International Journal of Mathematics and Statistics Studies

Vol.3, No.5, pp.11-17, September 2015

Published by European Centre for Research Training and Development UK (www.eajournals.org)

OPTIMAL PORTFOLIO MIX FOR MULTIGROW INSURANCE COMPANY IN GHANA USING LINEAR PROGRAMMING

Douglas Kwasi Boah
Department of Mathematics, University for Development Studies
P. O. Box 24 Navrongo - Ghana

Isaac Kwasi Adu
Department of Mathematics, Valley View University, Techiman Campus,
P. O. Box 183 B/A-Ghana

Vincent Tulasi
Department of Statistics and Mathematics, Ho Polytechnic
P. O. Box HP 217 V/R – Ghana.

ABSTRACT: *In this paper, the concept of Linear Programming (LP) was applied to Multigrow Insurance Company in Ghana which had a portfolio problem. The company had obtained GH₵ 200000 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. Based on the data collected, the problem was formulated as a Linear Programming Problem and solved using Management Scientist Version 5 Software. Optimal portfolio mix was obtained for the Insurance Company. Finally, the total optimal return on the investments of the company was found to be GH₵ 15980. It is strongly recommended that the Company should adhere to the proposed optimal portfolio mix and also employ at least one operations researcher to assist the Company in its activities.*

KEYWORDS: Portfolio, Investment, Linear Programming, Optimal Portfolio Mix, Optimal Return.

INTRODUCTION

In finance, a portfolio is a collection of investments held by an investment company, hedge fund, financial institution or individual (Investopedia, 2011). These investments often include stocks, which are investments in individual businesses; bonds, which are investments in debt that are designed to earn interest; and mutual funds, which are essentially pools of money from many investors that are invested by professionals or according to indices. It is a generally accepted principle that a portfolio is designed according to the investor's risk tolerance, time frame and investment objectives. The monetary value of each asset may influence the risk/reward ratio of the portfolio and is referred to as the asset allocation of the portfolio (Investopedia, 2011). When determining a proper asset allocation one aims at maximizing the expected return.

Multigrow Insurance Company which was established in 2007 and whose main office is in Adum-Kumasi (Ghana) had a portfolio problem. The company had obtained GH₵ 200000 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. The objective of the study was to find an optimal portfolio mix for the company so as to maximize return.

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

LITERATURE

A lot of researchers have done works on portfolio among which are the following. Konno (1990) looked at 'Piecewise linear risk function and portfolio optimization'. Konno and Yamazaki (1991) presented 'Mean-absolute deviation portfolio optimization model and its application to Tokyo stock exchange'. King (1993) presented 'Asymmetric risk measures and tracking models for portfolio optimization under uncertainty'. Speranza (1996) presented a heuristic algorithm for a portfolio optimization model applied to the Milan stock market. Young (1998) looked at 'A minimax portfolio selection rule with linear programming solution'. Chang et al (2000) looked at 'Heuristics for cardinality constrained portfolio optimization'. Kellerer et al (2000) worked on 'Selecting portfolios with fixed costs and minimum transaction lots'. Jobst et al (2001) presented 'Computational aspects of alternative portfolio selection models in the presence of discrete asset choice constraints'. Konno and Wijayanayake (2001) looked at 'Portfolio optimization problem under concave transaction costs and minimal transaction unit constraints'. Schaerf 2002) presented 'Local search techniques for constrained portfolio selection problems'. Crama and Schyns (2003) presented 'Simulated annealing for complex portfolio selection problems'. Maringer and Kellerer (2003) looked at 'Optimization of cardinality constrained portfolios with a hybrid local search algorithm'. Mitra et al (2003) presented a review of portfolio planning: models and systems. Li et al (2006) worked on 'Optimal lot solution to cardinality constrained mean variance formulation for portfolio selection'. Fernandez and Gomez (2007) worked on 'Portfolio selection using neural networks'. Garlappi et al (2007) looked at 'Portfolio selection with parameter and model uncertainty: a multi-prior approach'. Golosnoy and Okhrin (2007) presented 'Multivariate shrinkage for optimal portfolio weights'. Kan and Zhou (2007) worked on 'Optimal portfolio choice with parameter uncertainty'. Mansini et al (2007) looked at 'Conditional Value at Risk and related linear programming models for portfolio optimization'. Shawa et al (2008) presented 'Lagrangian relaxation procedure for cardinality-constrained portfolio optimization'. DeMiguel et al (2009) presented a generalized approach to portfolio optimization: Improving performance by constraining portfolio norms'.

METHODOLOGY

The concept of Linear Programming was applied to Multigrow Insurance Company in Ghana which had a difficulty in determining how much to invest in each of five investment areas in order to maximize return on GH₵ 200000.00 cash. Linear programming (LP), also called linear optimization, is a method used to achieve the best outcome for an objective (such as maximum profit or minimum cost) in a mathematical model whose requirements are represented by linear relationships. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality or inequality constraints. The general form of the LP model is stated as:

Optimize $f(x) = \sum_{j=1}^{n} c_j x_j$

Subject to;
$$\begin{aligned} &\textstyle\sum_{j=1}^{n} a_{ij} x_j \leq b_i && 1 \leq i \leq p \\ &\textstyle\sum_{j=1}^{n} a_{ij} x_j = b_i && p+1 \leq i \leq k \\ &\textstyle\sum_{j=1}^{n} a_{ij} x_j \geq b_i && k+1 \leq i \leq m \end{aligned}$$

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

$$x_j \geq 0 \qquad\qquad 1 \leq j \leq n \qquad\qquad [1]$$

where $f(x)$ is the objective function, x_j is the j^{th} decision variable, c_j is the j^{th} cost coefficient, a_{ij} is the j^{th} technological coefficient in the i^{th} constraint, b_i is the i^{th} *right-hand-side parameter* (resource availability) and *p*, *k*, *m*, and *n* are integers. The general form LP [1] can be transformed into the standard form as:

Optimize $f(x) = \sum_{j=1}^{n} c_j x_j$

Subject to $\quad \sum_{j=1}^{n} a_{ij} x_j = b_i \qquad 1 \leq i \leq m$

$$x_j \geq 0 \qquad\qquad 1 \leq j \leq n \qquad\qquad [2]$$

The standard form LP [2] is obtained by adding to or subtracting from each inequality constraint slack or surplus variables. A slack variable is a non-negative variable which when added to the left-hand-side (LHS) of a less-than-or-equal-to constraint transforms it into an equality constraint. A surplus variable on the other hand transforms a greater-than-or-equal-to constraint into an equality constraint. The standard form LP is necessary for the application of solution algorithms, since the algorithms work only with equality conditions (Williams, 2013). The objective function may either be maximized or minimized. There are four main assumptions inherent in a LP model that must be taken into account in any application. They are proportionality, additivity, divisibility, and certainty (Hillier and Lieberman, 2000).

Secondary data (The Company's projected annual rates of return) was collected from the Manager of the Company as shown in Table 1

Table 1:

INVESTMENT	RATE OF RETURN (in percentage)
Mobile Oil	10.5
Shell Oil	7.2
Anglo Gold	7.4
Tarkwa Gold	6.4
Government Bonds	4.4

The Company's Projected Annual Rates of Return (Multigrow Insurance Company, 2015).

The company had imposed the following investment guidelines:

1. None of the industries should receive more than 50% of the total investment
2. Government bonds should be at least 25% of the mining industry's investments
3. The investment in mobile oil though has the highest return, is a high risk one and therefore should not be more than 60% of the total oil industry investment.

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

RESULTS

Problem Formulation

Based on the data collected, the problem was formulated as a Linear Programming Problem as follows:

Let x_1 = amount of cedis to be invested in Mobile Oil

x_2 = amount of cedis to be invested in Shell Oil

x_3 = amount of cedis to be invested in Anglo Gold

x_4 = amount of cedis to be invested in Tarkwa Gold

x_5 = amount of cedis to be invested in Government Bonds

Objective function: $P = 0.105\,x_1 + 0.072\,x_2 + 0.074x_3 + 0.064x_4 + 0.044x_5$

Constraint functions:

Available funds: $x_1 + x_2 + x_3 + x_4 + x_5 = 200{,}000$

Oil industry investment: $x_1 + x_2 \leq 100{,}000$

Mining industry investment: $x_3 + x_4 \leq 100{,}000$

Government bond investment: $x_5 \geq 0.25\,(x_3 + x_4)$ OR

$-0.25x_3 - 0.25x_4 + x_5 \geq 0$

Investment in mobile oil: $x_1 \leq 0.6\,(x_1 + x_2)$ OR $0.4x_1 - 0.6x_2 \leq 0$

The Linear Programming (LP) model is then given as:

Maximize $P = 0.105\,x_1 + 0.072\,x_2 + 0.074x_3 + 0.064x_4 + 0.044x_5$

Subject to:

$x_1 + x_2 + x_3 + x_4 + x_5 = 200{,}000$

$x_1 + x_2 \leq 100{,}000$

$x_3 + x_4 \leq 100{,}000$

$-0.25x_3 - 0.25x_4 + x_5 \geq 0$

$0.4x_1 - 0.6x_2 \leq 0$

$x_1, x_2, x_3, x_4, x_5 \geq 0.$

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

Management Scientist Version 5 Software developed by Anderson et al (2000) was used to solve the resulting linear programming model. Optimal portfolio mix and optimal return on the investments were obtained for the company as shown below.

Optimal Solution

Objective Function Value = 15980.000

Variable	Value	Reduced Costs
x_1	60000.000	0.000
x_2	40000.000	0.000
x_3	80000.000	0.000
x_4	0.000	0.010
x_5	20000.000	0.000

DISCUSSION

It follows that, the optimal portfolio mix for the insurance company is as follows. Multigrow Insurance Company should invest an amount of GH₵ 60000.00, GH₵ 40000.00, GH₵ 80000.00 and GH₵ 20000.00 in Mobile Oil, Shell Oil, Anglo Gold and Government Bonds respectively. Also, the company should not invest in Tarkwa Gold. Finally, the optimal return on the investments will be GH₵ 15980.00 if the insurance company goes by the proposed optimal portfolio mix.

CONCLUSION

The concept of Linear Programming (LP) was applied to Multigrow Insurance Company in Ghana which had a portfolio problem. The company had obtained GH₵ 200000.00 cash but had a difficulty in determining how much to invest in each of five investment areas in order to maximize return. Based on the data collected, the problem was formulated as a Linear Programming Problem and solved using Management Scientist Version 5 Software. Optimal portfolio mix was obtained for the Insurance Company. The Multigrow Insurance Company should invest an amount of GH₵ 60000.00, GH₵ 40000.00, GH₵ 80000.00 and GH₵ 20000.00 in Mobile Oil, Shell Oil, Anglo Gold and Government Bonds respectively. Also, the company should not invest in Tarkwa Gold. Finally, the total optimal return on the investments of the company was found to be GH₵ 15980. It is strongly recommended that Multigrow Insurance Company should adhere to the proposed optimal portfolio mix and also employ at least one operations researcher to assist the Company in its activities.

ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)

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ISSN 2053-2229 (Print), ISSN 2053-2210 (Online)