

Full Length Research Paper

Magnetohydrodynamics (MHD) boundary layer stagnation point flow with radiation and chemical reaction towards a heated shrinking porous surface

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This paper is an investigation of the effects of chemical reaction on two dimensional steady stagnation point flow of an electrically conducting, incompressible, and viscous fluid with radiation towards a heated shrinking porous surface. A chemically reactive species is emitted from the vertical surface into the flow field. The governing partial differential equations are solved using the Newton-Raphson shooting method along with the fourth-order Runge-Kutta integration. Velocity, temperature and concentration profiles are presented graphically. Numerical results for the skin friction coefficient, the rate of heat transfer represented by the local Nusselt number and the rate of mass transfer represented by the local Sherwood number are presented in tables and discussed quantitatively. The effects of magnetic field parameter, the velocity ratio parameter, the radiation parameter, the suction parameter, Schmidt number, Eckert number, Prandtl number and reaction rate parameter on the flow field are discussed.

Key words: Porous medium, stagnation point, magnetohydrodynamics (MHD), shrinking surface, radiation.

INTRODUCTION

Heat transfer is an important area of fluid dynamic research. The presence of magnetic field in a body of fluid has now been known to have significant practical applications in science, engineering and industry. It is commonly encountered in nuclear power plants, cooling of transmission lines and in electric transformers. Some investigations have been conducted to study the effects of radiation on electrically conducting fluids due to its wide applications in space technology.

The problem of radiation on magnetohydrodynamic (MHD) free convection flows under different surface or boundary conditions using different mathematical techniques have been reported in the literature. For instance, Seini and Makinde (2013) investigated the MHD boundary layer flow due to exponential stretching surface with radiation and chemical reaction and observed that the rate of heat transfer at the surface was adversely affected by increases in the transverse magnetic field

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parameter and the radiation parameter. Mahapatra and Nandy (2011) presented a momentum and heat transfer solution to MHD axisymmetric stagnation-point flow over a shrinking sheet. Mahapatra et al. (2011) analysed the steady two-dimensional MHD stagnation-point flow of an electrically conducting fluid over a shrinking sheet with a uniform transverse magnetic field whilst Jafar et al. (2011) investigated the MHD stagnation point flow over a nonlinearly stretching/shrinking sheet. In a related study, Javed et al. (2012) analysed the heat transfer in a viscous fluid over a non-linear shrinking sheet in the presence of a magnetic field and obtained dual solutions for the shrinking sheet problem whilst Zeeshan et al. (2012) investigated the porosity and magnetohydrodynamic flow of non-Newtonian nanofluid in coaxial cylinders using the Homotopy Analyses Method (HAM). Similar results have been reported by many authors including Ibrahim and Makinde (2010a, b; 2011a, b), who considered the MHD flow under varied boundary conditions.

Makinde and Charles (2010) conducted computational dynamics on hydromagnetic stagnation point flow towards a stretching sheet and observed that the cooling rate of the stretching sheet in an electrically conducting fluid subjected to magnetic field could be controlled and a final product with desired characteristics can be achieved. Ibrahim and Makinde (2010a) investigations on MHD boundary layer flow of chemically reacting fluid with heat and mass transfer past a stretching sheet also concluded that both the magnetic field strength and the uniform heat source had significant impact in cooling surfaces. Rana and Bhargava (2012) analysed the steady laminar boundary layer flow resulting from non-linear stretching of a flat surface in a nanofluid whilst Hameed (2012) numerically analysed the steady non-Newtonian flows with heat transfer analysis, MHD and nonlinear slip effects. Ellahi (2013) provided analytical solutions to the effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe whilst Sheikholeslami et al. (2014) analysed the effects of heat transfer in the flow of nanofluids over a permeable stretching wall in a porous medium and concluded that increasing the nanoparticle volume fraction had the effect of decreasing the momentum boundary layer thickness and entropy generation rate but increases the thermal boundary layer thickness.

The presence of chemical reaction and non-uniform heat source over an unsteady stretching surface was investigated by Seini (2013) who observed that the heat and mass transfer rates and the skin friction coefficient increases as the unsteadiness parameter increased and decreases as the space-dependent and temperature-dependent parameters for heat source/sink increased. Alireza et al. (2013) then presented an analytical solution for MHD stagnation point flow and heat transfer over a permeable stretching sheet with chemical reaction. Arthur

and Seini (2014) recently analyzed the MHD thermal stagnation point flow towards a stretching porous surface whilst Seini and Makinde (2014) analyzed the boundary layer flow problem near stagnation-points on a vertical surface with slip in the presence of transverse magnetic field. Sheikholeslami et al. (2014) then investigated the effects of MHD on Cu-water nanofluid flow and heat transfer by means of Control-Volume Finite-Element Method (CVFEM). Similarly, Ellahi et al. (2013) studied the non-Newtonian nanofluid flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity using the Homotopy Analysis Method (HAM). Furthermore, Ellahi et al. (2014a, b) investigated the effects of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct and also analyzed the steady flows in viscous fluid with heat and mass transfer with slip effects using the Spectral Homotopy Analyses Method (SHAM) and obtained interesting results for the generalised Couette flow problem.

To the best knowledge of the authors, only a limited number of researchers have attempted to solve the problem of radiation effects on MHD boundary layer stagnation point flow towards a heated shrinking sheet, notably Muhammad and Shahzad (2011). Stagnation point flow with radiation towards a shrinking sheet is quite useful and important from the practical point of view. In this paper, an attempt is made to investigate the effect of radiation on chemically reacting MHD boundary layer flow towards a heated shrinking porous surface due to its numerous industrial applications involving cooling.

FORMULATION OF THE PROBLEM

Consider a steady two-dimensional flow of an incompressible and electrically conducting fluid towards a stagnation point on a porous stretching sheet in the presence of radiation and magnetic field of strength B_0 , applied in the positive y direction as shown in Figure 1.

The tangential velocity u_w , and the free stream velocity U_∞ were assumed to vary proportional to the distance x from the stagnation point so that $u_w(x) = ax$ and $U_\infty(x) = bx$. The induced magnetic field due to the motion of the electrically conducting fluid and the pressure gradient are neglected. The tangential temperature is maintained at the prescribed constant value T_w .

The boundary layer equations for a steady incompressible viscous hydrodynamic fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U - u), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_0}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} (U - u)^2 - \frac{\partial q_r}{\partial y}, \quad (3)$$

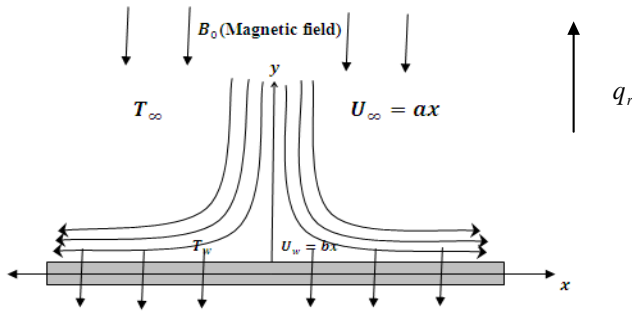


Figure 1. Schematic diagram of the problem.

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \gamma(C - C_\infty). \tag{4}$$

Boundary conditions

$$y = 0, u(x,0) = bx, v(x,0) = -V, T(x,0) = T_w, C(x,0) = C_w, \tag{5}$$

$$y \rightarrow \infty, u(x, \infty) \rightarrow U = ax, T(x, \infty) = T_\infty, C(x, \infty) = C_\infty,$$

where $b > 0$ is the shrinking rate, T_w is the heated wall surface temperature and T_∞ is the temperature of the fluid outside the boundary layer ($T_w > T_\infty$), C_w is the wall surface concentration and C_∞ is the concentration of the fluid outside the boundary layer ($C_w > C_\infty$).

Using the Rosseland approximation for radiation, Ibrahim and Makinde (2011b) simplified the heat flux as

$$q_r = -\frac{4\sigma}{3k} \frac{\partial T^4}{\partial y} \tag{6}$$

where k and σ are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature differences within the flow such as the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we get;

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Using Equations (6) and (7), Equation (3) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{k_0}{\rho c_p} + \frac{16}{3} \frac{\sigma T_\infty^3}{\rho c_p k} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} (U - u)^2, \tag{8}$$

Where $k = \frac{3Nr}{3Nr + 4}$

In order to obtain the velocity, temperature and concentration fields for the problem, Equations (1), (2), (4) and (8) are solved subject to the appropriate boundary conditions given in Equation (5) by employing the following similarity variables.

$$\eta = y \sqrt{\frac{a}{\nu}}, \quad u(x, y) = axf'(\eta), \quad v = -\sqrt{a\nu}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$p(x, \infty) = P_0 - \frac{\rho a^2}{2}(x^2 + y^2), \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{9}$$

where η is a similarity parameter, P_0 is the stagnation pressure,

a is the strength of the stagnation point with dimension $\frac{1}{t}$.

Equation (9) satisfies Equation (1) identically by noting the usual definition for a stream function as:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x},$$

hence, the velocity field represents a possible fluid motion. Equations (2), (4) and (9) are then transformed into

$$f''' + M^2(1 - f') + 1 = f'^2 - ff'', \tag{10}$$

$$\theta'' + \text{Pr}kf\theta' + \text{Pr}EcM^2f'^2 = 0, \tag{11}$$

$$\phi'' + \text{Sc}\phi' - \text{Sc}\beta\phi = 0, \tag{12}$$

where $M = \sqrt{\frac{\sigma_e B_0^2}{\rho a}}$ and $\text{Pr} = \frac{\mu c_p}{k_0}$ are the magnetic

parameter and Prandtl number respectively. Ec , Sc and β are the Eckert number, the Schmidt number and the reaction rate

parameter. $k = \frac{3Nr}{3Nr + 4}$ and Nr is the radiation parameter. The boundary conditions given in Equation (5) are also transformed into

$$f'(0) = \frac{b}{a} = B, f(0) = fw, \theta(0) = 1, \phi(0) = 1,$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0. \tag{13}$$

NUMERICAL PROCEDURE

The governing Equations (10), (11) and (12) are highly non-linear. Most physical systems are inherently nonlinear in nature and are of great interest to engineers, physicist and mathematicians, problems involving nonlinear ordinary differential equations are difficult to solve and give rise to interesting phenomena such as chaos. We employ the Runge-Kutta integration along with the Newton Raphson algorithm to obtain approximate solutions. In this method, we let:

Table 1. Comparison of shear stress and heat transfer for $M = 1$ $Nr = 3$ $Pr = 0.7$ with various values of B .

B	Shear stress ($f''(0)$)		Rate of Heat Transfer ($-\theta'(0)$)	
	Muhammad and Shahzad (2011)	Present Study	Muhammad and Shahzad (2011)	Present Study
0.25	1.877455	1.877560	0.412813	0.413620
0.50	2.120114	2.120190	0.378822	0.379139
0.75	2.307090	2.307127	0.342202	0.342030
1.0	2.429972	2.429962	0.302334	0.301637

Table 2. Results of skin friction coefficient, Nusselt number and Sherwood number for varying parameter values when $Pr = 0.71$, $Ec = 0.1$, $Sc = 0.24$.

M	β	F_w	B	Nr	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0.1	0.1	0.1	3	1.505368	0.327923	0.323899
2	0.1	0.1	0.1	3	2.178774	-0.17930	0.323899
3	0.1	0.1	0.1	3	2.977204	-1.09646	0.323899
1	0.5	0.1	0.1	3	1.505368	0.327923	0.487086
1	1.0	0.1	0.1	3	1.505368	0.327923	0.624423
1	0.1	0.5	0.1	3	1.717241	0.438848	0.323893
1	0.1	1.0	0.1	3	2.008972	0.594699	0.323899
1	0.1	0.1	0.5	3	0.896951	0.354065	0.487086
1	0.1	0.1	0.8	3	0.375867	0.369433	0.487086
1	0.1	0.5	0.1	4	1.505368	0.366143	0.487086
1	0.1	0.5	0.1	5	1.505368	0.351018	0.487086

$$\begin{aligned}
 x_1 &= f, \\
 x'_1 &= x_2 = f', \\
 x'_2 &= x_3 = f'', \\
 x'_3 &= f''' = x_2^2 - x_1 x_3 - 1 - M^2(1 - x_2), \\
 x'_4 &= x_5 = \theta', \\
 x'_5 &= -Pr k f x_5 - Pr Ec M^2 x_2^2, \\
 x'_6 &= x_7 = \phi, \\
 x'_7 &= x_8 = \phi', \\
 x'_8 &= \phi'' = -Sc x_8 + Sc \beta x_7.
 \end{aligned}
 \tag{14}$$

RESULTS AND DISCUSSION

Table 1 compares results of this study and that of Muhammad and Shahzad (2011). It is observed that the numerical results are consistent with their work and hence validate our numerical procedure.

The results of varying parameter values on the local skin friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively

proportional to $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ are shown in Table 2. It is observed that increasing the magnetic field strength parameter increases the skin friction coefficient at the surface due to the presence of the Lorenz force. It however reduces the rate of heat transfer and does not affect the rate of mass transfer for obvious reasons. With the case of increasing the reaction rate parameter, the rate of mass transfer at the surface increases, however, the skin friction coefficient and the rate of heat transfer are not affected by the reaction rate parameter. It is interesting to note that increasing the suction parameter do not only increase the skin friction coefficient but also the rate of heat and mass transfer. Increasing the velocity ratio parameter decreases the skin friction coefficient and increases the rate of heat transfer on the surface. Furthermore, increasing the radiation parameter causes a reduction in the rate of heat transfer at the surface whereas both the coefficient of skin friction and the rate of mass transfer are not affected.

Effects of parameter variation on the velocity profiles

The effect of varying various parameters on the velocity

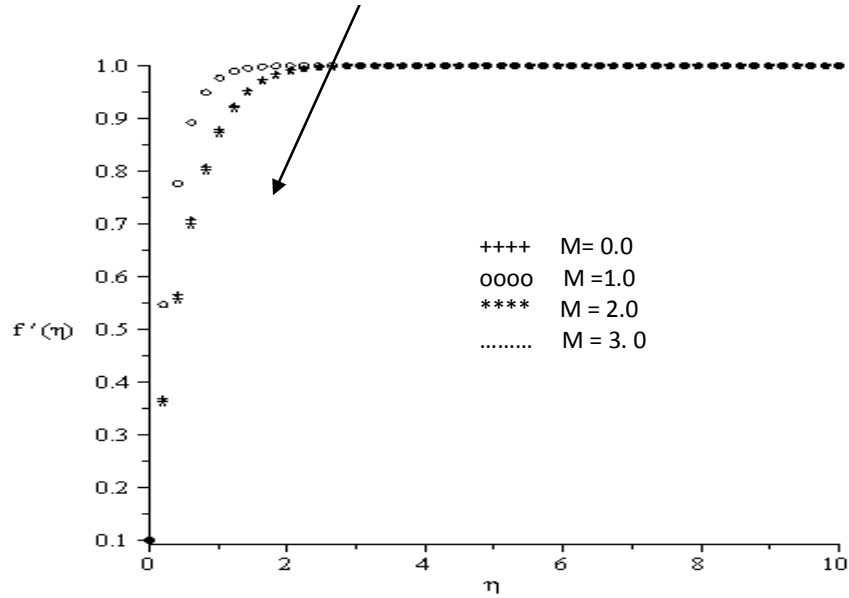


Figure 2. Velocity profiles for varying values of magnetic parameter (M).

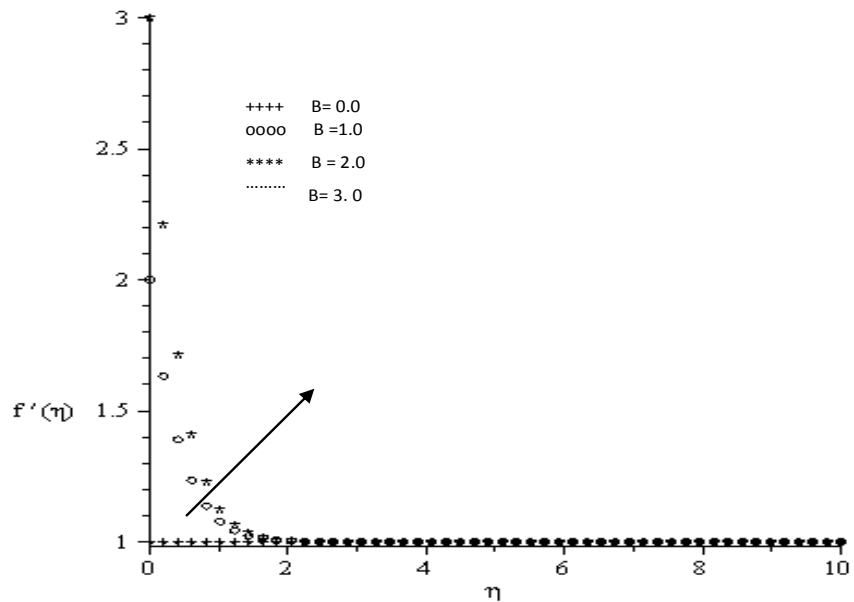


Figure 3. Velocity profiles for varying values of velocity ratio parameter (B).

boundary layer are depicted in Figures 2 to 4. The velocity profiles are observed to decrease when the magnetic field parameter is increased (Figure 2). This phenomenon is due to the fact that increasing the magnetic field strength increases the Lorentz force which creates an opposing force to the fluid transport. In Figures 3 and 4, it is observed that increasing the velocity ratio parameter (B) and the suction parameter (fw) increased the velocity boundary layer.

Effects of parameter variation of temperature profiles

Figures 5 to 10 depicts the effects of the magnetic field parameter, velocity ratio parameter, radiation parameter, Eckert number and Prandtl number respectively on the temperature profiles. It is observed that both the Eckert number and the magnetic parameter contribute to greater thermal boundary layer thickness. The reverse is true for increasing radiation parameter, velocity ratio parameter,

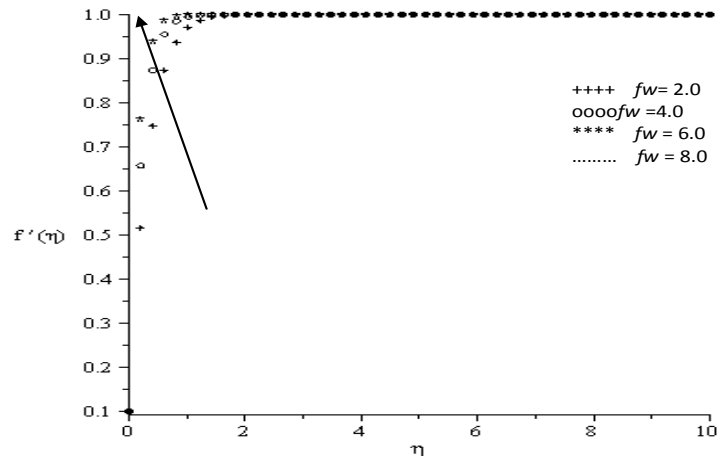


Figure 4. Velocity profiles for varying values of Suction parameter (f_w).

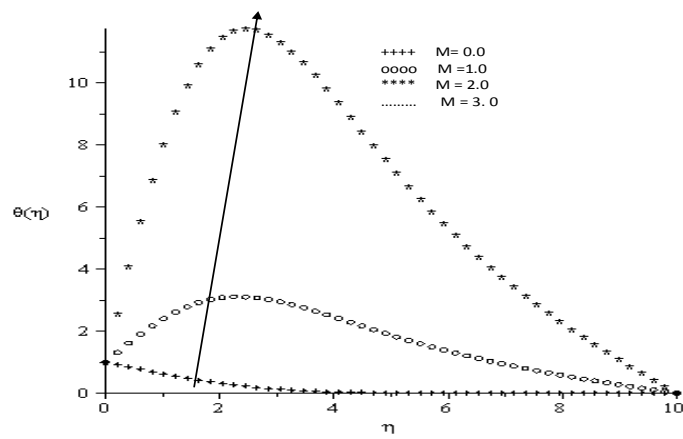


Figure 5. Temperature profiles for varying values of magnetic parameter (M).

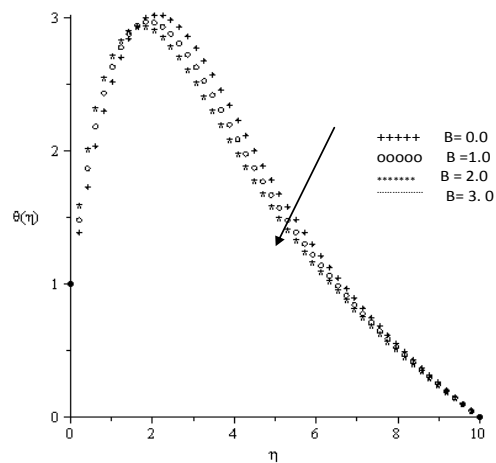


Figure 6. Temperature profiles for varying values of velocity ratio parameter (B).

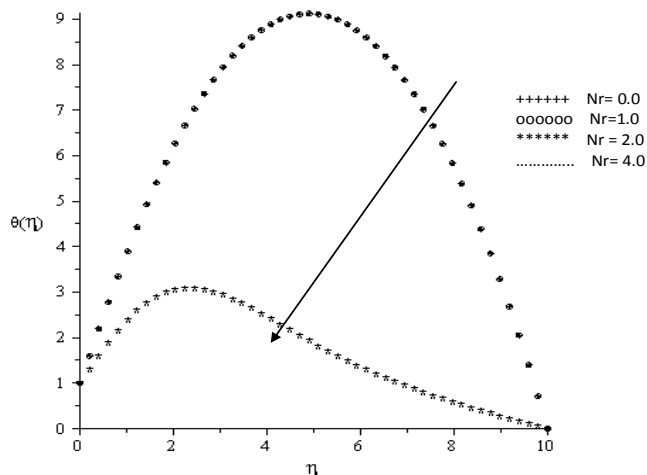


Figure 7. Temperature profiles for varying values of radiation parameter (Nr).

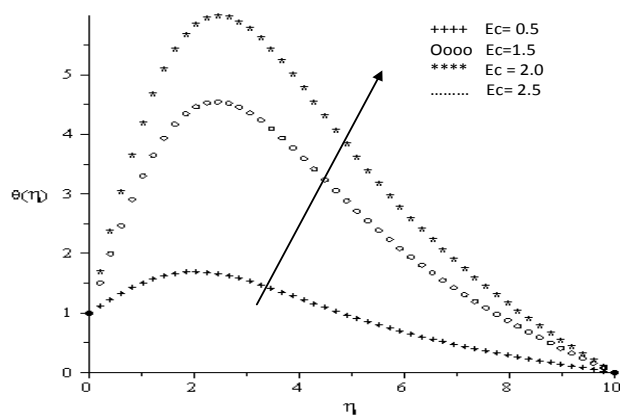


Figure 8. Temperature profiles for varying values of Eckert number (Ec).

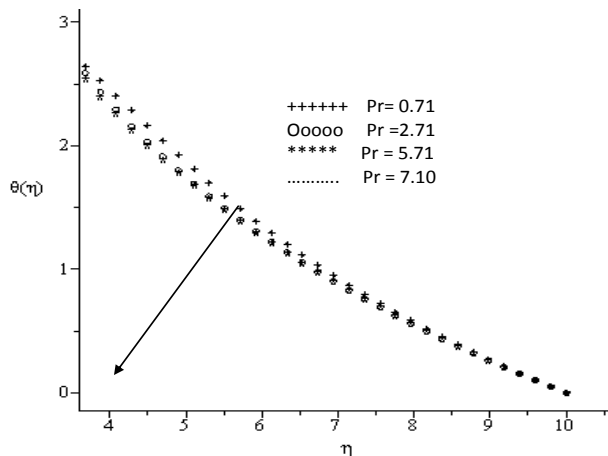


Figure 9. Temperature profiles for varying values of Prandtl number (Pr).

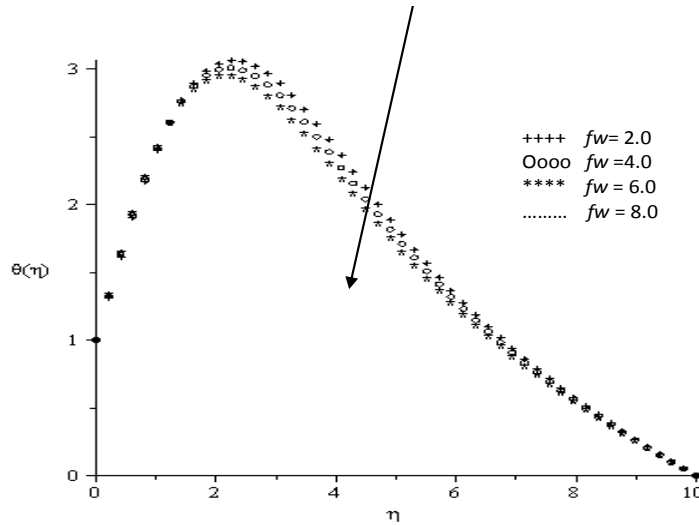


Figure 10. Temperature profiles for varying values of suction parameter (f_w).

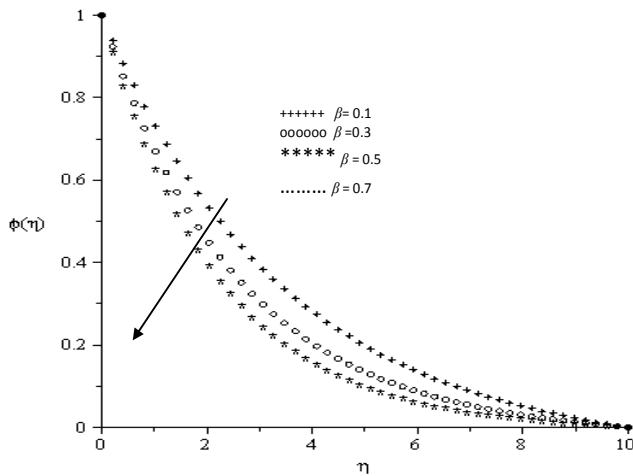


Figure 11. Concentration profiles for varying values of the reaction rate parameter (β).

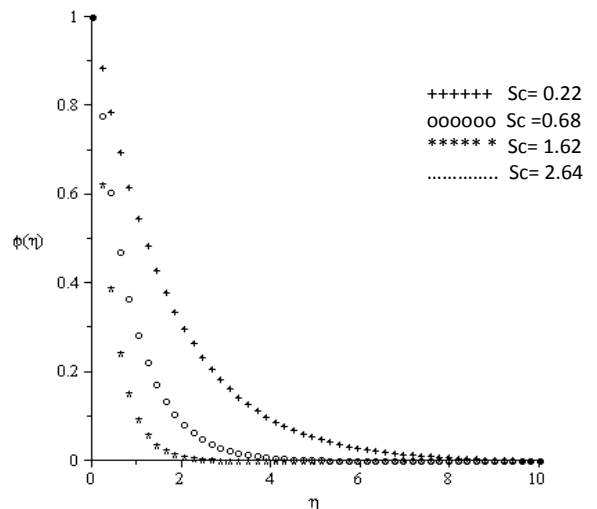


Figure 12. Concentration profiles for varying Schmidt number.

suction parameter and the Prandtl number. For the Prandtl number, it could be due to the fact that decreasing the thermal diffusivity results in the heat being diffused away from the surface which leads to increases in the temperature gradient at the surface.

Effects of parameter variation on the concentration profiles

It is observed in Figures 11 and 12 that increasing the reaction rate parameter and the Schmidt numbers reduces the concentration boundary layers.

CONCLUSIONS

The MHD boundary layer stagnation point flow with radiation and chemical reaction towards a heated shrinking porous surface has been investigated. Numerical results have been compared to earlier results published in the literature and the agreement was good. Our results reveal that:

- (i) The velocity profiles of the flow increases with increasing suction parameter (f_w) and the velocity ratio (B). It however decreases with increasing values of the magnetic parameter (M).

(ii) The thermal boundary layer is observed to decrease with increasing values of Pr , f_w , Nr and B . It however observed to increase with increasing values of M and Ec .

(iii) The concentration boundary layer decreases with increasing reaction rate parameter (β) and Schmidt number.

(iv) The skin friction at the surface increases for increasing parameter values of M and f_w but decreases for increasing B .

Conflict of Interest

The authors have not declared any conflict of interest.

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