Turkish Journal of Analysis and Number Theory, 2014, Vol. 2, No. 6, 226-229
Available online at http://pubs.sciepub.com/tjant/2/6/7

DOI:10.12691/tjant-2-6-7

Generalized Inequalities Related to the Classical Euler's Gamma Function

Kwara Nantomah*

Department of Mathematics, University for Development Studies, Navrongo Campus, Navrongo UE/R, Ghana
*Corresponding author: mykwarasoft@yahoo.com, knantomah@uds.edu.gh

Received October 10, 2014; Revised November 26, 2014; Accepted December 14, 2014

Abstract This paper presents some inequalities concerning certain ratios of the classical Euler's Gamma function. The results generalized some recent results.

Keywords: *Gamma function, q-Gamma function, k-Gamma function, (p,q)-Gamma function, (q,k)-Gamma function, inequality*

Cite This Article: Kwara Nantomah, "Generalized Inequalities Related to the Classical Euler's Gamma Function." *Turkish Journal of Analysis and Number Theory*, vol. 2, no. 6 (2014): 226-229. doi: 10.12691/tjant-2-6-7.

1. Introduction

We begin by outlining the following basic definitions well-known in literature.

The celebrated classical Euler's Gamma function, $\Gamma(t)$ is defined for $t>0$ as

$$\Gamma(t)=\int_0^\infty e^{-x}x^{t-1}dx.$$

The q-Gamma function, $\Gamma_q(t)$ is defined for $q\in(0,1)$ and $t>0$ as (see [2])

$$\Gamma_q(t)=(1-q)^{1-t}\prod_{n=1}^{\infty}\frac{1-q^n}{1-q^{t+n}}.$$

Also, the k-Gamma function, $\Gamma_k(t)$ was defined by Diaz and Pariguan [1] for $k>0$ and $t>0$ as

$$\Gamma_k(t)=\int_0^\infty e^{-\frac{x^k}{k}}x^{t-1}dx.$$

Diaz and Teruel [5] further defined the (q,k)-Gamma function $\Gamma_{(q,k)}(t)$ for $q\in(0,1)$, $k>0$ and $t>0$ as

$$\Gamma_{(q,k)}(t)=\frac{(1-q^k)_{q,k}^{\frac{t}{k}-1}}{(1-q)^{\frac{t}{k}-1}},$$

where

$$(t)_{n,k}=t(t+k)(t+2k)\cdots(t+(n-1)k)=\prod_{j=0}^{n-1}(t+jk)$$

is the k-generalized Pochhammer symbol.

Furthermore, Krasniqi and Merovci [4] defined the (p,q)-Gamma function $\Gamma_{(p,q)}(t)$ for $p\in N$, $q\in(0,1)$ and $t>0$ as

$$\Gamma_{(p,q)}(t)=\frac{[p]_q^t[p]_q!}{[t]_q[t+1]_q\cdots[t+p]_q},$$

where

$$[p]_q=\frac{1-q^p}{1-q}.$$

The psi function, $\psi(t)$ otherwise known as the digamma function is defined as the logarithmic derivative of the Gamma function. That is,

$$\psi(t)=\frac{d}{dt}\text{In}\Gamma(t)=\frac{\Gamma'(t)}{\Gamma(t)}.$$

The q-digamma function, k-digamma function, (p,q)-diagamma function and (q,k)-digamma function are similarly defined as follows:

$$\psi_q(t)=\frac{d}{dt}\text{In}\Gamma_q(t)=\frac{\Gamma_q'(t)}{\Gamma_q(t)},$$

$$\psi_k(t)=\frac{d}{dt}\text{In}\Gamma_k(t)=\frac{\Gamma_k'(t)}{\Gamma_k(t)},$$

$$\psi_{(p,q)}(t)=\frac{d}{dt}\text{In}\Gamma_{(p,q)}(t)=\frac{\Gamma_{(p,q)}'(t)}{\Gamma_{(p,q)}(t)}$$

and

$$\psi_{(q,k)}(t)=\frac{d}{dt}\text{In}\Gamma_{(q,k)}(t)=\frac{\Gamma_{(q,k)}'(t)}{\Gamma_{(q,k)}(t)}.$$

It is common knowledge that these functions exhibit the following series charaterizations (see also [7-12]):

$$\psi_q(t) = -\text{In}(1-q) + (\text{In}q)\sum_{n=1}^{\infty}\frac{q^{nt}}{1-q^n} \quad (1)$$

$$\psi_k(t) = \frac{\text{In}k - \gamma}{k} - \frac{1}{t} + \sum_{n=1}^{\infty}\frac{t}{nk(nk+t)} \quad (2)$$

$$\psi_{(p,q)}(t) = \text{In}[p]_q + (\text{In}q)\sum_{n=1}^{p}\frac{q^{nt}}{1-q^n} \quad (3)$$

$$\psi_{(q,k)}(t) = \frac{-\text{In}(1-q)}{k} + (\text{In}q)\sum_{n=1}^{\infty}\frac{q^{nkt}}{1-q^{nk}} \quad (4)$$

where $\gamma = 0.577215664901532...$ represents the Euler-Mascheroni's constant.

Of late, the following double inequalities were presented in [7] by the use of some monotonicity properties of some functions related with the Gamma function.

$$\frac{(1-q)^{-t}\Gamma_q(\alpha)}{[p]_q^t\Gamma_{(p,q)}(\alpha)} \geq \frac{\Gamma_q(\alpha+t)}{\Gamma_{(p,q)}(\alpha+t)} \geq \frac{(1-q)^{1-t}\Gamma_q(\alpha+1)}{[p]_q^{t-1}\Gamma_{(p,q)}(\alpha+1)} \quad (5)$$

for $t \in (0,1)$, $\alpha > 0$, $p \in N$ and $q \in (0,1)$.

$$\frac{(1-q)^{-t}\Gamma_q(\alpha)}{(1-q)^{-\frac{t}{k}}\Gamma_{(q,k)}(\alpha)} \geq \frac{\Gamma_q(\alpha+t)}{\Gamma_{(q,k)}(\alpha+t)} \geq \frac{(1-q)^{1-t}\Gamma_q(\alpha+1)}{(1-q)^{\frac{1}{k}(1-t)}\Gamma_{(q,k)}(\alpha+1)} \quad (6)$$

for $t \in (0,1)$, $\alpha > 0$, $q \in (0,1)$ and $k \geq 1$.

$$\frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}}\Gamma_k(\alpha)}{(\alpha+t)[p]_q^t\Gamma_{(p,q)}(\alpha)} < \frac{\Gamma_k(\alpha+t)}{\Gamma_{(p,q)}(\alpha+t)} < \frac{(\alpha+1)k^{\frac{t-1}{k}} e^{\frac{\gamma(1-t)}{k}}\Gamma_k(\alpha+1)}{(\alpha+t)[p]_q^{t-1}\Gamma_{(p,q)}(\alpha+1)} \quad (7)$$

for $t \in (0,1)$, $\alpha > 0$, $p \in N$, $q \in (0,1)$ and $k > 0$.

$$\frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}}\Gamma_k(\alpha)}{(\alpha+t)(1-q)^{-\frac{t}{k}}\Gamma_{(q,k)}(\alpha)} < \frac{\Gamma_k(\alpha+t)}{\Gamma_{(q,k)}(\alpha+t)} < \frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}}\Gamma_k(\alpha+1)}{(\alpha+t)(1-q)^{\frac{1-t}{k}}\Gamma_{(q,k)}(\alpha+1)} \quad (8)$$

for $t \in (0,1)$, $\alpha > 0$, $q \in (0,1)$ and $k > 0$.

Results of this form can also be found in [8,9,10,11,12]. By utilizing similar techniques as in the previous results, this paper seeks to provide some generalizations of the above inequalities. We present our results in the following sections.

2. Supporting Results

We begin with the following Lemmas.

Lemma 2.1. Suppose that $\alpha > 0$, $\beta > 0$, $u \geq w > 0$, $t > 0$, $p \in N$ and $q \in (0,1)$. Then,

$$u\text{In}(1-q) + w\text{In}[p]_q + u\psi_q(\alpha+\beta t) - w\psi_{(p,q)}(\alpha+\beta t) \leq 0.$$

Proof. From the characterization in equations (1) and (3) we obtain,

$$u\text{In}(1-q) + w\text{In}[p]_q + u\psi_q(t) - w\psi_{(p,q)}(t) = (\text{In}q)\left[u\sum_{n=1}^{\infty}\frac{q^{nt}}{1-q^n} - w\sum_{n=1}^{p}\frac{q^{nt}}{1-q^n}\right] \leq 0.$$

We conclude the proof by substituting t by $\alpha + \beta t$.

Lemma 2.2. Suppose that $\alpha > 0$, $\beta > 0$, $u \geq w > 0$, $t > 0$, $q \in (0,1)$ and $k \geq 1$. Then,

$$u\text{In}(1-q) - w\frac{\text{In}(1-q)}{k} + u\psi_q(\alpha+\beta t) - w\psi_{(q,k)}(\alpha+\beta t) \leq 0.$$

Proof. From the characterization in equations (1) and (4) we obtain,

$$u\text{In}(1-q) - w\frac{\text{In}(1-q)}{k} + u\psi_q(t) - w\psi_{(q,k)}(t) = (\text{In}q)\sum_{n=1}^{\infty}\left[u\frac{q^{nt}}{1-q^n} - w\frac{q^{nkt}}{1-q^{nk}}\right] \leq 0.$$

We conclude the proof by substituting t by $\alpha + \beta t$.

Lemma 2.3. Suppose that $\alpha > 0$, $\beta > 0$, $u > 0$ $w > 0$, $t > 0$, $k > 0$, $p \in N$ and $q \in (0,1)$. Then,

$$w\text{In}[p]_q - u\frac{\text{In}k}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha+\beta t} + u\psi_k(\alpha+\beta t) - w\psi_{(p,q)}(\alpha+\beta t) > 0.$$

Proof. From the characterization in equations (2) and (3) we obtain,

$$w\text{In}[p]_q - u\frac{\text{In}k}{k} + \frac{u\gamma}{k} + \frac{u}{t} + u\psi_q(t) - w\psi_{(p,q)}(t) = u\sum_{n=1}^{\infty}\frac{t}{nk(nk+t)} - w\sum_{n=1}^{p}\frac{q^{nt}}{1-q^n} > 0.$$

We conclude the proof by substituting t by $\alpha + \beta t$.

Lemma 2.4. Suppose that $\alpha > 0$, $\beta > 0$, $u > 0$ $w > 0$, $t > 0$, $q \in (0,1)$ and $k > 0$. Then,

$$-\frac{\text{In}(k^u(1-q)^w)}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha+\beta t} + u\psi_k(\alpha+\beta t) - w\psi_{(q,k)}(\alpha+\beta t) > 0.$$

Proof. From the characterization in equations (2) and (4) we obtain,

$$-\frac{\text{In}(k^u(1-q)^w)}{k}+\frac{u\gamma}{k}+\frac{u}{\alpha+\beta t}+u\psi_q(t)-w\psi_{(q,k)}(t)$$
$$=u\sum_{n=1}^{\infty}\frac{t}{nk(nk+t)}-w(\text{In}q)\sum_{n=1}^{\infty}\frac{q^{nkt}}{1-q^{nk}}>0.$$

We conclude the proof by substituting t by $\alpha+\beta t$.

3. Main Results

We now present our results in the following Theorems.

Theorem 3.1. Define a function E for $p\in N$ and $q\in(0,1)$ by

$$E(t)=\frac{(1-q)^{u\beta t}\Gamma_q(\alpha+\beta t)^u}{[p]_q^{-w\beta t}\Gamma_{(p,q)}(\alpha+\beta t)^w},t\in(0,\infty) \tag{9}$$

where u, w, α, β are positive real numbers such that $u\geq w$. Then, E is non-increasing on $t\in(0,\infty)$ and the inequalities:

$$\frac{(1-q)^{-u\beta t}\Gamma_q(\alpha)^u}{[p]_q^{w\beta t}\Gamma_{(p,q)}(\alpha)^w}\geq\frac{\Gamma_q(\alpha+\beta t)^u}{\Gamma_{(p,q)}(\alpha+\beta t)^w}\geq\frac{(1-q)^{u\beta(1-t)}\Gamma_q(\alpha+\beta)^u}{[p]_q^{w\beta(t-1)}\Gamma_{(p,q)}(\alpha+\beta)^w} \tag{10}$$

are valid for each $t\in(0,1)$.

Proof. Let $\lambda(t)=\text{In}E(t)$ for every $t\in(0,\infty)$. Then

$$\lambda(t)=\text{In}\frac{(1-q)^{u\beta t}\Gamma_q(\alpha+\beta t)^u}{[p]_q^{-w\beta t}\Gamma_{(p,q)}(\alpha+\beta t)^w}$$
$$=u\beta t\text{In}(1-q)+w\beta t\text{In}[p]_q$$
$$+\ u\text{In}\Gamma_q(\alpha+\beta t)-w\text{In}\Gamma_{(p,q)}(\alpha+\beta t)$$

Then,

$$\lambda'(t)=u\beta\text{In}(1-q)+w\beta\text{In}[p]_q$$
$$+\ u\beta\psi_q(\alpha+\beta t)-w\beta\psi_{(p,q)}(\alpha+\beta t)$$
$$=\beta\Big[u\text{In}(1-q)+w\text{In}[p]_q$$
$$+u\psi_q(\alpha+\beta t)-w\psi_{(p,q)}(\alpha+\beta t)\Big]\leq 0$$

as a result of Lemma 2.1. That implies λ is non-increasing on $t\in(0,\infty)$. Consequently, E is non-increasing on $t\in(0,\infty)$ and for each $t\in(0,1)$ we have,

$$E(0)\geq E(t)\geq E(1)$$

yielding equation (10).

Theorem 3.2. Define a function F for $q\in(0,1)$ and $k\geq 1$ by

$$F(t)=\frac{(1-q)^{u\beta t}\Gamma_q(\alpha+\beta t)^u}{(1-q)^{\frac{u\beta t}{k}}\Gamma_{(q,k)}(\alpha+\beta t)^w},t\in(0,\infty) \tag{11}$$

where u, w, α, β are positive real numbers such that $u\geq w$. Then, F is non-increasing on $t\in(0,\infty)$ and the inequalities:

$$\frac{(1-q)^{-u\beta t}\Gamma_q(\alpha)^u}{(1-q)^{-\frac{w\beta t}{k}}\Gamma_{(q,k)}(\alpha)^w}\geq\frac{\Gamma_q(\alpha+\beta t)^u}{\Gamma_{(q,k)}(\alpha+\beta t)^w}\geq\frac{(1-q)^{u\beta(1-t)}\Gamma_q(\alpha+\beta)^u}{(1-q)^{\frac{w\beta}{k}(1-t)}\Gamma_{(q,k)}(\alpha+\beta)^w} \tag{12}$$

are valid for each $t\in(0,1)$.

Proof. Let $\eta(t)=\text{In}F(t)$ for every $t\in(0,\infty)$. Then

$$\eta(t)=\text{In}\frac{(1-q)^{u\beta t}\Gamma_q(\alpha+\beta t)^u}{(1-q)^{\frac{u\beta t}{k}}\Gamma_{(q,k)}(\alpha+\beta t)^w}$$
$$=u\beta t\text{In}(1-q)-\frac{w\beta t}{k}\text{In}(1-q)$$
$$+\ u\text{In}\Gamma_q(\alpha+\beta t)-w\text{In}\Gamma_{(q,k)}(\alpha+\beta t)$$

Then,

$$\eta'(t)=u\beta\text{In}(1-q)-\frac{w\beta}{k}\text{In}(1-q)$$
$$+\ u\beta\psi_q(\alpha+\beta t)-w\beta\psi_{(q,k)}(\alpha+\beta t)$$
$$=\beta\Big[u\text{In}(1-q)-w\frac{\text{In}(1-q)}{k}$$
$$+u\psi_q(\alpha+\beta t)-w\psi_{(q,k)}(\alpha+\beta t)\Big]\leq 0$$

as a result of Lemma 2.2. That implies η is non-increasing on $t\in(0,\infty)$. Consequently, F is non-increasing on $t\in(0,\infty)$ and for each $t\in(0,1)$ we have,

$$F(0)\geq F(t)\geq F(1)$$

yielding equation (12).

Theorem 3.3. Define a function G for $t\in(0,\infty)$, $p\in N$, $q\in(0,1)$ and $k>0$ by

$$G(t)=\frac{(\alpha+\beta t)^u k^{-\frac{u\beta t}{k}}e^{\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha+\beta t)^u}{[p]_q^{-w\beta t}\Gamma_{(p,q)}(\alpha+\beta t)^w} \tag{13}$$

where u, w, α, β are positive real numbers. Then, G is increasing on $t\in(0,\infty)$ and the inequalities:

$$\frac{\alpha^u k^{\frac{u\beta t}{k}}e^{-\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha)^u}{(\alpha+\beta t)^u[p]_q^{w\beta t}\Gamma_{(p,q)}(\alpha)^w}<\frac{\Gamma_k(\alpha+\beta t)^u}{\Gamma_{(p,q)}(\alpha+\beta t)^w}<\frac{(\alpha+\beta)^u k^{\frac{u\beta(t-1)}{k}}e^{\frac{u\beta\gamma(1-t)}{k}}\Gamma_k(\alpha+\beta)^u}{(\alpha+\beta t)^u[p]_q^{w\beta(t-1)}\Gamma_{(p,q)}(\alpha+\beta)^w} \tag{14}$$

are valid for each $t\in(0,1)$.

Proof. Let $\mu(t)=\text{In}G(t)$ for every $t\in(0,\infty)$. Then

$$\mu(t)=\text{In}\frac{(\alpha+\beta t)^u k^{-\frac{u\beta t}{k}}e^{\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha+\beta t)^u}{[p]_q^{-w\beta t}\Gamma_{(p,q)}(\alpha+\beta t)^w}$$

$$= u\text{In}(\alpha+\beta t) - \frac{u\beta t}{k}\text{In}k + \frac{u\beta\gamma t}{k} + w\beta t\text{In}[p]_q + u\text{In}\Gamma_k(\alpha+\beta t) - w\text{In}\Gamma_{(p,q)}(\alpha+\beta t).$$

Then,

$$\mu'(t) = w\beta\text{In}[p]_q - u\beta\frac{\text{In}k-\gamma}{k} + \frac{u\beta}{\alpha+\beta t} + u\beta\psi_k(\alpha+\beta t) - w\beta\psi_{(p,q)}(\alpha+\beta t)$$

$$= \beta\left[w\text{In}[p]_q - u\frac{\text{In}k}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha+\beta t} + u\psi_k(\alpha+\beta t) - w\psi_{(p,q)}(\alpha+\beta t)\right] > 0$$

as a result of Lemma 2.3. That implies μ is non-increasing on $t\in(0,\infty)$. Consequently, G is non-increasing on $t\in(0,\infty)$ and for each $t\in(0,1)$ we have,

$$G(0) < G(t) < G(1)$$

yielding equation (14).

Theorem 3.4. Define a function H for $t\in(0,\infty)$, $q\in(0,1)$ and $k>0$ by

$$H(t) = \frac{(\alpha+\beta t)^u e^{\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha+\beta t)^u}{k^{\frac{u\beta t}{k}}(1-q)^{\frac{w\beta t}{k}}\Gamma_{(q,k)}(\alpha+\beta t)^w} \qquad (15)$$

where u, w, α, β are positive real numbers. Then, H is increasing on $t\in(0,\infty)$ and the inequalities:

$$\frac{\alpha^u e^{-\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha)^u}{(\alpha+\beta t)^u k^{-\frac{u\beta t}{k}}(1-q)^{-\frac{w\beta t}{k}}\Gamma_{(q,k)}(\alpha)^w} < \frac{\Gamma_k(\alpha+\beta t)^u}{\Gamma_{(q,k)}(\alpha+\beta t)^w} < \frac{(\alpha+\beta)^u e^{\frac{u\beta\gamma(1-t)}{k}}\Gamma_k(\alpha+\beta)^u}{(\alpha+\beta t)^u k^{\frac{u\beta(1-t)}{k}}(1-q)^{\frac{w\beta(1-t)}{k}}\Gamma_{(q,k)}(\alpha+\beta)^w} \qquad (16)$$

are valid for each $t\in(0,1)$.

Proof. Let $\delta(t) = \text{In}H(t)$ for every $t\in(0,\infty)$. Then

$$\delta(t) = \text{In}\frac{(\alpha+\beta t)^u e^{\frac{u\beta\gamma t}{k}}\Gamma_k(\alpha+\beta t)^u}{k^{\frac{u\beta t}{k}}(1-q)^{\frac{w\beta t}{k}}\Gamma_{(q,k)}(\alpha+\beta t)^w}$$

$$= u\text{In}(\alpha+\beta t) + \frac{u\beta\gamma t}{k} - \frac{u\beta t}{k}\text{In}k - \frac{w\beta t}{k}\text{In}(1-q) + u\text{In}\Gamma_k(\alpha+\beta t) - w\text{In}\Gamma_{(q,k)}(\alpha+\beta t).$$

Then,

$$\delta'(t) = -\beta\frac{\text{In}(k^u(1-q)^w)}{k} + \frac{u\beta\gamma}{k} + \frac{u\beta}{\alpha+\beta t} + u\beta\psi_k(\alpha+\beta t) - w\beta\psi_{(q,k)}(\alpha+\beta t)$$

$$= \beta\left[-\frac{\text{In}(k^u(1-q)^w)}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha+\beta t} + u\psi_k(\alpha+\beta t) - w\psi_{(q,k)}(\alpha+\beta t)\right] > 0$$

as a result of Lemma 2.4. That implies δ is non-increasing on $t\in(0,\infty)$. Consequently, H is non-increasing on $t\in(0,\infty)$ and for each $t\in(0,1)$ we have,

$$H(0) < H(t) < H(1)$$

yielding equation (16).

4. Conclusion

If we fix $u = w = \beta = 1$ in inequalities (10), (12), (14) and (16), then we respectively obtain the inequalities (5), (6), (7) and (8) as special cases. By this, the previous results [7] have been generalized.

Competing Interests

The authors have no competing interests.

References

[1] R. Diaz and E. Pariguan, *On hypergeometric functions and Pachhammer k-symbol*, Divulgaciones Matematicas 15(2)(2007), 179-192.

[2] T. Mansour, *Some inequalities for the q-Gamma Function*, J. Ineq. Pure Appl. Math. 9(1)(2008), Art. 18.

[3] F. Merovci, *Power Product Inequalities for the Γ_k Function*, Int. Journal of Math. Analysis, 4(21)(2010), 1007-1012.

[4] V. Krasniqi and F. Merovci, *Some Completely Monotonic Properties for the (p, q)-Gamma Function*, Mathematica Balkanica, New Series 26(2012), 1-2.

[5] R. Diaz and C. Teruel, *q,k-generalized gamma and beta functions*, J. Nonlin. Math. Phys. 12(2005), 118-134.

[6] V. Krasniqi, T. Mansour and A. Sh. Shabani, *Some Monotonicity Properties and Inequalities for Γ and ζ Functions*, Mathematical Communications 15(2)(2010), 365-376.

[7] K. Nantomah, *On Certain Inequalities Concerning the Classical Euler's Gamma Function*, Advances in Inequalities and Applications, Vol. 2014 (2014) Art ID 42.

[8] K. Nantomah and M. M. Iddrisu, *Some Inequalities Involving the Ratio of Gamma Functions*, Int. Journal of Math. Analysis 8(12)(2014), 555-560.

[9] K. Nantomah, M. M. Iddrisu and E. Prempeh, *Generalization of Some Inequalities for theRatio of Gamma Functions*, Int. Journal of Math. Analysis, 8(18)(2014), 895-900.

[10] K. Nantomah and E. Prempeh, *Generalizations of Some Inequalities for the p-Gamma, q-Gamma and k-Gamma Functions*, Electron. J. Math. Anal. Appl. 3(1)(2015),158-163.

[11] K. Nantomah and E. Prempeh, *Some Sharp Inequalities for the Ratio of Gamma Functions*, Math. Aeterna, 4(5)(2014), 501-507.

[12] K. Nantomah and E. Prempeh, *Generalizations of Some Sharp Inequalities for the Ratio of Gamma Functions*, Math. Aeterna, 4(5)(2014), 539-544.