



Flow over unsteady stretching surface with chemical reaction and non-uniform heat source

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ABSTRACT

Fluid flow over unsteady stretching surface with chemical reaction and non-uniform heat source in a quiescent medium extending to infinity was investigated. The boundary layer problem was modelled as partial differential equations and transformed to a set of ordinary differential equations using similarity variables. The three important parameters namely, the unsteadiness parameter (S), space-dependent parameter (A*) and temperature-dependent parameter (B*) for heat source/sink were included in the problem. The velocity, temperature and concentration profiles were obtained using the Runge–Kutta–Fehlberg method with the shooting techniques. The results obtained show that heat and mass transfer rates, $-\theta'(0)$ and $-\phi'(0)$, respectively and the skin friction coefficient, $f''(0)$, increased as the unsteadiness parameter increases and decreased as the space-dependent and temperature-dependent parameters for heat source/sink increase.

Full Length Research Article

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INTRODUCTION

A great number of fluid dynamic problems with significance to aerodynamic applications are inherently unsteady. Most of these flow problems are encountered during bluff body wakes, turbulent boundary layers, chemically reactive flows, turbine and rotor flows, as well as aeroelastic problems. The heat transfer problem in quiescent fluids driven by a continuously stretching surface is one important industrial problem. This problem arises during the drawing of polymer sheets, the continuous extrusion of filaments from a die, the cooling of metallic plates in a bath, aerodynamic extrusion of a plastic sheet, continuous casting, rolling, annealing and tinning of copper wires. During these processes, some of the mechanical properties of the elements are greatly affected due to the rate of cooling.

Sakiadis (1961) is credited for his pioneering works in boundary layer research generated by a continuous solid surface moving with a constant velocity. Some researchers including Vajravelu and Roper (1999), Vajravelu (2001), Ali and Magyari (2007), Sajid and Hayat (2008), and Ibrahim and Makinde (2010a, b; 2011) investigated the heat transfer problem in a stretching

sheet with a linear, power-law or exponential surface velocity and a uniform or different surface temperature conditions. The problem was extended by Abo–Eldahab and Aziz (2004) to include space-dependent exponentially decaying with internal heat generation or absorption. Abel et al. (2007) and Bataller (2007) analysed the effects of non-uniform heat source on viscoelastic fluid flow and heat transfer over a stretching sheets. Other researchers including Pantokratoras (2008), Mukhopadhyay et al. (2005), and Mukhopadhyay and Layek (2008) extended the problem to include the effects of variable fluid properties on the flow over a stretching sheet. In most of these investigations, the flow and temperature fields were considered at steady state. Some other researchers (Dandapat et al., 2003, 2007; Andersson, 2000; Ali and Magyari, 2007; Tsai et al., 2008) have studied the problem for unsteady stretching surface condition without considering the effects of chemically reactive species. Makinde and Chinyoka (2012) analysed the unsteady flow of a variable viscosity reactive fluid in a slit with wall suction or injection whilst Devakar and Iyengar (2013) recently investigated the

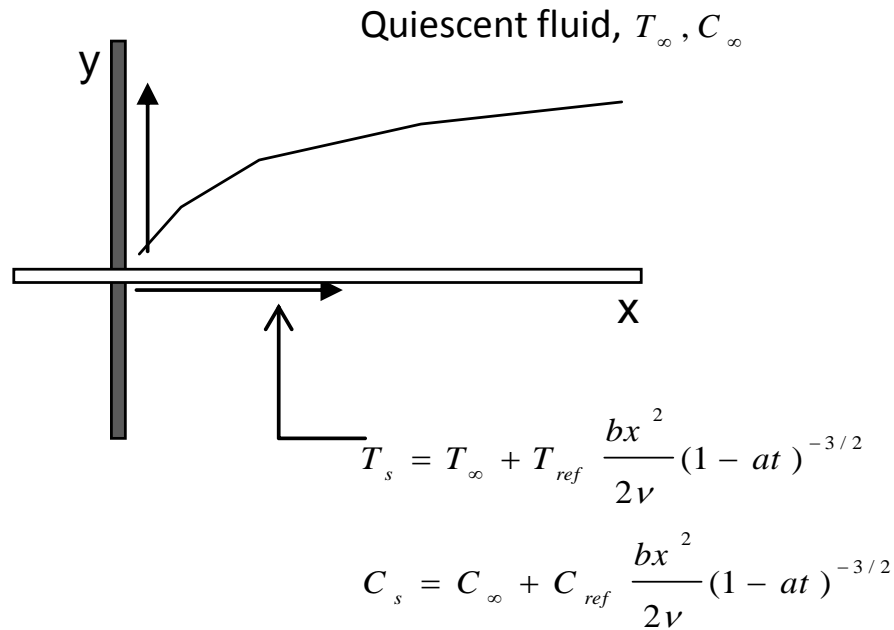


Figure 1. Schematic Diagram of the Flow Problem.

unsteady flows of a micropolar fluid between parallel plates using state space approach.

In this paper, the heat and mass transfer over unsteady stretching surface in a quiescent fluid extending to infinity in the presence of chemical reaction and non-uniform heat source/sink was investigated. The continuity, momentum, energy and concentration equations are transformed into a two-point boundary value problem using similarity analysis. The problem is then solved numerically by the Runge–Kutta–Fehlberg method with the shooting technique.

PROBLEM FORMULATION

The flow problem depicting unsteady, two dimensional incompressible and viscous flow on a horizontal thin elastic sheet that issues from a narrow slot at the origin and is continuously being stretched with a velocity $u_s = bx/(1-at)$, Tsai et al. (2008) (where a and b are positive constants, and $t < 1/a$) in the positive x -direction, (Figure 1). The fluid is considered to be Newtonian with constant temperature (T_∞) and concentration (C_∞) away from the surface.

The surface is assumed to have a non-uniform internal heat generation/absorption and the surface temperature and concentration varies with the coordinate x and time t . The governing equations for unsteady first-order chemical reactions are represented by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} q''', \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma(C - C_\infty), \quad (4)$$

Where t is time, u and v represent the velocity components in x and y directions respectively. T and C represent the fluid temperature and concentration, respectively. The fluid density, kinematic viscosity, specific heat capacity at constant pressure, thermal conductivity and the rate of chemical reaction are respectively represented by $\rho, \nu, c_p, k, \gamma$ while q''' is the non-uniform heat generated (> 0) or absorbed (< 0) per unit volume. The value of q''' is chosen approximately in accordance with Abo-Eldahab and Aziz

(2004),

$$q''' = \left(\frac{ku_s}{x\nu} \right) [A^* (T_s - T_\infty) e^{-\eta} + B^* (T - T_\infty)], \quad (5)$$

Where A^* and B^* are parameters of space-dependent and temperature-dependent heat generation/absorption. It is noted that both A^* and B^* are positive to internal heat source and negative to internal heat sink. The sheet surface temperature (T_s) and concentration (C_s) are considered as functions of distance, x and time, t as follows:

$$T_s = T_\infty + T_{ref} \frac{bx^2}{2\nu} (1 - at)^{-3/2};$$

$$C_s = C_\infty + C_{ref} \frac{bx^2}{2\nu} (1 - at)^{-3/2}, \quad (6)$$

Where T_{ref} and C_{ref} are constant reference temperature and concentration, respectively.

In this study, the sheet is assumed to be heated and its temperature and concentration is higher compared to the free stream temperature (T_∞) and concentration, (C_∞). The associated boundary conditions are:

$$u = u_s(x, t), v = 0, T = T_s(x, t), C = C_s(x, t) \text{ at } y = 0,$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty, \quad (7)$$

Where u_s is the velocity on the surface of sheet.

By introducing the following dimensionless parameters, the problem is transformed to ordinary differential equations.

$$\eta = \sqrt{\frac{b}{\nu(1-at)}} y, \quad \psi = \sqrt{\frac{\nu b}{1-at}} x f(\eta),$$

$$\theta = \frac{T - T_\infty}{T_s - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_s - C_\infty}, \quad (8)$$

Where ψ is the stream function, $f(\eta)$ is a dimensionless stream function, θ and ϕ are the dimensionless temperature and concentration respectively and η is the similarity variable. The continuity equation is identically satisfied when the velocity components are defined in the usual way as:

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (9)$$

Equations 2 to 4 are then transformed to dimensionless form;

$$f''' + ff'' - (f')^2 - S \left(\frac{1}{2} \eta f'' + f' \right) = 0, \quad (10)$$

$$\theta'' - \text{Pr} \left[\frac{1}{2} S (3\theta + \eta \theta') + 2f'\theta - f\theta' \right] + A^* e^{-\eta} + B^* \theta = 0, \quad (11)$$

$$\phi'' + Sc [f\phi' - 2f'\phi - \frac{1}{2} S (3\phi + \eta \phi') - \beta\phi] = 0, \quad (12)$$

and the associated boundary conditions become

$$f'(0) = 1, f(0) = 0, \theta(0) = 1, \phi(0) = 1,$$

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \quad (13)$$

Where the primes denote differentiation with respect to η , $S (= a/b)$ is the unsteadiness parameter, $\text{Pr} (= \rho c_p \nu / k)$ is the Prandtl number, $Sc (= \nu / D)$ is

the Schmidt number and $\beta = \frac{\gamma}{b} (1 - at)$ is the instantaneous reaction rate parameter. Note that the problem reduces to steady state when $S = 0$. Of practical importance in engineering are the local skin friction coefficient, local Nusselt number and the local Sherwood numbers which are respectively defined as:

$$C_f = \frac{\tau_s}{\rho U_s^2}, \quad Nu = \frac{xq_s}{k(T_s - T_\infty)}, \quad Sh = \frac{xq_m}{D_m(C_s - C_\infty)}, \quad (14)$$

Where τ_s is the plate surface shear stress, q_s is the surface heat flux and q_m is the surface mass flux, which are given by:

$$\tau_s = \mu \frac{\partial u}{\partial y} \Big|_{y=0}, \quad q_s = -k \frac{\partial T}{\partial y} \Big|_{y=0}, \quad q_m = -D_m \frac{\partial C}{\partial y} \Big|_{y=0}. \quad (15)$$

Substituting Equation (15) into equation (14) and simplifying:

$$\text{Re}_x^{1/2} C_f = f''(0), \quad \text{Re}_x^{-1/2} Nu = -\theta'(0), \quad \text{Re}_x^{-1/2} Sh = -\phi'(0), \quad (16)$$

Table 1. Comparison of dimensionless temperature gradient $\theta'(0)$ when $S = 0, \eta = 0, \beta = 0,$ and $A^* = 0$.

B^*	Pr	$\theta'(0)$			
		Vajravelu and Roper (1999)	Liu (2004)	Tsai et al. (2008)	Present study
-1	1	-1.710937	-1.71094	-1.710937	-1.710934
-2	2	-2.486000	-	-2.485997	-2.485997
-3	3	-3.028179	-	-3.028177	-3.028177
-4	4	-3.585194	-	-3.585192	-3.585193
-5	5	-4.028535	-4.02854	-4.028540	-4.028535

Where $Re_x = U_s x / \nu$ is the flow local Reynolds number.

NUMERICAL PROCEDURE

The set of Equations 10 to 12 subject to the boundary conditions (12) are solved numerically by the Runge–Kutta–Fehlberg method with the shooting technique. Computations of the local skin-friction coefficient, the local Nusselt number and the local Sherwood numbers are done and presented in tables. The velocity, temperature and concentration profiles were obtained graphically.

The dimensionless higher order differential equations are reduced to a system of first order differential equations by letting:

$$f = x_1, f' = x_2, f'' = x_3, \theta = x_4, \theta' = x_5, \phi = x_6, \phi' = x_7 \tag{17}$$

Thus, the corresponding system of first order differential equations is:

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= x_3, \\ x_3' &= -x_1 x_3 + x_2^2 + S \left(\frac{1}{2} \eta x_3 - x_2 \right), \\ x_4' &= x_5, \\ x_5' &= Pr \left[\frac{1}{2} S (3x_4 + \eta x_5 + 2x_2 x_4 - x_1 x_5) - A^* e^{-\eta} - B^* x_4 \right], \\ x_6' &= x_7, \\ x_7' &= -Sc [x_1 x_7 - 2x_2 x_6 - \frac{1}{2} S (3x_6 + \eta x_7) - \beta x_6], \end{aligned} \tag{18}$$

subject to the boundary conditions

$$\begin{aligned} x_1(0) &= 0, x_2(0) = 1, x_3(0) = s_1, x_4(0) = 1, \\ x_5(0) &= s_2, x_6(0) = 1, x_7(0) = s_3, \end{aligned} \tag{19}$$

In the shooting method, the unspecified initial conditions;

$s_1, s_2,$ and s_3 in Equation (19) are assumed and Equation (18) integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value. If a difference exists, improved values of the missing initial conditions must be obtained and the process is repeated. The computations were done by a written program, which uses a symbolic and computational computer language MAPLE, Heck (2003). A step size of $\Delta \eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ to each group of parameters S, A^*, B^*, Pr and Sc are determined when the values of unknown boundary conditions at $\eta = 0$ does not change to successful loop with error less than 10^{-7} . From the process of numerical computation, the local skin friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $F''(0), -\theta'(0)$ and $-\phi'(0)$ are worked out and their numerical values presented in a tabular form.

RESULTS AND DISCUSSION

To validate the accuracy of the numerical procedure, the results of $\theta'(0)$ are compared with previously published data (Table 1), for the case of steady state ($S = 0$) and $A^* = 0$. The results shown in the table is consistent with earlier established data.

Table 2 shows the effects of varying various controlling parameters on the local skin-friction and the rate of heat and mass transfers at the sheet surface. It is observed that both the skin-friction coefficient and the rate of mass transfer do not change with Prandtl (Pr) numbers. A similar observation was made for the space-dependent (A^*) and temperature-dependent (B^*) parameters. It is

Table 2. Effects of parameter variation on $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$.

Pr	A*	B*	S	Sc	β	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	0.05	0.05	1.0	0.24	1	1.414214	1.435639	0.9694777
3.0	0.05	0.05	1.0	0.24	1	1.414214	3.239729	0.9694777
7.1	0.05	0.05	1.0	0.24	1	1.414214	5.117666	0.9694777
0.71	0.1	0.05	1.0	0.24	1	1.414214	1.389026	0.9694777
0.71	1.0	0.05	1.0	0.24	1	1.414214	0.549986	0.9694777
0.71	0.05	0.1	1.0	0.24	1	1.414214	1.415432	0.9694777
0.71	0.05	1.0	1.0	0.24	1	1.414214	0.861092	0.9694777
0.71	0.05	0.05	2.0	0.24	1	1.732051	1.774437	1.1383073
0.71	0.05	0.05	3.0	0.24	1	2.000000	2.054749	1.2859428
0.71	0.05	0.05	1.0	2.14	1	1.414214	1.435639	3.1170885
0.71	0.05	0.05	1.0	2.64	1	1.414214	1.435639	3.4783306
0.71	0.05	0.05	1	0.24	2	1.414214	1.435639	1.0952491
0.71	0.05	0.05	1	0.24	3	1.414214	1.435639	1.2060021
0.71	0.05	0.05	1	0.24	4	1.414214	1.435639	1.3062607

clear from the table that the rate of heat transfer, which represents the Nusselt number increases with increasing values of Pr and reduces with increasing values of A* and B*. The unsteadiness parameter (S) is observed to have an effect of increasing the skin-friction coefficient and the rate of heat and mass transfers at the surface. The Schmidt number (Sc) and the reaction rate parameter are observed to influence only the rate of mass transfer for obvious reasons.

Figures 2 to 12 depict graphical representations of the various controlling parameters on the velocity, temperature and concentration profiles. In Figures 2 and 3, the distribution of the dimensionless velocity profile $f'(\eta)$ with increasing values of the unsteadiness parameter (S) and the dimensionless variable (η) are shown. It is seen that the velocity profile decreases with both the unsteadiness parameter (S) and the dimensionless variable (η) for the reason that unsteadiness will result in higher wall friction coefficient which will tend to reduce the velocity of flow.

Furthermore, it is clear from Figures 4 and 5 that increasing the unsteadiness parameter and the Prandtl number decreases the temperature profiles for the same reasons. The temperature profiles for different space-dependent and temperature-dependent parameters for heat source/sink are presented in Figures 6 and 7, respectively. It is observed that both A* and B* increases the temperature profiles. The heat generation source ($A^* > 0$ and $B^* < 0$) leads to a larger thermal diffusion boundary layer that may increase the thermal boundary layer thickness; on the contrary, the layer thickness decreases for heat absorption sink ($A^* < 0$ and $B^* > 0$).

From Equation (16), the values of $-f''(0)$, $\theta'(0)$ and $\phi'(0)$ represent the magnitude of the skin-friction coefficient as well as the heat and mass flux at the surface of the sheet. It is noted that a positive $-\theta'(0)$ and $-\phi'(0)$ denote heat and mass transfer from the sheet surface to fluid stream. Increasing the dimensionless variable (η) is observed to increase the temperature and concentration profiles (Figures 8 and 11).

In Figures 9 and 10, the concentration boundary layers are observed to decrease with increasing values of the unsteadiness parameter and the Schmidt number. Same is observed in Figure 12 which represents the concentration profile when the reaction rate parameter is increased for the first – order reaction. This is caused by the destructive nature of the chemical reaction within the boundary layer.

Conclusion

In this paper, the partial differential equations modelling the unsteady flow problem is transformed to non-linear systems of ordinary differential equations using similarity analysis. The problem involved heat and mass transfer in an incompressible, quiescent Newtonian fluid flow caused solely by a unsteady stretching of a horizontal sheet with non-uniform internal heat generation/absorption in the presence of chemically reactive species. The heat and mass transfer rates, $-\theta'(0)$ and $-\phi'(0)$ respectively and the skin friction

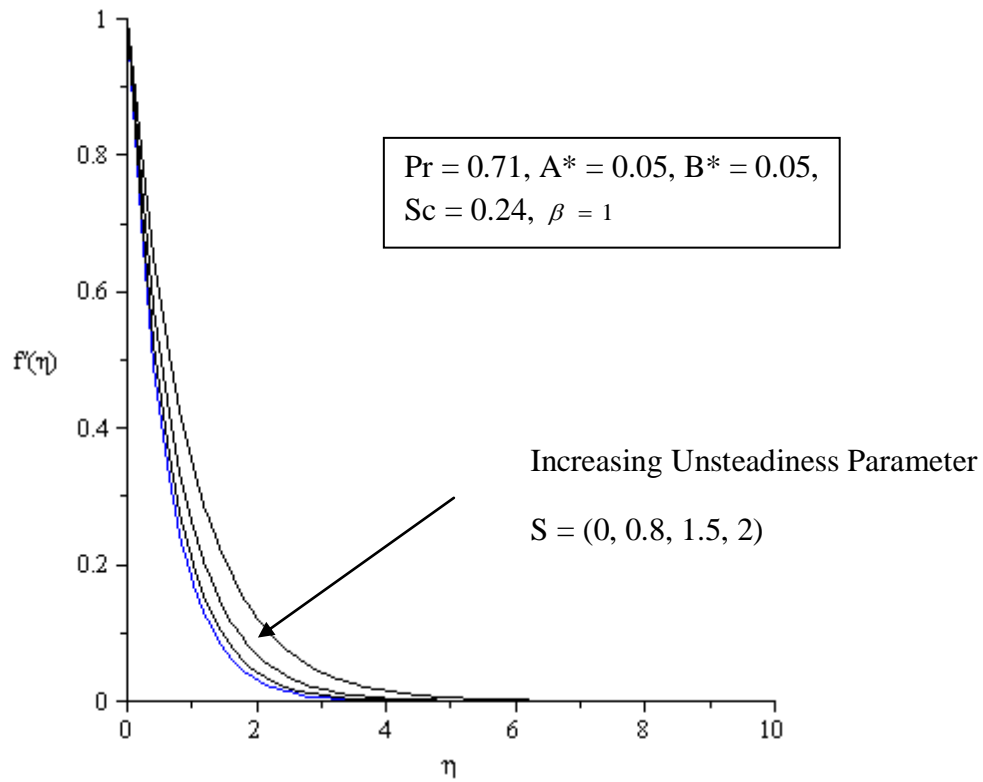


Figure 2. Velocity profile for varying values of the unsteadiness parameter (S).

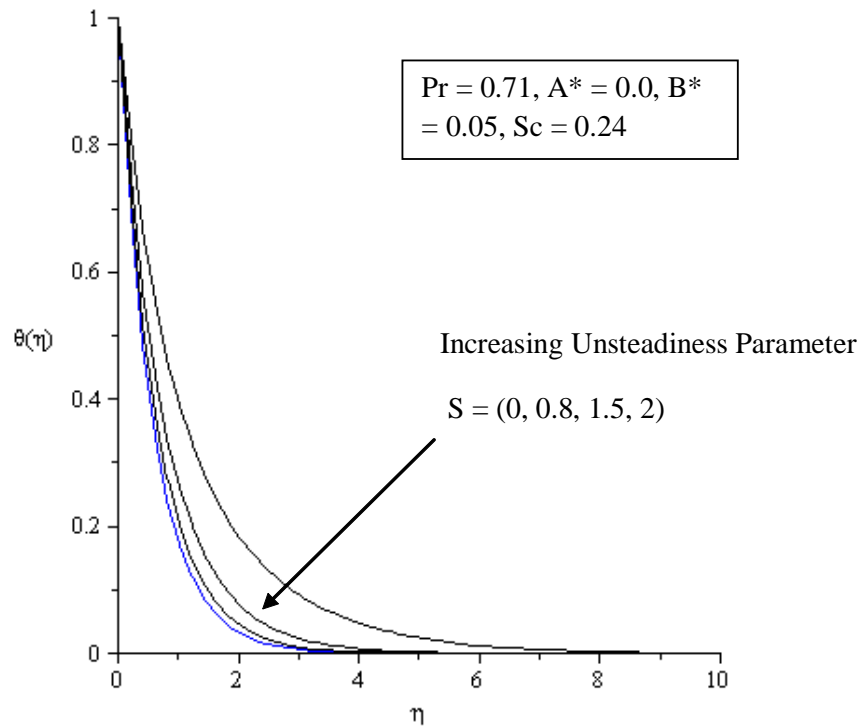


Figure 3. Velocity profile for varying values of η .

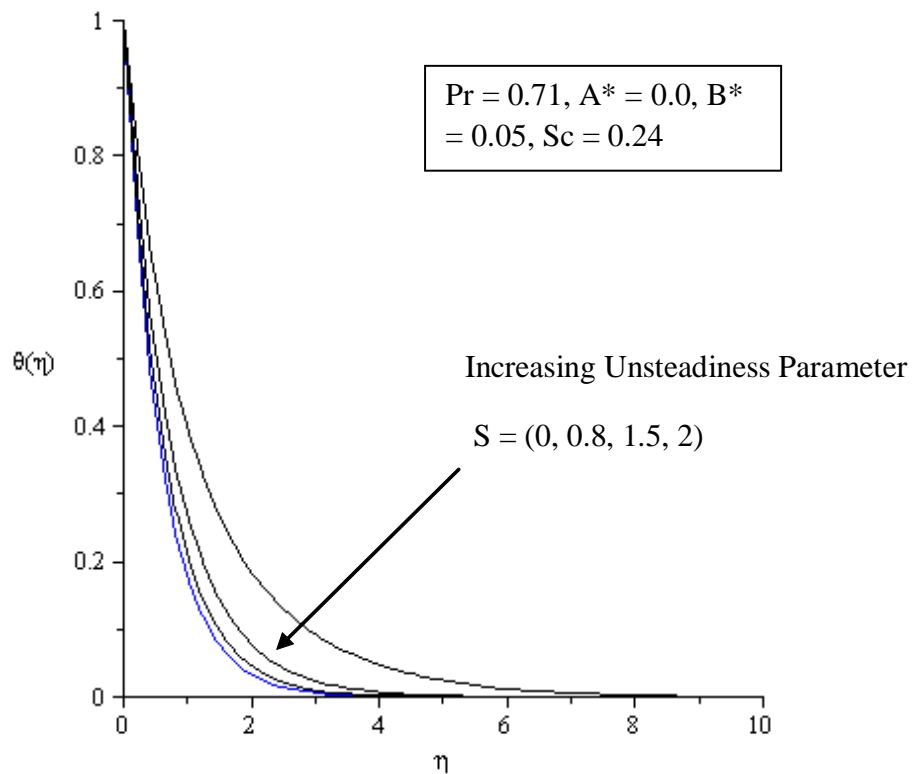


Figure 4. Temperature profiles for varying values of the unsteadiness parameter (S).

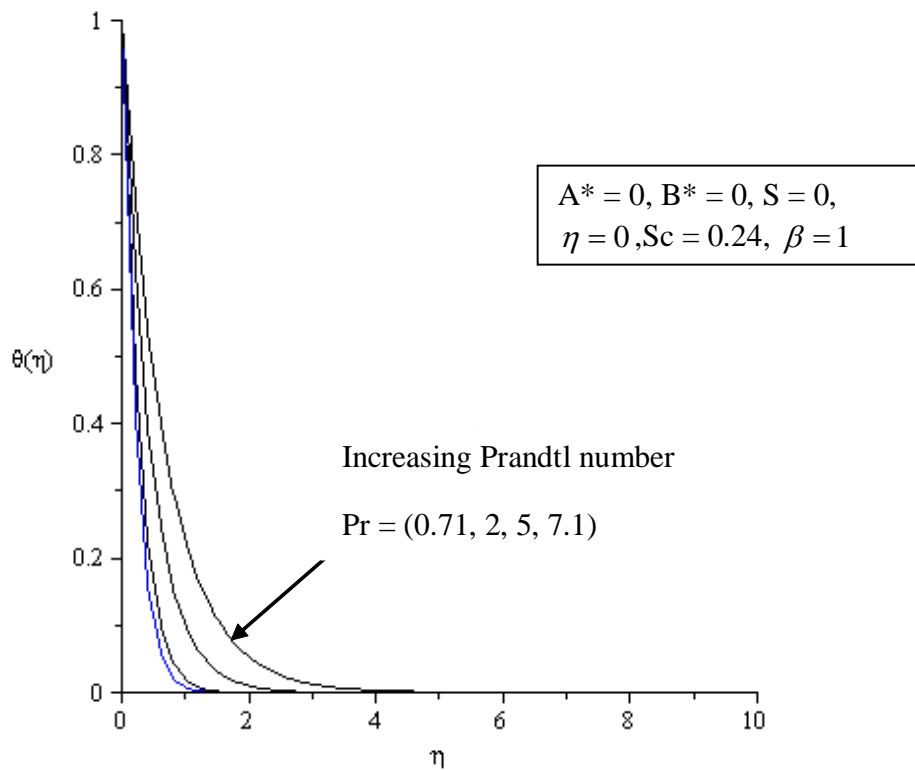


Figure 5. Temperature profiles for varying values of Pr.

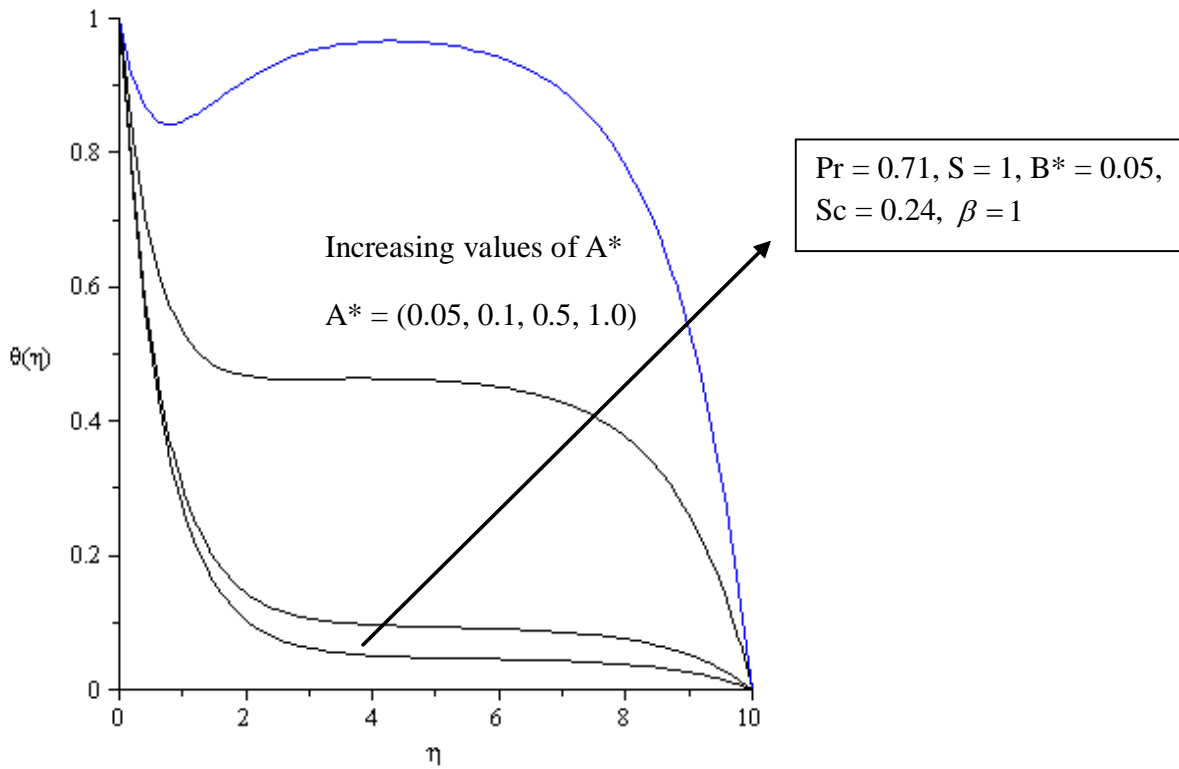


Figure 6. Temperature profiles for varying values of A^* .

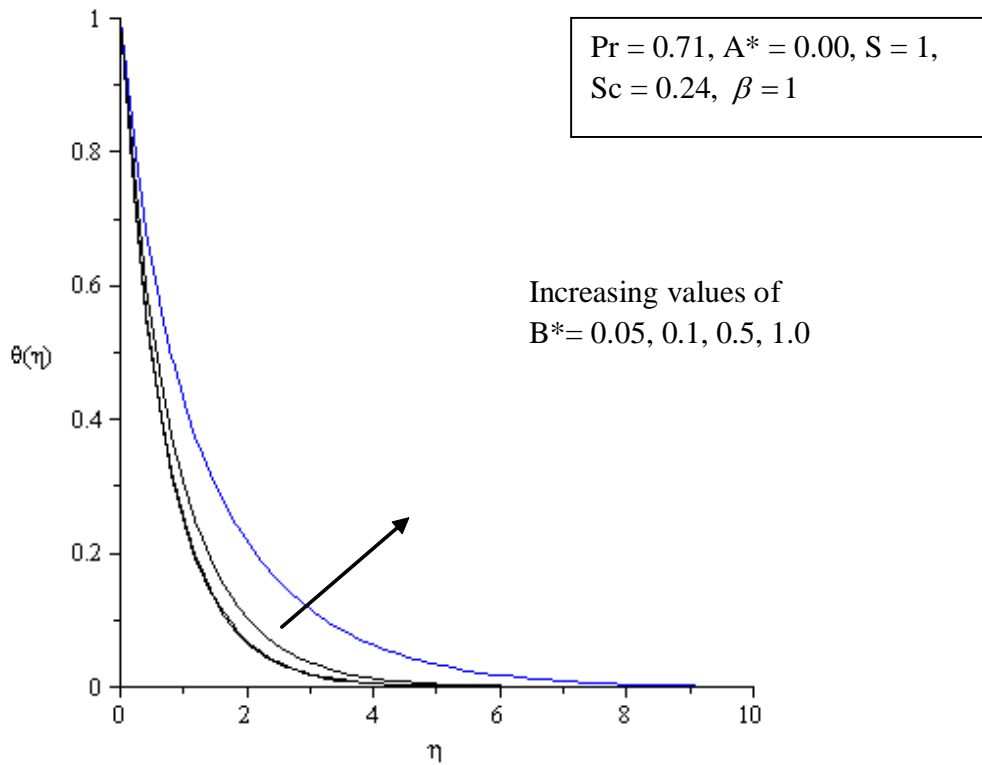


Figure 7. Temperature profiles for varying values of B^* .

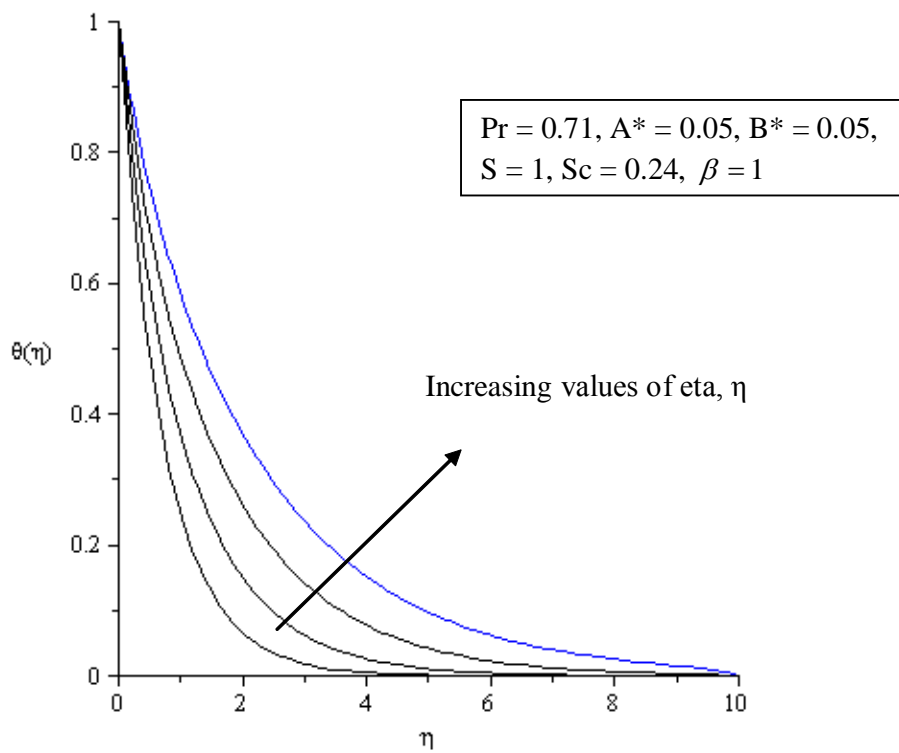


Figure 8. Temperature profile for varying values of η .

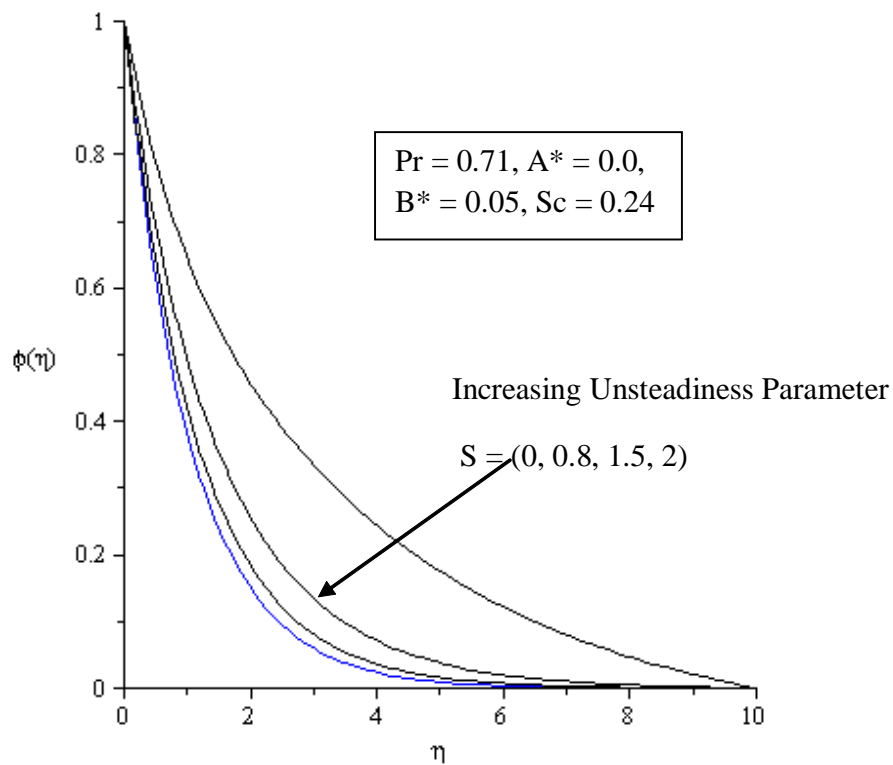


Figure 9. Concentration profiles for varying values of unsteadiness parameter (S).

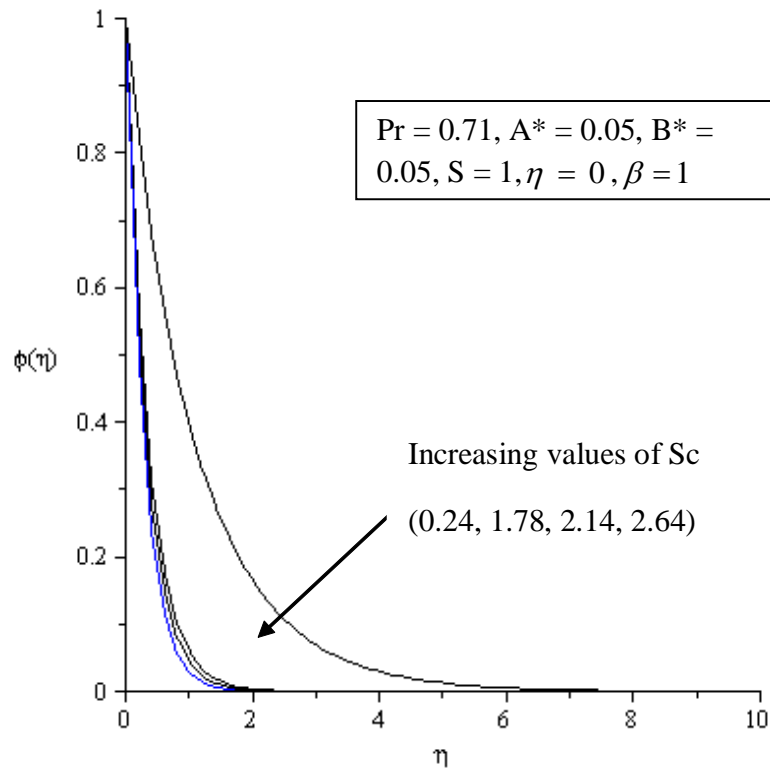


Figure 10. Concentration for varying values of Schmidt number (Sc).

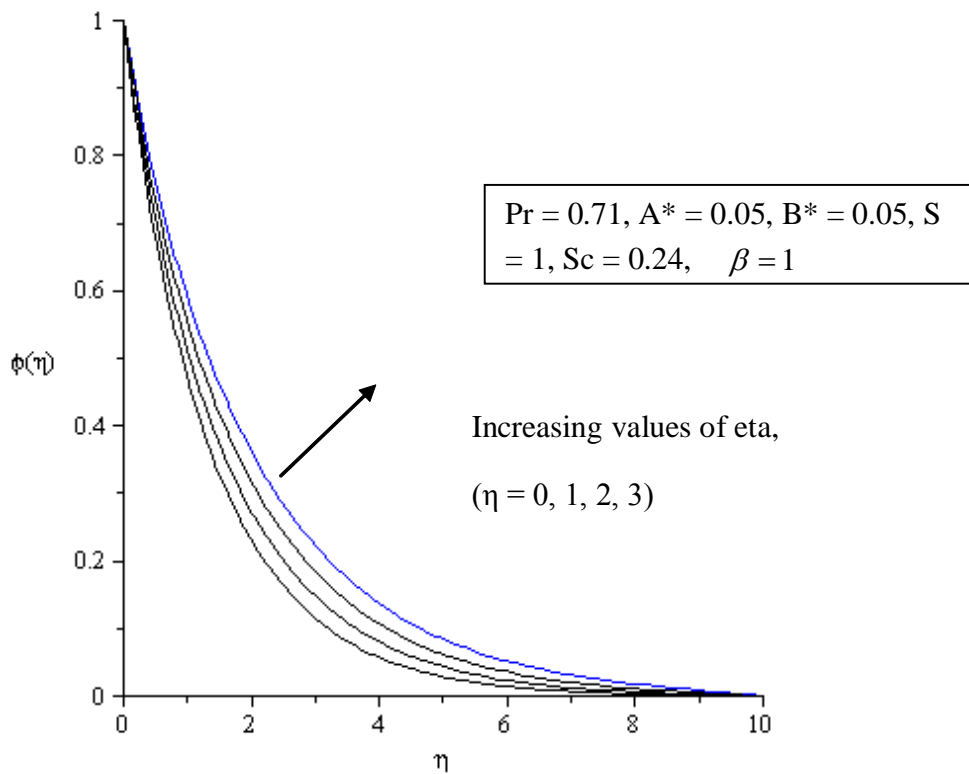


Figure 11. Concentration profiles for varying values of η .

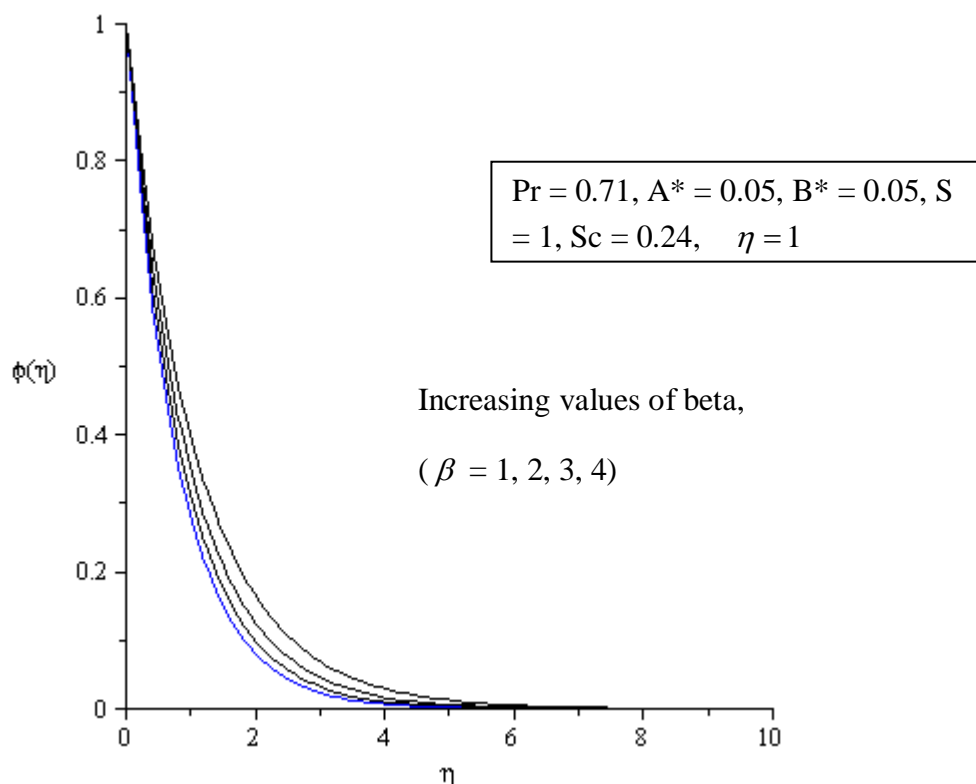


Figure 12. Concentration profiles for varying reaction rate parameter (β).

coefficient, $f''(0)$ were investigated and observed to increase as the unsteadiness parameter increases. The rate of heat transfer, $-\theta'(0)$ is observed to be the only parameter affected by the space-dependent (A^*) and temperature dependent (B^*) parameters for heat source/sink. The graphical illustrations also include the effects of chemical reaction on the boundary layer near the surface.

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APPENDIX

Nomenclature:

- x, y Cartesian Coordinates variables
- u, v Velocity components in the x - and y - coordinate axes
- T_∞ Temperature of fluid medium far away from the plate surface,
- C_∞ Concentration of the fluid medium far away from the plate surface,
- T Fluid temperature,
- T_s Plate surface temperature,
- C Fluid concentration,
- C_s Plate surface concentration,
- D Coefficient of mass diffusivity,
- t Time (seconds)
- c_p Specific heat capacity at constant pressure,
- u_s Velocity of stretching surface,
- T_m Mean fluid temperature,
- C_m Mean fluid concentration
- T_{ref} Reference temperature,
- C_{ref} Reference concentration,
- q''' Non-uniform heat source
- A^* Space-dependent parameter
- B^* Temperature-dependent parameter
- Pr Prandtl number,
- Sc Schmidt number,
- q_w Surface heat flux,

- q_m Surface mass flux,
- Re_x Local Reynolds number,
- C_f Local skin friction coefficient,
- Nu_x Local Nusselt number,
- Sh_x Local Sherwood number,

Greek Symbols

- η Dimensionless coordinate variable
- τ_w Plate surface shear stress,
- ρ Fluid density,
- ν Kinematic viscosity,
- α Thermal diffusivity,
- ψ Dimensionless stream function
- ϕ Dimensionless concentration,
- θ Dimensionless temperature,

Subscripts

- w Wall conditions
- ∞ Conditions at infinity