacting MHD boundary layer flow of heat and mass transfer past a low-heat-resistant sheet moving vertically downwards, Scientific Research and Essays Vol. 6(22), pp. 4762-4775
[16] S.Y. Ibrahim and O.D. Makinde (2011b), Radiation effect on chemically reacting MHD boundary layer flow of heat and mass transfer through a porous vertical flat plate. International Journal of the Physical Sciences Vol. 6(6), pp. 1508-1516
[17] Y.I. Seini (2013), Flow over unsteady stretching surface with chemical reaction and non-uniform heat source. Journal of Eng. and Manufacturing Technology, JEMT 1 (2013) 24 – 35.
[18] Y.I. Seini and O.D. Makinde (2013a), Hydromagnetic Flow with Dufour and Soret Effects past a Vertical Plate Embedded in Porous Media, Mathematical Theory and Modeling, Vol.3, No.12, 2013, 47 – 64.
[19] Y.I. Seini and O.D. Makinde (2013b), "MHD Boundary Layer Flow Due to Exponential Stretching Surface with Radiation and Chemical reaction". Math Prob. Eng. Vol. 2013, Article ID 163614, 1 - 7. http://dx.doi.org/10.1155/2013/163614

[20] Y.I. Seini and O.D. Makinde (2014), Boundary layer flow near stagnation-points on a vertical surface with slip in the presence of transverse magnetic field. International Journal of Numerical Methods for Heat & Fluid Flow Vol. 24 No. 3, pp. 643-653.