Sensitivity Analysis of a Mixed Integer Linear Programming Model For Optimal Hydrothermal Energy Generation For Ghana

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Abstract: This paper examines further a Mixed Integer Linear Programming model constructed for optimal hydrothermal energy generation for Ghana as in [1]. Post Optimal Analysis is carried out on the model in order to assess its stability to slight variations of some input parameters such as minimum level running costs, extra hourly running costs above minimum level and start up costs of each generator on one hand and load demands and reserve margins on the other. The results show that the firm could minimize its cost of power generation if its input parameters were comparable to those lying between the 10 percent and -10 percent range.The10 percent and -10 percent range yielded a range of investment plans for the firmand also provided a basis for the selection of the best optimal solution.

Keywords: Stability, Post Optimality Analysis, Scheduling, Marginal Cost

1. Introduction

This study is a further development to an earlier work [1], in which mixed integer linear programming (MILP) was applied as a modeling tool to a power generation scheduling problem of a major power producer in Ghana. The aimwas to determine an optimal power production schedule that meets daily load demands at minimum cost of production and also to ascertain the marginal cost of producing electricity per day and therefore thetariff rate. The goal of this study is to perform post optimality analysis on the proposed model in [1] in order to ascertain its robustness, a range of variability under which the input parameters such as cost of running each generator at the minimum level, extra hourly cost of running each generator above its minimum level, start-up cost of each generator, load demand and reserve margin can change without affecting the optimum feasibility as well as provide a range of investment plans for the firm. Hydro-thermal power generation scheduling is a multifaceted problem consisting of Unit Commitment and Economic Dispatch problems. Unit Commitment refers to the problem of deciding on the startup and shutdown of the generators while Economic Dispatch refers the problem of deciding on the loading levels of each of the committed generators to generate enough power to satisfy load demand, budgetary and operational constraints at minimum production cost [2]. MILP has gained widespread usage in solving hydrothermal and unit commitment problems in the power sectordue to the recent improved capabilities of commercial solvers, the increased computational power of modern computers, their modeling capabilities and adaptability and ability to provide global optimal solutions. Delarue *et al.,*[3], Nadia*et al.,*[4], Ana and Pedroso [5] and Morales-Espana *et al.,* [6] employed MILP technique in solving varying power generation problems. The next section reviews the power generation problem and the formulated MILP model as discussed in [1] and followed by the post optimality analysis. The results and discussion section present details of the output levels of the generators resulting from the sensitivity analysis, the marginal costs of generation and some discussions. Remarks to conclude the discussions as well as point out direction for future work are made in the last section.

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2. The Power Generation Problem

The power generation firm operates eight power plants comprising two Hydroelectric (H_i, i=1, 2) and six Thermal (T_i, i=1,…, 6). These plants are committed to meeting the daily electricity load demands at some daily operational cost. The eight power plants together have twenty-four generators, ten (10) of which are for hydroelectric power generation and 14 for thermal. Each generator has to work between a minimum and a maximum level. There is an hourly cost of running each generator at its minimum level. In addition there is an extra hourly cost for each megawatt (MW) of power generated above the minimum level. Startup of a generator also involved cost. In addition to meeting the estimated daily electricity load demands, there must be sufficient generators working at any time of the day to make it possible to meet an increase in load. This increase would have to be met by the generators already operating within their permitted limits. There must be enough reserve (spinning reserve) to cater for unexpected increase in load demands or breakdown of any generator. The desire of the firm is to meet the daily load demands of consumers at minimum cost of operation of the power plants. The MILP model as in [1] is presented as follows:

Minimize $Cost = \sum_{i=1}^{24}\sum_{j=1}^{8}\left(C_j L_i n_{ij} + C_j^I L_i y_{ij} + C_j^{II} Z_{ij}\right)$

Subject to

$$y_{ij} \leq (M_j - m_j)n_{ij}$$
$$x_{ij} = m_j n_{ij} + y_{ij}$$

$$\sum_{j=1}^{8}(m_j n_{ij} + y_{ij}) \geq D_i$$
$$\sum_{j=1}^{8} M_j n_{ij} \geq (D_i + R_i)$$
$$Z_{ij} \geq n_{ij} - n_{(i-1)j}$$
$$x_{ij}, y_{ij} \geq 0$$
$$n_{ij}, Z_{ij} \geq 0, Z^{+} \quad (1)$$

Where:

x_{ij} : Power output in MW from each generator of plant j, $(j = 1, \dots, 8)$ in period i $(i = 1, \dots, 24)$

y_{ij} : Excess power output in MW from each generator of plant j, $j = 1, \dots, 8$ in period $i, (i = 1, \dots, 24)$ above minimum level.

n_{ij} : Number of generators of plant j working in period i.

Z_{ij} : Number of generators of plant j to start in period i.

D_i : Demand of power in MW in period *i.*

R_i : Reserve margin of power in MW in period *i.*

M_j : Maximum level of power output of each generator of plant *j.*

m_j : Minimum level of power output of each generator of plant *j.*

C_j : Hourly cost (in GH ¢) of running each generator of plant *j* at the minimum level.

C_j^{I} : Extra hourly cost (in GH ¢) of running each generator of plant *j* above minimum level.

C_j^{II}: Cost of starting up each generator of plant *j*.

L_i : Length of each period *i*.

4. Post Optimality Analysis

Sensitivity analysis was performed on the model to ascertain its robustness and to find a range of possible values under which the input parameters can change without affecting the optimum feasibility [7]. Since the parameters of the model are usually approximations of their exact value, analysis of their sensitivity to slight variations is crucial towards finding an implementable solution. Parameters such as cost of running each generator at the minimum level, extra hourly cost of running each generator above its minimum level, start-up cost of each generator, load demands and reserve margins were varied 5% upward and downward and later 10% upward and downward to reflect a reasonable level of variation that can potentially occur.Parameters such as the number of generators available, minimum and maximum level of each generator were assumed fixed. Therefore, there were four resultant Scenarios. In scenario one, the load demands and reserve margins were varied by 5% upwards and downwards and 10% upwards and downwards. The cost input parameters were varied by 5% upwards and downwards and 10% upwardsand downwards in scenario two. The cost factors, load demand and reserve margin were varied simultaneously in scenario three. Kpong and TT2PP were assumed shut down for maintenance works and due to shortage of crude oil/gas respectively in scenario four. All the scenarios resulted to twenty-five cases as depicted in Table.In the Table, the first column indicates the scenarios, the second the percentage variation of load demands (%LD), the third the percentage variation ofspinning reserves (%SR) and the last column the percentage variation of cost parameters (%CP). The five and ten percent upward and downward variations are denoted respectively by 5 and -5 and 10 and -10 in "% varied" column. The entry "0" indicates the original parameter value.

Table 1: Cases of Scenarios

Cases of Scenarios	%LD	%SR	%CP	Cases of Scenarios	%LD	%SR	%CP
Case 1 of scenario 1	5	5	0	Case 5 of scenario 3	-5	-5	5
Case 2 of scenario 1	-5	-5	0	Case 6 of scenario 3	-5	-5	-10
Case 3 of scenario 1	10	10	0	Case 7 of scenario 3	-5	-5	10
Case 4 of scenario 1	-10	-10	0	Case 8 of scenario 3	10	10	10
Case 1 of scenario 2	0	0	5	Case 9 of scenario 3	10	10	-10
Case 2 of scenario 2	0	0	5	Case 10 of scenario 3	10	10	5
Case 3 of scenario 2	0	0	10	Case 11 of scenario 3	-10	-10	5
Case 4 of scenario 2	0	0	-10	Case 12 of scenario 3	-10	-10	10
Case 1 of scenario 3	5	5	5	Case 13 of scenario 3	-10	-10	-10
Case 2 of scenario 3	5	5	-10	Case 14 of scenario 3	-10	-10	-5
Case 3 of scenario 3	5	5	10	Case 15 of scenario 3	5	5	-5
Case 4 of scenario 3	-5	-5	-5	Case 16 of scenario 3	10	10	-5
Scenario 4	Kpong and TT2PP were assumed shut down						

The details of adjusted load demands and the key parameters of the problem are presented in Tables 2 and 3 (with costs in Ghana Cedis (GH¢)). Table 2 shows the periods (in hourly interval) and their corresponding load demands (LD) and spinning reserve (SR). Table 3 also show the power plants, the cost per minimum level of operation of the generators (CM), the cost per hour of generating above the minimum level (CA) and the start-up costs (SC). The interval [1, 2) am, indicates the period starting from 1 am and ending before 2 am and so on.

Table 2: Adjustment figures of load demands and reserve margins

	5% Upward			**5% Downward**	
Period(am, pm)	LD(MW)	SR (MW)	Period(am, pm)	LD(MW)	SR(MW)
[1, 2)	1594.95	98.70	[1, 2)	1443.05	89.30
[2, 3)	1564.50	129.15	[2, 3)	1415.50	116.85
[3, 4)	1546.65	147.00	[3, 4)	1399.35	133.00
[4, 5)	1554.00	139.65	[4,5)	1406.00	126.35
[5, 6)	1580.25	113.40	[5, 6)	1429.75	102.60
[6, 7)	1663.20	72.45	[6, 7)	1504.80	65.55
[7, 8)	1580.25	155.4	[7, 8)	1429.75	140.60
[8, 9)	1629.60	143.85	[8, 9)	1474.40	130.15
[9, 10)	1668.45	105.00	[9, 10)	1509.55	95.00
[10, 11)	1696.80	76.65	[10, 11)	1535.20	69.35
[11, 12)	1703.10	70.35	[11, 12)	1540.90	63.65
[12, 1)	1720.95	52.50	[12, 1)	1557.05	47.50
[1, 2)	1697.95	75.60	[1, 2)	1536.30	68.40
[2, 3)	1715.7	57.75	[2, 3)	1552.30	52.25
[3, 4)	1726.2	166.95	[3, 4)	1561.80	151.05
[4, 5)	1741.95	151.20	[4,5)	1576.05	136.80
[5, 6)	1735.65	157.50	[5, 6)	1570.35	142.50
[6, 7)	1718.85	174.30	[6, 7)	1555.15	157.7
[7, 8)	1863.75	67.20	[7, 8)	1686.15	60.80
[8, 9)	1874.25	56.70	[8, 9)	1695.75	51.30
[9, 10)	1865.85	65.10	[9, 10)	1688.75	58.90
[10, 11)	1824.90	106.05	[10, 11)	1651.10	95.95
[11, 12)	1788.15	142.80	[11, 12)	1617.85	129.20
[12, 1)	1682.10	91.35	[12, 1)	1521.90	82.65
	10% Downward			**10% Upward**	
Period(am, pm)	LD(MW)	SR(MW)	Period(am, pm)	LD(MW)	SR(MW)
[1, 2)	1670.90	103.40	[1, 2)	1670.90	103.40
[2, 3)	1639.00	135.30	[2, 3)	1639.00	135.30
[3, 4)	1620.30	154.00	[3, 4)	1620.30	154.00
[4,5)	1628.00	146.30	[4,5)	1628.00	146.30
[5, 6)	1655.50	118.80	[5, 6)	1655.50	118.80
[6, 7)	1742.40	75.90	[6, 7)	1742.40	75.90
[7, 8)	1655.50	162.80	[7, 8)	1655.50	162.80
[8, 9)	1707.20	150.70	[8, 9)	1707.20	150.70
[9, 10)	1747.90	110.00	[9, 10)	1747.90	110.00
[10, 11)	1777.60	80.30	[10, 11)	1777.60	80.30
[11, 12)	1784.20	73.70	[11, 12)	1784.20	73.70
[12, 1)	1802.90	55.00	[12, 1)	1802.90	55.00
[1, 2)	1778.70	79.20	[1, 2)	1778.70	79.20
[2, 3)	1797.40	60.50	[2, 3)	1797.40	60.50
[3, 4)	1808.40	174.90	[3, 4)	1808.40	174.90
[4,5)	1824.90	158.40	[4,5)	1824.90	158.40
[5, 6)	1818.30	165.00	[5, 6)	1818.30	165.00
[6, 7)	1800.70	182.60	[6, 7)	1800.70	182.60
[7, 8)	1952.50	70.40	[7, 8)	1952.50	70.40
[8, 9)	1963.50	59.40	[8, 9)	1963.50	59.40
[9, 10)	1954.70	68.20	[9, 10)	1954.70	68.20
[10, 11)	1911.80	111.10	[10, 11)	1911.80	111.10
[11, 12)	1873.30	149.60	[11, 12)	1873.30	149.60
[12, 1)	1762.20	95.70	[12, 1)	1762.20	95.70

Table 3: Adjusted figures of the cost parameters

	5% Upward				5% Downward		
PP	CM GH¢	CA GH¢	SC GH¢	PP	CM GH¢	CA GH¢	SC GH¢
H1	4578.3040	35.7483	6031.1096	H1	4142.2751	32.3437	5456.7183
H2	2519.1968	73.2900	2908.0494	H2	2279.2733	66.3100	2631.0923
T1	24302.7301	240.6936	26500.9380	T1	21988.1844	217.7704	23977.0391
T2	36312.0944	3362.8947	40505.3954	T2	32853.7997	328.3333	36647.7387
T3	25446.2225	231.2268	30173.8637	T3	23022.7728	209.2052	27300.1624
T4	10550.1168	317.7195	11769.5584	T4	95045.3437	287.4605	10648.6481
T5	9943.3449	222.9119	12195.6555	T5	8996.3596	201.6822	11034.1645
T6	65633.4981	592.5192	75985.8568	T6	59382.6888	536.0888	68749.1085
	10% Upward				10% Downward		
PP	CM GH¢	CA GH¢	SC GH¢	PP	CM GH¢	CA GH¢	SC GH¢
H1	4796.3184	37.4506	6318.3052	H1	3924.2606	30.6414	5169.5226
H2	2639.1585	76.7800	3046.5278	H2	2159.3115	62.8200	2492.6138
T1	25460.0030	252.1552	27762.8874	T1	20830.9115	206.3088	22715.0897
T2	38041.2417	380.1754	42434.2237	T2	31124.6523	311.0526	34718.9103
T3	26657.9474	242.2376	31610.7143	T3	21811.0479	198.1944	25863.3117
T4	11052.5033	332.8490	12330.0136	T4	9042.9572	272.3310	10088.1929
T5	10416.8375	233.5267	12776.4010	T5	8522.8670	191.0673	10453.4190
T6	68758.9028	620.7344	79604.2309	T6	56257.2841	507.8736	65130.7344

5. Results and Discussions

5.1 Generators and their output levels

The output of the optimization algorithm (using LPsolve version: 5.5.2) are presented in Tables 4 to 7(a andb) below. In each Table, the first column indicates the production periods; the second the power plants (PP) to commit to power generation; the third the number of generators of a power plant to be working (GW) in any period: the fourth the number of generators to start up (GS) in any period (Zero entry in the fourth column means no new generator should be added to those already working whiles nonzero entry indicates the number of generators that should be added to those already working): the fifth, the total power outputs (TPO) from the committed generators and the sixth, the load demands (LD) in any period

Table 4 (a): Generators and output levels 1 for case 1 of scenario 1.

Period (am)	PP	GW	GS	TPO (MW)	LD (MW)	Period (am, pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1594.95	[7, 8)	H1	6	0	845.25	1580.25
	H2	4	0	124.95			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	1	110	
	T4	2	0	60			T4	2	0	60	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	874.5	1564.50	[8, 9)	H1	6	0	864.6	1629.60
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	1	90	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	856.65	1546.65	[9, 10)	H1	6	0	900	1668.45
	H2	4	0	120			H2	4	0	123.45	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	0	90	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
	H1	6	0	864			H1	6	0	900	

[4, 5)	H2	4	0	120	1554.00	[10, 11)	H2	4	0	151.8	1696.80
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	0	90	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	890.25	1580.25	[11, 12)	H1	6	0	900	1703.10
	H2	4	0	120			H2	4	0	158.1	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	0	90	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	900	1663.20	[12, 1)	H1	6	0	900	1720.95
	H2	4	0	148.2			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	121.45	
	T4	2	0	60			T4	3	0	90	
	T5	1	1	45			T5	1	0	49.5	
	T6	0	0	0			T6	0	0	0	

From Table 4 (a), at period [1, 2) am; six, four, three and two generators from H1, H2, T1 and T4 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 900 MW, 124.95 MW, 300 MW, 60 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. Zero entries in column four mean no new generator from those plants should be added to those already working. At period [2, 3) am; six, four, three and two generators from H1, H2, T1 and T4 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 874.5 MW, 120 MW, 300 MW, 60 MW 100 MW and 110 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [3, 4) am; six, four, three and two generators from H1, H2, T1 and T4 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 856.65 MW, 120 MW, 300 MW, 60 MW 100 MW and 110 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [4, 5) am; six, four, three and two generators from H1, H2, T1 and T4 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 864 MW, 120 MW, 300 MW, 60 MW, 100 MW and 110 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [5, 6) am; six, four,three and two generators from H1, H2, T1 and T4 respectively and a generator each from T2 and T3 should be committed to power generation. Their respective outputs; 890.25 MW, 120 MW, 300 MW, 60 MW 100 MW and 110 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2, T5 and T6 should be on standby for emergency use. At period [6, 7) am; six, four and three generators from H1, H2, T1 and T4 respectively and a generator each from T2, T3 and T5 should be committed to power generation. Their respective outputs; 843 MW, 148.2 MW, 300 MW, 60 MW, 100 MW, 110 MW and 45 MW satisfy exactly the load demand at that period. Four generators from T4 and a generator each from T2 and T6 should be on standby for emergency use. One recorded in column four against T5 indicates that a new generator from that plant has to be added to those already working. Similar interpretations follow for the outputs displayed in Tables (Tables 4(a) to 7(b)) below.

Table 4 (b): Generators and output for Case 1 of Scenario 1.

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)	Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1697.95	[7, 8)	H1	6	0	900	1863.75
	H2	4	0	152.85			H2	4	0	158.75	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	3	0	90			T4	5	2	150	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	900	1715.7	[8, 9)	H1	6	0	900	1874.25
	H2	4	0	160			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	2	0	200	
	T3	1	0	116.2			T3	1	0	114.75	
	T4	3	0	90			T4	5	0	150	

	T5	1	0	49.5			T5	1	0	49.5	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	861.2	1726.2	[9, 10)	H1	6	0	900	1865.85
	H2	4	0	120			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	2	1	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	3	0	90			T4	5	0	150	
	T5	1	0	45			T5	1	0	45.85	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	876.95	1741.95	[10, 11)	H1	6	0	899.9	1824.90
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	3	0	90			T4	5	0	150	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	870.65	1735.65	[11, 12)	H1	6	0	863.15	1788.15
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	3	0	90			T4	5	0	150	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	853.85	1718.85	[12, 1)	H1	6	0	900	1682.10
	H2	4	0	120			H2	4	0	137.1	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	3	0	90			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

Table 5 (a): Generators and output for Case 2 of Scenario 1.

Period (am)	PP	GW	GS	TPO (MW)	LD (MW)	Period (am)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	868.05	1443.05	[7, 8)	H1	6	0	824.75	1429.75
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	1	0	30	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	840.5	1415.50	[8, 9)	H1	6	0	839.4	1474.40
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	2	1	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	824.35	1399.35	[9, 10)	H1	6	0	874.55	1509.55
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	831	1406.00	[10, 11)	H1	6	0	900	1535.20
	H2	4	0	120			H2	4	0	120.2	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

[5, 6)	H1	6	0	869.75	1429.75	[11, 12)	H1	6	0	900	1540.90
	H2	4	0	120			H2	4	0	125.9	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	1	1	30			T4	2	0	60	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	899.8	1504.80	[12, 1)	H1	6	0	900	1557.05
	H2	4	0	120			H2	4	0	142.05	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	1	0	30			T4	2	0	60	
	T5	1	1	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

Table 5 (b): Generators and output for Case 2 of Scenario 1.

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)	Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1536.30	[7, 8)	H1	6	0	900	1686.15
	H2	4	0	121.15			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	1	0	100	
	T3	1	0	110			T3	1	0	116.75	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	900	1552.30	[8, 9)	H1	6	0	900	1695.75
	H2	4	0	137.5			H2	4	0	160	
	T1	3	0	300			T1	3	0	300.25	
	T2	0	0	0			T2	1	0	100	
	T3	1	0	110			T3	1	0	126	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	826.8	1561.80	[9, 10)	H1	6	0	900	1688.75
	H2	4	0	120			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	1	1	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	118.65	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	841.05	1576.05	[10, 11)	H1	6	0	150	1651.10
	H2	4	0	120			H2	4	0	136.1	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	835.35	1570.35	[11, 12)	H1	6	0	882.85	1617.85
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	1	0	100	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	820.15	1555.15	[12, 1)	H1	6	0	886.9	1521.90
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	1	0	100			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

Table 6 (a): Generators and output for Case 3 of Scenario 1.

Period (am)	PP	GW	GS	TPO (MW)	LD (MW)	Period (am)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	865.9	1670.90	[7, 8)	H1	6	0	850.5	1655.50
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	1	0	30			T4	1	0	30	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	834	1639.00	[8, 9)	H1	6	0	842.2	1707.20
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	1	0	30			T4	3	2	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	845.3	1620.30	[9, 10)	H1	6	0	882.9	1747.90
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	853	1628.00	[10, 11)	H1	6	0	900	1777.60
	H2	4	0	120			H2	4	0	132.6	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	865.5	1655.50	[11, 12)	H1	6	0	900	1784.20
	H2	4	0	120			H2	4	0	139.2	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	2	2	60			T4	3	0	90	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	900	1742.40	[12, 1)	H1	6	0	900	1802.90
	H2	4	0	157.4			H2	4	0	157.9	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	1	0	30			T4	3	0	90	
	T5	1	1	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

Table 6 (b): Generators and output for Case 3 of Scenario 1.

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)	Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1778.70	[7, 8)	H1	6	0	900	1952.50
	H2	4	0	133.7			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	113	
	T4	3	0	90			T4	4	0	120	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	1	1	110	
[2, 3)	H1	6	0	900	1797.40	[8, 9)	H1	6	0	900	1963.50
	H2	4	0	152.4			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	124	

	T4	3	0	90			T4	4	0	120	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	1	0	110	
[3, 4)	H1	6	0	853.4	1808.40	[9, 10)	H1	6	0	900	1954.70
	H2	4	0	120			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	115.2	
	T4	6	3	180			T4	4	0	120	
	T5	1	0	45			T5	1	0	49.5	
	T6	0	0	0			T6	1	0	110	
[4, 5)	H1	6	0	869.9	1824.90	[10, 11)	H1	6	0	900	1911.80
	H2	4	0	120			H2	4	0	126.8	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	6	0	180			T4	4	0	120	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	1	0	110	
[5, 6)	H1	6	0	863.3	1818.30	[11, 12)	H1	6	0	868.3	1873.30
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	6	0	180			T4	4	0	120	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	1	0	110	
[6, 7)	H1	6	0	845.7	1800.70	[12, 1)	H1	6	0	897.2	1276.20
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	2	0	200			T2	2	0	200	
	T3	1	0	110			T3	1	0	110	
	T4	6	0	180			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

Table 7 (a): Generators and output for Case 4 of Scenario 1.

Period	PP	GW	GS	TPO (MW)	LD (MW)	Period (am)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	892.1	1367.10	[7, 8)	H1	6	0	824.5	1354.50
	H2	4	0	120			H2	4	0	120	
	T1	2	0	200			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	1	0	45			T5	0	0	0	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	866	1341.00	[8, 9)	H1	6	0	851.8	1396.80
	H2	4	0	120			H2	3	0	90	
	T1	2	0	200			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	1	0	45			T5	1	1	45	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	850.7	1325.70	[9, 10)	H1	6	0	885.1	1430.10
	H2	4	0	120			H2	3	0	90	
	T1	2	0	200			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	857	1332.00	[10, 11)	H1	6	0	900	1454.40
	H2	4	0	120			H2	3	0	99.4	
	T1	2	0	200			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	1	0	45			T5	1	0	45	

	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	824.5	1354.50	[11, 12)	H1	6	0	900	1459.80
	H2	4	0	120			H2	3	0	104.8	
	T1	3	1	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	0	0	0			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	875.6	1425.60	[12, 1)	H1	6	0	900	1475.10
	H2	4	0	120			H2	3	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	0	0	0	
	T5	0	0	0			T5	1	0	45.1	
	T6	0	0	0			T6	0	0	0	

Table 7 (b): Generators and output for Case 4 of Scenario 1.

Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)	Period (pm)	PP	GW	GS	TPO (MW)	LD (MW)
[1, 2)	H1	6	0	900	1455.30	[7, 8)	H1	6	0	900	1597.50
	H2	3	0	100.3			H2	4	0	152.5	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	3	1	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[2, 3)	H1	6	0	900	1470.60	[8, 9)	H1	6	0	900	1606.50
	H2	3	0	115.6			H2	4	0	160	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	0	0	0			T4	3	0	90	
	T5	1	0	45			T5	1	0	46.5	
	T6	0	0	0			T6	0	0	0	
[3, 4)	H1	6	0	844.6	1479.60	[9, 10)	H1	6	0	900	1599.30
	H2	4	1	120			H2	4	0	154.3	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	2	2	60			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[4, 5)	H1	6	0	858.1	1493.10	[10, 11)	H1	6	0	899.2	1564.20
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[5, 6)	H1	6	0	852.7	1487.70	[11, 12)	H1	6	0	867.7	1532.70
	H2	4	0	120			H2	4	0	120	
	T1	3	0	300			T1	3	0	300	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	3	0	90	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	
[6, 7)	H1	6	0	838.3	1473.30	[12, 1)	H1	6	0	900	1441.80
	H2	4	0	120			H2	4	0	126.8	
	T1	3	0	300			T1	2	0	200	
	T2	0	0	0			T2	0	0	0	
	T3	1	0	110			T3	1	0	110	
	T4	2	0	60			T4	2	0	60	
	T5	1	0	45			T5	1	0	45	
	T6	0	0	0			T6	0	0	0	

5.2 Marginal cost of Electricity Generation

The marginal costs for producing electricity for the various periods indicate the amount by which the optimal generation cost will change if their respective load demands experience a unit change [8]. The marginal costs (MC) associated with each of the production periods and the ranges of the load demands for which they are valid for cases 1,2,3 and 4 of scenario 1 are presented in Tables 8 (a and b) below. In each Table, the first column indicates the production periods. The second, the marginal costs associated with each of the production periods. The third and fourth columns record the minimum and maximum ranges of the load demands for which the marginal costs are valid.

Table 8 (a): Marginal cost for case 1 of scenario 1.

Marginal cost for case 1 of scenario 1					Marginal cost for case 2 of scenario 1				
Period (am, pm)	MC (Gh¢)	Min (MW)	LD (MW)	Max (MW)	Period (am, pm)	MC (Gh¢)	Min (MW)	LD (MW)	Max (MW)
[1, 2)	69.80	1590.0	1594.95	1630.0	[1, 2)	34.05	1325.0	1443.05	1475.0
[2, 3)	34.05	1440.0	1564.50	1590.0	[2, 3)	34.05	1325.0	1415.50	1475.0
[3, 4)	34.05	1440.0	1546.65	1590.0	[3, 4)	34.05	1325.0	1399.35	1475.0
[4,5)	34.05	1440.0	1554.00	1590.0	[4,5)	34.05	1325.0	1406.00	1475.0
[5, 6)	34.05	1440.0	1580.25	1590.0	[5, 6)	34.05	1310.0	1429.75	1460.0
[6, 7)	69.80	1635.0	1663.20	1675.0	[6, 7)	34.05	1355.0	1504.80	1505.0
[7, 8)	34.05	1485.0	1580.25	1635.0	[7, 8)	34.05	1355.0	1429.75	1505.0
[8, 9)	34.05	1485.0	1629.60	1635.0	[8, 9)	34.05	1385.0	1474.40	1535.0
[9, 10)	69.80	1665.0	1668.45	1705.0	[9, 10)	34.05	1385.0	1509.55	1535.0
[10, 11)	69.80	1665.0	1696.80	1705.0	[10, 11)	69.80	1535.0	1535.20	1575.0
[11, 12)	69.80	1665.0	1703.10	1705.0	[11, 12)	69.80	1535.0	1540.90	1575.0
[12, 1)	220.22	1709.5	1720.95	1725.5	[12, 1)	69.80	1535.0	1557.05	1575.0
[1, 2)	69.80	1665.0	1697.95	1705.0	[1, 2)	69.80	1535.0	1536.30	1575.0
[2, 3)	220.22	1709.5	1715.7	1725.5	[2, 3)	69.80	1535.0	1552.30	1575.0
[3, 4)	34.05	1615.0	1726.2	1765.0	[3, 4)	34.05	1485.0	1561.80	1635.0
[4,5)	34.05	1615.0	1741.95	1765.0	[4,5)	34.05	1485.0	1576.05	1635.0
[5, 6)	34.05	1615.0	1735.65	1765.0	[5, 6)	34.05	1485.0	1570.35	1635.0
[6, 7)	34.05	1615.0	1718.85	1765.0	[6, 7)	34.05	1485.0	1555.15	1635.0
[7, 8)	69.80	1825.0	1863.75	1865.0	[7, 8)	220.22	1679.5	1686.15	1695.5
[8, 9)	220.22	1869.5	1874.25	1885.5	[8, 9)	229.24	1695.5	1695.75	1725.5
[9, 10)	212.30	1865.0	1865.85	1869.5	[9, 10)	220.22	1679.5	1688.75	1695.5
[10, 11)	34.05	1675.0	1824.90	1825.0	[10, 11)	69.80	1635.0	1651.10	1675.0
[11, 12)	34.05	1675.0	1788.15	1825.0	[11, 12)	34.05	1485.0	1617.85	1635.0
[12, 1)	69.8	1665.0	1682.10	1705.0	[12, 1)	34.05	1385.0	1521.90	1535.0

The MCs are valid as long as their associated load demands lie within the specified minimum and maximum ranges. For instance, for period [1, 2) am, a marginal cost of GH¢ 69.80 is valid for that period since the load demand of 1594.95 MW lie within the range of 1590 MW to 1630 MW. This means that, any unit change in load demand changes the optimal generation cost by GH¢ 69.80. The average marginal cost for producing electricity in a day for case 1 of scenario 1 is GH¢ 76.67. This marginal cost indicates the minimum tariff that is appropriate for the firm to charge consumers fora megawatt of power. Similar interpretations follow for the rest of the periods, which all record load demands lying between their respective minimum and maximum values.

Table 8 (b): Marginal cost for producing electricity for case 3 of scenario 1.

Marginal cost for case 3 of scenario 1					Marginal cost for case 4 of scenario 1				
Period (am, pm)	MC (Gh¢)	Min (MW)	LD (MW)	Max (MW)	Period (am, pm)	MC (Gh¢)	Min (MW)	LD (MW)	Max (MW)
[1, 2)	34.05	1555.0	1670.90	1705.0	[1, 2)	34.05	1225	1367.10	1375.0
[2, 3)	34.05	1555.0	1639.00	1705.0	[2, 3)	34.05	1225	1341.00	1375.0
[3, 4)	34.05	1525.0	1620.30	1675.0	[3, 4)	34.05	1225	1325.70	1375.0
[4,5)	34.05	1525.0	1628.00	1675.0	[4, 5)	34.05	1225	1332.00	1375.0
[5, 6)	34.05	1540.0	1655.50	1690.0	[5, 6)	34.05	1280	1354.50	1430.0
[6, 7)	69.80	1705.0	1742.40	1745.0	[6, 7)	34.05	1300	1425.60	1450.0
[7, 8)	34.05	1555.0	1655.50	1705.0	[7, 8)	34.05	1280	1354.50	1430.0
[8, 9)	34.05	1615.0	1707.20	1765.0	[8, 9)	34.05	1295	1396.80	1445.0
[9, 10)	34.05	1615.0	1747.90	1765.0	[9, 10)	34.05	1295	1430.10	1445.0
[10, 11)	69.80	1765.0	1777.60	1805.0	[10, 11)	69.80	1445	1454.40	1475.0

[11, 12)	69.80	1765.0	1784.20	1805.0	[11, 12)	69.80	1445	1459.80	1475.0
[12, 1)	69.80	1765.0	1802.90	1805.0	[12, 1)	212.30	1475	1475.10	1479.5
[1, 2)	69.80	1765.0	1778.70	1805.0	[1, 2)	69.80	1445	1455.30	1475.0
[2, 3)	69.80	1765.0	1797.40	1805.0	[2, 3)	69.80	1445	1470.60	1475.0
[3, 4)	34.05	1705.0	1808.40	1855.0	[3, 4)	34.05	1385	1479.60	1535.0
[4,5)	34.05	1705.0	1824.90	1855.0	[4,5)	34.05	1385	1493.10	1535.0
[5, 6)	34.05	1705.0	1818.30	1855.0	[5, 6)	34.05	1385	1487.70	1535.0
[6, 7)	34.05	1705.0	1800.70	1855.0	[6, 7)	34.05	1385	1473.30	1535.0
[7, 8)	220.22	1949.5	1952.50	1965.5	[7, 8)	69.80	1565	1597.50	1605.0
[8, 9)	220.22	1949.5	1963.50	1965.5	[8, 9)	212.30	1605	1606.50	1609.5
[9, 10)	220.22	1949.5	1954.70	1965.5	[9, 10)	69.80	1565	1599.30	1605.0
[10, 11)	69.80	1905.0	1911.80	1945.0	[10, 11)	34.05	1415	1564.20	1565.0
[11, 12)	34.05	1755.0	1873.30	1905.0	[11, 12)	34.05	1415	1532.70	1565.0
[12, 1)	34.05	1615.0	1762.20	1765.0	[12, 1)	69.80	1435	1441.80	1475.0

5.3 Discussions

It is evident from the scenarios considered that cases with the same load demands and reserve margins present the same output levels of the generators committed to power production and therefore have the same power generation schedule. For instance, case 1 of scenario 1 and cases; 1, 2, 3 and 15 of scenario 3 have the same operating generators and output levels because the load demands and reserve margins in the respective cases were adjusted 5% upward. Similar interpretations follow for cases with the same percentage of upward or downward adjustment of the load demands and reserve margins. Also cases: 1 to 4 of scenario 2 has the same power generation schedule (same operating generators and output levels) as the original problem in [1]. The similarity in their generation schedule is due to the fact that they all have the same load demand and reserve margin pattern. Furthermore, it is evident that cases with the same cost factors present similar pattern of marginal cost of producing electricity. For instance, case 1 of scenario 2 and cases; 1, 5, 10 and 11 of scenario 3 have similar pattern of marginal cost of producing electricity due to the fact that the cost parameters of the respective cases were adjusted 5% upward. Similar interpretations follow for cases with the same percentage of upward or downward adjustment of the cost parameters. Moreover, it is observed in scenario 4 that load demands for the respective periods were satisfied when plants H2 and T5 were assumed shut down for maintenance works due to shortage of crude oil/gas respectively. The results of all the scenarios considered have similar interpretation as those of the original problem [1]. The optimal cost from run of the optimization algorithm using the original data was GH¢ 4,806,855.99. The optimal costs of the twenty-five cases considered are presented in Table 9 below. In the Table, the first column indicates the scenarios, the second the optimal costs associated with the cases considered, the third the percentage increase or decrease (%ID) above or below the optimal cost of the original problem and the optimal costs ranges in the last column.

Table 9: Summary of Results

ROW	SCENARIOS	OCGH¢	ID%GH¢	OPTIMAL COST RANGE	
				Lower GH¢	Upper GH¢
1	Case 1 of scenario 2 Case 2 of scenario 2 Case 3 of scenario 2 Case 4 of scenario 2	5,047,197.18 4,713,193.23 5,305,997.74 4,326,170.82	5.00 -1.95 10.38 -10.00	4,326,170.82	5,305,997.74
2	Case 1 of scenario 1 Case 1 of scenario 3 Case 2 of scenario 3 Case 3 of scenario 3 Case 15 of scenario 3	5,436,043.39 5,707,844.96 4,892,439.49 5,998,104.09 5,903,130.68	13.09 18.74 1.78 24.78 22.81	4,892,439.49	5,998,104.09
3	Case 2 of scenario 1 Case 4 of scenario 3 Case 5 of scenario 3 Case 6 of scenario 3	4,199,880.34 4,171,057.39 4,409,873.76 3,779,893.35	-12.63 -13.23 -8.26 -21.36	3,779,893.35	4,630,021.23

	Case 7 of scenario 3	4,630,021.23	3.68		
4	Case 3 of scenario 1 Case 8 of scenario 3 Case 9 of scenario 3 Case 10 of scenario 3 Case 16 of scenario 3	6,333,317.24 6,992,633.28 5,704,916.99 6,649,981.51 8,763,512.91	31.76 45.47 18.68 38.34 82.31	5,704,916.99	8,763,512.91
5	Case 4 of scenario 1 Case 11 of scenario 3 Case 12 of scenario 3 Case 13 of scenario 3 Case 14 of scenario 3	3,613,382.76 3,794,051.21 3,993,177.95 3,268,174.42 3,487,785.24	-24.83 -21.07 -16.93 -32.01 -27.44	3,268,174.42	3,993,177.95
6	Scenario 4	6,131,123.95	27.55		

The best optimal solution was given by case thirteen of scenario three, which yielded a minimum production cost of Gh¢3,268,174.42 (reduced by 32%) and the worst by case sixteen of scenario three, which also yielded a minimum production cost of Gh¢8,763,512.91 (increased by 82.31%). The cases in the various rows have the same number of generators and output levels of generators committed to power generation but differ in their optimal generation costs as shown in Table 9. For instance, the cases in row 1, have the same power generation schedule but differ in their optimal generation costs; so do the cases in rows 2, 3, 4 and 5. The ranges specifiedprovide investment plans for the firm.

6. Conclusions

The best optimal solution was given by case 13 of scenario 3, which yielded a minimum production cost of GH¢ 3,268,174.42 and the worst by case 16 of scenario 3, which also yielded a minimum production cost of GH¢ 8,763,512.91. Thus, the firm could minimize the cost of power generation if its cost parameters (the cost of running each generator at the minimum level, extra hourly cost of running each generator above the minimum level and start-up cost of each generator), load demands and reserve margins were comparable to those lying between the 10 percent and -10 percent range. Any scenario selected should provide an optimum investment for the firm.

Future Work

A number of features and characteristics of the hydrothermal power systems such as: the stochastic nature of electricity demand and the probabilistic reserve margin requirement constraints were omitted in the formulation of the model so a future paper will include them.

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