UNIVERSITY FOR DEVELOPMENT STUDIES, TAMALE

MODELLING PRICE VOLATILITY OF THREE MAJOR CEREALS IN THE NORTHERN REGION OF GHANA

AMADU YAKUBU

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BY

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JULY, 2016

DECLARATION

Student

I hereby declare that this thesis is the result of my own work and to the best of my knowledge, it contains no material previously presented for the award of any other degree in this university or elsewhere except where due acknowledgement has been made in the text.

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Supervisor

I hereby declare that the preparation and presentation of the thesis was duly supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies:

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Name: Dr. Albert Luguterah

ABSTRACT

Cereals are important crops that feed over billions of households worldwide. They have been used extensively for both human consumption and feeding of livestock. In Ghana, cereals such as rice, maize and millet are staple food of great socio-economic importance and they contribute significantly to agriculture Gross Domestic Product (GDP) and the economy of the country. In this study, we developed an ARIMA (p, q)-GARCH (m, s) model to model the volatility of the returns of rice, maize and millet in the Northern region of Ghana. Data on monthly returns of rice, maize and millet from the Ministry of Food and Agriculture were used for the modeling. The results revealed that ARIMA (0, 1)-GARCH (1, 0) was the best model for modeling the volatility of rice returns. Also, ARIMA (0, 1)-GARCH (1, 0) emerged as the best model for modeling the volatility of millet returns. Furthermore, ARIMA (1, 1)-GARCH (1, 0) was the best model for modeling the volatility of maize returns. Diagnostic checks of the three models with the Ljung-Box test and ARCH-LM test revealed that all the models were free from higher-order serial correlation and conditional heteroscedasticity respectively. The dynamic relationship between the returns of the cereals was also investigated using Vector Autoregressive model. VAR (2) and VAR (3) models were fitted to the data. Base on the Likelihood Ratio Test, VAR (3) model was the best for modeling the dynamic relationship between the returns of the cereals. The diagnostic checks revealed that VAR (3) model was adequate. The VAR (3) model was then used to make inference about the relationship between the returns of these cereals. The Granger causality test revealed a bilateral relationship between the returns of rice and that of millet whiles the returns of maize was independent of the returns of rice and millet. The IRF and FEVD analysis both affirm that there exists a dynamic relationship between the returns of the three cereals.

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Again, I owe the gratitude to Dr. Abu Moomin, head, Department of Horticulture whose care and support have brought me this far. Lastly, I would like to thank the management of Ministry of Food and Agriculture, Northern Regional office for providing me with the data to carry out this research.

DEDICATION

This work is dedicated to my dear parents Mr Sayibu Amadu and Haruna Mariama.

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LIST OF ACRONYMS

AAA	Additive error, Additive trend, Additive seasonality
ACF	Autocorrelation Function
ADF	Augmented Dickey-Fuller
ADR	American Depositary Receipt
AIC	Akaike Information Criterion
AICc	Akaike Information Criterion corrected
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARCH-LM	Autoregressive Conditional Heteroscedasticity Lagrange Multiplier
ARFIMA	Autoregressive Fractional Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
CBOT	Chicago Board of Trade
CGARCH	Component Generalised Autoregressive Conditional Heteroscedasticity
CIVETS	Colombia, Indonesia, Vietnam, Egypt, Turkey and South Africa
CUSUM	Cumulative Sum
CV	Coefficient of Variation
DCC	Dynamic Conditional Correlation

df	Degrees of freedom
DF	Dickey-Fuller
DHF	Dickey, Hasza and Fuller
ECM	Error Correction Model
ECVAR	Error Correction Vector Autoregrssive
EGARCH	Exponential Generalised Autoregressive Conditional Heteroscedasticity in mean
EU	European Union
FAIR	Federal Agriculture Improvement and Reforms
FEVD	Forecast Error Variance Decomposition
FIGARCH	Fractional Integrated Generalised Autoregressive Conditional Heteroscedasticity
FIAPARCH	Fractional Integrated Asymmetric Power Autoregressive Conditional Heteroscedasticity
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GDP	Gross Domestic Product
GED	Generalised Error Distribution
HQIC	Hannan-Quinn Information Criteria
IGARCH	Integrated Generalised Autoregressive Conditional Heteroscedasticity
IRF	Impulse Response Function
KPSS	Kwiatkowski-Phillips-Schmidt-Shin

LRT	Likelihood Ratio Test
MA	Moving Average
MATIF	Marché à Terme International de France
MLE	Maximum Likelihood Estimates
MGARCH	Multivariate Generalised Autoregressive Conditional Heteroscedasticity
MoFA	Ministry of Food and Agriculture
MTGARCH	Multivariate Threshold Generalised Autoregressive Conditional Heteroscedasticity
MVGARCH	Multivariate Generalised Autoregressive Conditional Heteroscedasticity
NPCs	National Product Classification for Services
NRD	Natural Resources Damage
OLS	Ordinary Least Squares
PACF	Partial Autocorrelation Function
PCV	Periodic Conditional Volatility
PGARCH	Periodic Generalised Autoregressive Conditional Heteroscedasticity
POTS	Partial Overlapping Time Series
PP	Phillip-Perron
SSA	Sub-Saharan Africa
SSI	Shanghai Stock Index
STS	Structural Time Series

SUR	Seemingly Unrelated Regression
SV	Stochastic Volatility
TGARCH	Threshold Generalised Autoregressive Conditional Heteroscedasticity
US	United States
VAR	Vector Autoregressive Model
VARFIMA	Vector Autoregressive Model Fractional Integrated Moving Average
VARMA	Vector Autoregressive Moving Average
WTO	World Trade Organization
WTI	West Texas Intermediate

CHAPTER ONE

INTRODUCTION

1.0 Background of the Study

Cereals are important crops that feed over billions of households worldwide. They have been used extensively for both human consumption and feeding of livestock. For instance, the consumption of maize worldwide is more than 116 million tons with Africa consuming 30% and Sub-Saharan Africa (SSA), 21% (IITA, 2009). Also, Khush (2009) inferred that rice feeds more than a third of the world's population. In Ghana, cereals such as rice, maize and millet are staple food of great socio-economic importance and they contribute significantly to agriculture Gross Domestic Product (GDP) and the economy of the country. Specifically, they are used in the preparation of local dishes and drinks for human consumption.

Uncertainties in prices of cereals are therefore major issues of concern for both producers and consumers. In Ghana for instance, food prices for rice, maize and other cereals increased from 20% to 30% between the last few months of 2007 and the beginning of 2008 (Wodon*et al*., 2008). This is an important concern regarding the future of cereal prices and their likely effect on food security in Ghana. The Food and Agriculture Organization (FAO) special report shows that, prices of grains in Ghana are subject to very strong seasonal fluctuation, particularly in Northern Ghana (FAO, 2002). That is, they fall sharply during the harvest period (August- October) and start rising from November until the end of the lean season (June-August). These uncertainties, characterised by unexpected price changes, may compel farmers to react by reducing output supply and investment in productive inputs (SealandShonkwiler,1987;Rezitisand Stavropoulos,2009;SckokaiandMoro,2009;Piot-Lepetit,2011;Tangermann,2011;Taya, 2012).

The effect of this price instability is that food production may decrease leading to food insecurity. According to the FAO (1996), food security requires that at all times, each individual has physical, social and economic access to sufficient, safe and nutritious food to meet their dietary needs and food preferences for an active and healthy life. In this definition, the FAO recognised among others, the importance of price in ensuring food security. It was also evident in the World Bank report (2008) that food security has become a major global concern since the mid-1970s due to rapidly increasing prices that led to global food insecurity.

Although the World Bank recognises that the world has enough food to feed everyone, they noted that about 852 million people are chronically hungry due to extreme poverty while up to 2 billion people are food insecure intermittently due to varying degrees of poverty. The highest incidence of food insecurity or undernourishment is in SSA where one in every three persons suffers from chronic hunger (World Bank, 2008). The situation in Ghana is however not that different. Although the country has made considerable progress in terms of poverty reduction over the past fifteen years, about 1.2 million people, representing 5% of the population are still food insecure and an additional 2 million people are vulnerable to become food insecure following any natural or man-made shock (World Food Programme, 2009).

One important factor that can quickly affect the food security status of the poor is the price of food commodities needed by them. Knowing the dangers associated with price instability of food commodities all over the world, it is imperative to investigate the uncertainties in these prices in order to help make policies to curb this problem. This study therefore models the volatility in the prices of three major cereals (corn, rice and millet) in the Northern region of Ghana.

1.1 Problem Statement

Commodity prices in general are volatile and in particular agricultural commodity prices are renowned for their continuously volatile nature (Newbery,1989). However, the degree of commodity price fluctuations or volatility is generally the concern that has attracted increasing attention in recent economic and financial literature and has been recognized asone of the most important economic phenomena (Engle, 1982). It has been argued that price volatility reduces welfare and competition by increasing consumer costs (Zheng*et al*., 2008).Similarly price volatility increases risk and uncertainties associated with both production and consumption (Apergis and Rezitis, 2011).

Though cereal market policies in Ghana may have undergone dramatic changesover the years, desirable outcomes in terms of intervention and prevention of market price volatility are still less satisfactory. For instance, in Ethiopia, pricevolatility in the markets of major cereal crops remains unsatisfactorily high in despite dramatic change in policies (Rashid and Meron, 2007). Hence, it is important that measures including research efforts are triggered to offset any such occurrence. This is important for cereals such as rice, maize and millet markets which form the major staple food in the Northern region which is also the mainstay of agriculture in Ghana (GSS, 2012). However, it is the second poorest region (50.4%) in the country (GSS, 2014). This means that any occurrence of price volatility may not only affect food production, but also the demand for food by the people.

Ironically, only little attention has been given to agriculturalfood price volatility in the Northern region and Ghana at large. Thus this study seeks to investigate the uncertainties in the prices of three major cereals in the Northern region.

1.2 Research Questions

i What is the domestic market price volatility on the selected cereals?

ii Which of the selected commodity is highly volatile in price?

iii What model best fits the domestic price volatility in the region?

iv Which model is appropriate for describing the short run relationship between the prices of the selected cereals?

1.3 General Objective

The main objective of this study is to model the price volatility of the three major cereals in the Northern region of Ghana.

1.4 Specific Objectives

i. To develop appropriate models for domestic price volatility for the cereals.
ii. To investigate the half-life volatility of each cereal.
iii. To determine the volatility associated with each cereal.
iv. To fit Vector Autoregressive (VAR) model for describing the short run relationship between the returns of these cereals.

1.5 Significance of the Study

The findings of this study could be used by the Ministry of Food and Agriculture (MoFA) to make policies to curb the problems of food price volatility in the region. In addition, this study could provide basis for further researches on price volatility of food commodities in Ghana.

1.6 Structure of the Thesis

The thesis is organised into five chapters. Chapter one contains the introduction of the research work. Chapter two comprises of literature review. Chapter three outlines the methodology employed in this research while chapter four presents the analysis and discussion of results. Chapter five is devoted to conclusion and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews related works done on price volatility of cereals and some relevant time series methods that have been used in modelling price volatility. The chapter is divided into five main headings namely; empirical researches on cereals, empirical researches on volatility, review of time series methods, review of GARCH models and conclusion.

2.1 Empirical Researches on Cereals

A lot of researches have been carried out on cereals all over the world. Badmus andAriyo(2011)usedARIMAmodeltoforecastareaandproduction ofmaizeinNigeria.They estimatedARIMA (1, 1, and 1)andARIMA (2, 1, 2)forcultivatedareaandproduction respectively. Theresultshowed that maizeproduction forecastfortheyear2020 will beabout9952.72tonswithupper andlowerlimits of 6479.8and13425.64thousandtonsrespectively.Themodelalsoshowsthatthemaizearea wouldbe 9229.74thousandhectareswithlowerand upperlimitof7087.67and11371.81thousandhectaresrespectivelyby2020.

Next, Suleman *et al.* (2013) used Vector Autoregressive (VAR) model to investigate the relationship between the production growth rates of three major cereals in Ghana. The VAR model favoured VAR at lag 1 which indicated that, in addition to the bivariate unidirectional production growth rate causalities; there is also a bilateral causality between production growth rate in Millet and production growth rate in Milled Rice and a Rice to Corn unidirectional production growth rate causality.

Also, Suleman andSarpong(2012a)employedtheBox-Jenkins approachtomodelmilledrice production inGhanausingtimeseriesdatafrom1960to2010.TheanalysisrevealedthatARIMA (2, 1, 0)wasthebestmodelforforecasting milledriceproduction.

Again, Najeeb*etal.*(2005)employedBox-Jenkinsmodeltoforecastwheatareaandproduction in Pakistan.TheirstudyshowedthatARIMA (1, 1, 1)andARIMA (2, 1, 2)weretheappropriate modelsforwheatareaandproductionrespectively.

Further, Anokye and Oduro (2014) studied the price dynamics of maize in Ghana using cobweb models derived from linear demand and nonlinear supply function and then compare with that from linear demand and supply functions which are constructed from real economic price and production data of maize. The results from the linear cobweb model provided unstable equilibrium state of prices towards the zero equilibrium price as well as the supply. Thus the system is unstable and no equilibrium price is achieved towards the equilibrium point, Pe = 0 which is also not realistic because of producers' sensitivity to price. However, the nonlinear cobweb model provided two equilibria of which one is also stable at the zero equilibrium price and the other unstable at non-zero equilibrium price which is realistic and a reflection of maize price system due to inflation and insufficiency of food supply at the markets in Ghana.

Again, Kuwornu *et al*. (2011) used the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) regression model to forecast foodstuff prices in Ghana over the period 1970 to 2006. The data that were used were monthly wholesale prices for maize, millet, and rice obtained from the Ghana Ministry of Food and Agriculture. The empirical results revealed that foodstuff prices exhibit high volatility with continual increasing prices over the studied period. The results of the out-sample forecast reveal that maize, millet and rice prices would increase by 23%, 11% and 10% respectively in the next month.

Next, FalakandEatzaz(2008)analysedfutureprospects ofwheatproduction inPakistan.They obtainedtheparametersoftheirforecastingmodelusingCobb-Douglas productionfunctionfor wheat,whilefuturevalues ofvarious inputs were obtainedasdynamicforecasts onthebasisof separateARIMAestimatesforeachinputandforeachProvince.

Further, KirttiandGoyari(2013)used kinkexponentialgrowthratemodeltoanalysegrowthratesofarea,production andyieldofmajor cropsinOdishaforpre-liberalizationandpost-liberalizationperiods. Theresultsshowed that allcrops, exceptriceexperienceddecelerationinareaduringpost-liberalizationperiod.Amongthosecrops, bajra,jowar,wheat,ragiandsmallmilletexperiencedahigherdeceleration. Eventhepositive growthrateofricearea wasverytrivial.

Again, Suleman and Sarpong (2012b) modeledandforecastproductionandconsumption ofcorninGhanausingARIMAmodels.ThestudyrevealedthatARIMA(2, 1, 1)andARIMA(1, 1, 0)were theappropriatemodelsforforecastingproductionandconsumption respectively. The forecastshowedan increasingpatterninconsumptionandproductionofcorn.

Moreover, Qureshi*etal.*(1992)analyzedthe relativecontributionof areaandyieldtototalproduction ofwheatandmaizein Pakistanandconcluded thattherewasmorethan100% increaseintotalwheatproductionthatcanbe attributed to yield enhancement.

Also, Karim *etal.*(2005)appliedregressionmodeling to forecastwheat production of Bangladeshdistricts. They usedsevenmodel selectioncriteriaandfoundthat differentmodelswereidentifiedfordifferent districtsfor wheatproduction forecasts. Theyfoundthatwheat production inBangladeshdistricts, that is Dinajpur,Rajshahi andRangpur, wouldbe1.54,0.35,0.31,and0.58milliontons,respectively,in2009/10.

Again, Iqbal*etal.*(2000)used ARIMA model forforecasting wheat areaandproduction inPakistan.TheyusedARIMA (1, 1, 1)modelforwheat areaforecasting andARIMA(2, 1, 2)modelforwheat productionforecasting.Theyfoundthatfor year 2000-2001, forecasts for wheatareawasabout8451.5thousand hectares.Wheatareaforecastfortheyear2022was 8475.1thousandhectares. The wheatproduction forecast showedanincreasing trend.Forthe year 2000-2001, aforecastof wheat production was about20670.8thousand tons whilewheatproductionforecastfortheyear2022 was about 29774.8thousandtons.

Also, Boken(2000)applieddifferenttimeseries models onwheatyieldtoforecast springwheatyield.He used Mean Square Error asdeterministic criteria toselectthebestmodelandfound thatquadratic modelisbestforwheat yieldforecasting.Whileonthebasisofstochasticcriteria, hefoundthatsimple averagemodel isbest.

Saeed*etal.* (2000)alsoappliedtimeseriesmodel toforecasts wheatproduction inPakistan. Theyusedthewheat production dataseriesfrom1947-48to1998-99. They suggested ARIMA (2, 2, 1)modelforwheatforecastofPakistan. They furtherforecasted wheat production for15years.

Next,SchmitzandWatts(1970)usedtimeseriesmodelingtopredictwheatyieldof fourcountries: Canada,UnitedStates,Australia and Argentinafortheperiod1950to1966.They compare parametrictimeseriesmodels with smoothing techniques and conclude thattrendmodels arebestforyieldforecasting.

Next, SabirandTahir(2012)forecasted wheatproduction, areaandpopulation fortheyear2011-12byusing exponentialsmoothing.Theyfoundthatthedemand for wheat was 12.70milliontonsforthepopulationof97.67 millionfortheyear2011-12.

Again, NehruandRajaram(2009)studiedwheatproductioninIndia using ARIMA technique. ARIMA(1, 1, 0)wasselectedasbestfittedmodelbasedonAkaike Information criterion (AIC) andShwartz Bayes Criterion (SBC).Thepredicted values for wheatproductionshowedthattherewillbesteadyincreasefrom 2008-09to2014-15.

Also, Singh*etal*.(2008)analysedthe effectofrainfallandtemperature effecton wheatyieldin southwesternregionof Punjab.Maximumtemperature, minimumtemperatureandrainfallfromDecemberto Marchforeach period offiveyears(1977-81to1997-2001) wereanalysed.Theresultsrevealed that temperaturesduringFebruary andMarch showedsignificanteffect on wheatyield.Thegrainyield revealedpositivecorrelationwithminimumtemperaturebutnotrends observedforotherparameters.

Further, Rahman (2010) modeled and forecasted the production of boro rice in Bangladesh by ARIMA approach. It was foundfromthestudythattheARIMA (0, 1, 0), ARIMA(0, 1, 3) and ARIMA(0, 1, 2) were thebestfor local,modern andtotalboro riceproduction respectively. Itwas observedfromthe analysisthatshorttermforecastsaremoreefficientforARIMAmodels.

Also, Hamjah (2014), modeled and forecasted the production of three varieties of rice in Bangladesh using ARIMA models. ThebestselectedARIMAmodelforAusproductionswas ARIMA(2,1,2),forAmanit was ARIMA(2,1,2)andforBoroit was ARIMA(1,1,3).Inthisstudy,comparisonbetweenthe originalseriesandforecastedseries were made which revealed that the fittedmodels were statistically wellbehavedtoforecastriceproductionsinBangladesh.

Again, Govardhana*et al*. (2014)analysedtrends and seasonalvariationsinmarketpricesof riceinGunturdistrictand Andhra Pradesh. Themodelsselectedforforecastingforwholesalepriceofricewas ARIMA(2,2,0)and ARIMA (0,1,1)inAndhraPradeshandGunturdistrictrespectively basedontheShwartz Bayes Criterion(SBC)andAkaikeInformationCriterion(AIC).Theforecastsof ricewholesale prices werefoundtobefairly accurateandshowedincreasedtrendsinbothAndhra PradeshandGuntur district.

Further, Bogahawatte(1998)employed theBoxJenkinsAutoregressive Integrated MovingAverage(ARIMA)approach tostudytheseasonalvariationsinretailandwholesalepricesof riceinmarkets ofColombo andfoundthat,seasonality inretailpriceswasmoreprominent thanthewholesaleprices.He alsoreportedthattheinteractionbetweenretail andwholesale prices andtheinfluence ofcurrent retailpriceonwholesale pricesofperiods t+1,t+2,t+3 weresignificant. Findingsofthestudy impliedthatanyincreaseinthesupplyofricedueto retailpriceinperiod't'willarrive inthemarket atperiod t+3,thus, preventing anyfurther increaseinprice.

Also, Mishra(1986)evaluated protection versesunder- pricingofagricultureinIndiafortwo major cereals, wheat and ricefrom 1955 to1980. Normal protection(NRD and NPC) wereestimatedforsixpointsateachquinaunnium.Besides showing changesovertimeinthelevels ofprotectionofexploitation, theaimwastoseewhethersomecomparative statementsvisa-a-visindustrializedandindustrializingcountriescouldalsobemade.

Again, Gilati *et al.* (1990) studied the effective incentives for wheat cultivars in India by selectingfourwheatgrowing states(Haryana, MadhyaPradesh,PunjabandUttarPradesh) under important hypothesis. The National Product Classification for services (NPCS) of four states averaged for the period1980-81 through1986-87werefoundto be0.84,0.75,0.85and0.77in thecaseofHaryana,Madhya Pradesh, Punjab and Uttar Pradesh respectively. These results indicated that wheat cultivatorsinIndiahadbeentaxedonpricingfrontcompared withimports.Butunderexport competition hypothesisonlyonestatenamelyPunjabwastakenforcalculation ofNPCs.It averaged1.34,whichimpliesthatcultivatorsinPunjabstatewereprotected.

2.2 Empirical Researches on volatility

Several researchers have worked on volatility using different approaches. Ghosh *et al.* (2010) employed Generalised Autoregressive Conditional Heteroscedastisity (GARCH), GARCH-dummy, Exponential GARCH in mean (EGARCH-M) and Ordinary Least Square (OLS) models to examine the price volatility and supply response for rice, jowar, bajra, maize, groundnut and cotton. Using annual prices, they check whether trade liberalization indeed exacerbates volatility of agricultural products in India. The results showed that prices of major agricultural products become more unstable in India after the signing of the World Trade Organisation (WTO) agreement.

Also, Swaray (2007) uses an Exponential GARCH (EGARCH) and a Threshold GARCH (TGARCH) model to assess the impact of the suspension of international commodity agreements on the asymmetry and persistency of the volatility. They employ monthly prices for cocoa, coffee, rubber, sugar and tin. The results demonstrate a rise in the asymmetry but a decrease in the persistence of the shocks.

Next, Oleg (2011) used Bivariate GARCH to model the conditional correlations between commodity futures and traditional asset classes in periods of high equity and bond volatility. The dataset consists of Shanghai Stock Index (SSI), China 10 year government bond index, and a set of commodities such as corn, cotton, oats, soybean meal, soybean oil, soybeans, sugar, copper and aluminium, and heating oil for the period 2006 to 2010. The conditional correlation between commodity futures and the Shanghai Stock Index (SSI) rises in period of recession when market risk rises. The negative correlation between treasury bonds and commodity futures rises with the bond volatility, suggesting that a bond and commodity portfolio should not be tilted more towards commodity futures in periods of high bond volatility.

Further, Busse*et al.* (2011) analysed the behaviour of price volatility of the European Union (EU) biofuel markets during and after the 2006 - 2008 event and investigated the correlation in price volatility of different commodities and their evolution over time. They use ARMA-GARCH(1,1) and dynamic conditional correlation Model (DCC) (categorised as MGARCH model) with rapeseed future price of "Marché à Terme International de France (MATIF)" in Paris, soybean spot price from Brazil, rapeseed oil prices and soybean oil prices from Rotterdam in Netherlands, and Brent crude oil one month forward prices. The results show a relatively high persistency in volatility in all series. They mention that the model neither allows for conclusions about causal mechanisms of volatility spillovers nor is it able to capture the magnitude of influence of one market on the other. They found a non-stable and increasing correlation between the returns of rapeseed in MATIF and crude oil prices. They concluded that the correlations of rapeseed price returns with vegetable oil and soybean price returns on the spot market are much lower than these with crude oil.

Again, Apergis and Rezitis (2003) employ a multivariate GARCH model including the Greece food price index. Macroeconomic variables such as money supply, income per capita, real ex-change rate, budget deficit/surplus during 1985–1999 were used in order to investigate the volatility spillovers effects between food and macroeconomic fundamentals. They found that additional to the effect of the volatility of food prices on its own volatility, significant and positive effects of macroeconomic volatility on food prices volatility can be recognised.

In addition, Khaligh*et al.* (2012) used Error Correction Vector Autoregressive (ECVAR) and Multivariable Generalised Autoregressive Conditional Heteroscedastisity (MVGARCH) to examine the relative uncertainty of prices in the agricultural input, agricultural output, and retail food markets, as well as the degree by which price uncertainty in one market affects price uncertainty in the others for poultry market in Iran. They have used agricultural input prices index, producer prices index, and retail prices index of poultry market during 1997 – 2010. They find that information generated by both agricultural input and retail food prices could lead to changes in the volatility of agricultural output prices.

Next, Apergis and Rezitis (2003b) also used ECVAR and MVGARCH to investigate the volatility spillover effects between agricultural inputs, output and retail food prices in Greece. They used Greece agricultural input price index, agricultural output prices index, retail food price index (1990=100) for the period 1985 - 1999. They conclude that the volatility of retail food prices had a larger impact than volatility of input prices on the volatility of output prices, indicating that demand-specific factors are stronger than cost factors in affecting the volatility of output prices.

Also, Serra (2011) used the smooth transition conditional correlation GARCH to assess the linkage between price volatility at different levels of the Spanish beef marketing chain. She used the farm-gate and consumer beef prices in Spain for the period 1996-2005. She concluded that during turbulent times, price volatilities can be negatively correlated and one cannot expect that, stabilizing one market will lead to stability in other related markets.

Moreover, Alom*et al.* (2011) use Multivariate Threshold GARCH (MTGARCH) to analyse the relationship of inter-country food price returns. The MTGARCH consists of mean and variance equations (two stage model). Therefore, the spillover effect of food price returns is analysed at the mean level of the returns and for the volatility of the returns separately.

A GARCH (1,1)-X model with the addition of lagged position variables to the normal GARCH (1,1) model has been used by Gilbert (2012) to model the impact of speculative trading on grain price volatilities. He uses the cash prices, four front futures contracts on the Chicago Board of

Trade (CBOT) for soft wheat, corn, soybeans and soybean oil. Additionally, he also used position data which are taken from CBOT Commitments of Traders report for the period on 2006-2011. The results do not present any statistical significant effects of financialisation on cash and future returns of Chicago grains and vegetable oil markets.

Next, Kang *et al.* (2009) investigated the efficacy of a volatility model for three crude oil markets; Brent, Dubai and West Texas Intermediate (WTI). They employed different competitive GARCH volatility models such as Component GARCH (CGARCH), Fractional Integrated GARCH (FIGARCH), Integrated GARCH (IGARCH) and GARCH to assess persistence in the volatility of the three crude oil prices. They find that the persistence coefficients are quite close to one in the standard GARCH(1,1) model, a fact that favours the IGARCH(1,1) specification. As the IGARCH(1,1) model nests the GARCH(1,1) models, the estimates of the IGARCH(1,1) model are quite similar to those of the GARCH(1,1) model. In the case of CGARCH (1, 1) and FIGARCH models, the estimated coefficients are smaller than that of the GARCH model, thereby indicating that the short-run volatility component is weaker. Hence, unlike the GARCH and IGARCH models, the CGARCH and FIGARCH models are able to capture volatility persistence due to the insignificance of diagnostic tests.

Also, Karali*et al.* (2011) use weekly data for soybean, corn and wheat in the US future market to apply a Stochastic Volatility (SV) and Bayesian Seemingly Unrelated Regression (SUR) method to prove whether modeling volatility as a stochastic instead of a deterministic variable leads to improved inference about its relationship with seasonality, storage, and time to delivery. The results revealed that as volatility decreases the closer the time to delivery for soybeans and for wheat and increases for corn. This study provides limited support for the theory of storage and for Samuelson's maturity hypothesis.

Again, Smith (2005) develops a Partially Overlapping Time Series (POTS) framework to model jointly volatility dynamics of traded futures contracts with different delivery dates. This model incorporates time-to-delivery, storability, seasonality and GARCH effects. Using United States corn futures, the author shows the dynamic structure of the data and reveals substantial inefficiency on the contract delivery. His results also provided evidence in favour of both the theory of storage and the relevance of the Samuelson effect.

Further, Lence and Hayes (2002) consider a 'Rational Expectations Storage model' to uncover the potential effects of the Federal Agriculture Improvement and Reform (FAIR) Act on the US markets for corn and soybeans. The results suggest that the price volatility that has been evident in the grain markets since the FAIR Act enactment was due to an unusual sequence of events that took place in the 1995 crop year.

Also, Yang *et al*. (2001) investigate the effects of the market-oriented US FAIR act 1996 on agricultural price volatility, using GARCH models for corn, oat, soybeans, wheat and cotton daily future prices. The results showed that agricultural liberalization policy causes: an increase in price volatility for the three major commodities (corn, soybean and wheat); a little change for oats; and a decrease for cotton.

Next, Onour and Sergi (2011) compare the performance of models, when considering a normal in- stead of a t-distribution to capture volatility in food commodity prices. They use monthly prices for wheat, rice, sugar, beef, coffee, and groundnut and conclude that the t-distribution model is superior to the normal distribution. This implies that the normality assumption of the residuals may lead to unreliable volatility results.

Long memory or long dependence processes in agricultural future prices is considered by Jin and Frechette (2004). They found out that a Fractional Integrated Generalised Autoregressive

Conditional Heteroscedastisity (FIGARCH) approach can be a better way to model long dependence inside the volatility by allowing for fractional integration in the variance equation. However, Elder and Jin (2007) argue that wavelet methodology can explain long memory processes in agricultural commodity futures better than the FIGARCH.

Next, Sephton (2009) developed the fractional integration idea by considering the leverage effect for the same dataset as Jin and Frechette (2004). He found out that Fractional Integrated Asymmetric Power Autoregressive Conditional Heteroscedastisity (FIAPARCH) explains the long dependence in futures prices for some of the crops better than FIGARCH as some agricultural commodities futures display asymmetric leverage effects.

Also, Power and Turvey (2010) assess the presence of the long- memory phenomenon in the volatility of energy and agricultural commodity prices using the improved Hurst coefficient estimator in a wavelet- based R/S analysis. He used daily future prices for coffee, cotton, sugar, cocoa, orange juice, wheat, live cattle, lean hogs, corn and soybeans, and found the evidence of long memory and non-constant Hurst parameter in most of the considered commodities.

Next, Egelkraut and García (2006) investigated the predictive accuracy of implied forward volatility for agricultural commodities with different seasonality. They used daily future prices for corn, soybeans, soybean meal, wheat, and hogs and their results indicated that the implied forward volatility has better predictive power for commodities whose uncertainty resolution is concentrated in space and time.

In addition, Giot (2003) evaluates the information content of the implied volatility for options on future contracts of cacao, coffee and sugar. It was shown that lagged squared returns slightly improve the information content provided by the lagged implied volatility in a GARCH framework. Moreover, he showed that Value at Risk models that rely on past implied volatility perform as well as with ARCH type modeled conditional variance, concluding that implied volatility for the considered commodities has high information content.

Also, Voituriez (2001) used the 'Trader Behaviour model' for the palm oil market to test the hypothesis that the overlapping of the operators' expectations (short versus long term expectation horizon) is triggering volatility changes. He used monthly prices and founds that volatility can increase as long as operators with a short term expectations horizon superimposes on the long term expectations trade, precluding the argument that larger markets reduce volatility.

Also, Taylor (2004) compares the performance of the Period GARCH (PGARCH) with alternative Periodic Conditional Volatility (PCV) models using 5-minute high frequency data of cocoa futures. When considering high-frequency commodity future returns, the periodicity in conditional return volatility is a key issue. He argues that not considering adequately the periodicities in high frequency data could lead to poor forecasts of future return volatility. Moreover he concluded that return volatility forecasts, obtained by the spline PGARCH model, are shown to be less accurate than those generated by PCV models, but if used in a Value at Risk framework, the spline model produces accurate and consistent VaR measures.

Again, Pietola*et al*. (2010) assess the empirical relationship between US weekly wheat prices, inventories, and volatility. They use a 'Co-integrated vector- autoregressive' model, and add price volatility in the form of the estimated variance to the basic model. The price movements and inventories have a significant negative relationship in the very short run, but this relation levels off over time. Thus, in the short run, increasing wheat prices coincide with decreasing inventories. Decreasing prices imply either inventory build-ups or postponement of inventory withdrawals.

Again, Fong and See (2002) employ a Markov regime-switching approach allowing for

GARCH-dynamics and sudden changes in both mean and variance in order to model the conditional volatility of daily returns on crude-oil futures prices. They documented that the regime-switching model performs better than non-switching models regardless of evaluation criterion out-of- sample forecast analysis.

Next, Vo (2009) also employs the concept of regime-switching stochastic volatility and explains the behaviour of crude oil prices of West Texas Intermediate (WTI) market in order to forecast their volatility. More specifically, the paper models the volatility of oil return as a stochastic volatility process whose mean is subject to shifts in regime.

In addition, Narayan and Narayan (2007) use the Exponential Generalized Conditional Heteroscedasticity (EGARCH) model with daily data for the period 1991–2006 with the intention of checking for evidence of asymmetry and persistence of shocks. In their work, volatility was characterised in various sub-samples to judge the robustness of their results. Across the various sub-samples, they show an inconsistent evidence of asymmetry and persistence of shocks and also across full sample period. Their evidence also suggests that shocks have permanent effects and asymmetric effects on volatility.

Again, Houand Suardi (2011) in their work consider an alternative approach involving nonparametric GARCH framework to model and forecast oil price return volatility. They focus on two crude oil markets, Brents and West Texas Intermediate (WTI) and their out-of-sample volatility forecast of the nonparametric GARCH model yields superior performance relative to an extensive class of parametric GARCH models.

Also, Hayat and Narayan (2010) employed the exponential smoothing time series model which they referred to as Additive error, Additive trend, Additive seasonality (AAA) in class one of Hyndman *et al.* (2005) to examine whether the volatility of the growth in the US oil stocks has changed overtime. They found that the growth in volatility of oil stocks had declined overtime. They also conducted a Monte Carlo simulation exercise to investigate whether the decline was real or an artefact of the growth definition. Their findings support the fact that the decline in growth volatility of oil stocks is anartefact of the growth definition. Hayat and Narayan (2011) however consider univariate time series models to examine whether supply and demand shocks explain the apparent decline in the volatility of the growth of the U.S. oil stocks since about the mid-1980s. They found that nearly 70% of the variation in the U.S. oil stock growth is explained by its supply and demand factors, each sharing about half of this variation.

Next, Singh *et al.* (2010) studied the price and volatility spillovers across North American, European and Asian stock markets by utilizing the VAR (15) and AR-GARCH models. By studying the stock markets of fifteen countries in these regions, they found that both return and volatility of one market is affected by the performance of those indices that either open or close before that respective index.

In addition, Korkmaz *et al.* (2012) studied the return and volatility spillovers among CIVETS countries (Colombia, Indonesia, Vietnam, Egypt, Turkey and South Africa). In this paper, by applying the causality-in- mean andcausality-in-variance tests, the authors found out that the contemporaneous spillover effect among these countries are generally low. However, the structure of the causal relationship suggests that there are some intra-regional and inter-regional interdependence in return and volatility.

Another paper by Poshakwale and Aquino (2008) studied the issue of volatility transmission between ADRs and their underlying stocks. By using the GARCH model, they investigated how changes in the volatility of ADR markets affected the volatility in the markets of the underlying stocks and vice-versa. They found out that there is a bi-directional volatility transmission between the ADR markets and the underlying stock markets.

Again, Fong and See (2001) examine issues in modelling the conditional variance of future returns considering regime switches in volatility. Using daily settlement prices of the Goldman Sachs Commodity spot Index and futures, they find that the simple GARCH model is not adequate in the presence of regime shifts since this characteristic dominates the GARCH effects.

Next, Black and Tonks (2000) use a multi-period futures model to test whether price volatility increases or decreases as the maturity date of the futures contract approaches. They found that if output uncertainty is resolved before the maturity of the contract, and if the retrade market is informationally efficient, then the Samuelson hypothesis of increasing volatility before maturity will not occur.

2.3 Review of Time Series Methods

2.3.1 Unit Root Tests

Time series data requires stationarity when modeling. This is very important for estimation and forecasting in time series analysis (Diebold and Kilian, 2001). Unfortunately, most time series data are found to be non-stationary. As a result of this, many researchers have developed models that are used to test for the stationarity of time series data.

Dickey and Fuller (1979) developed the Augmented Dickey-Fuller (ADF) test in which a null hypothesis is a non-stationary process with a unit root and an alternative hypothesis is a trend stationary process. Several methods for testing unit root have been developed; Nelson and Plosser (1982) used the tests developed by Dickey and Fuller to test the economic indicators of the American economy. They found that almost all economic time series such as the Gross National Product have unit root.

Also, Phillips and Perron (1988) weakened a strong assumption on the error term and extended the Dickey-Fuller test to a more general test (Philips-Perron (PP) test). However, the PP-test did not alter the result of Nelson and Plosser (1982), even using the same data as Nelson and Plosser(1982).

Another important contribution on unit root test was made by Kwiatkowski *et al.* (1992). They came out with a unit root test that reversed the null hypothesis and alternative hypothesis (KPSS test) and verified that only half of the economic time series had unit root using the same data set as Nelson and Plosser's (1982).

Moreover, Christiano (1992) criticized Perron's exogenous treatment of a structural change and devised a method with which structural changes with a drift term and a trend can be detected endogenously and proposed a test whose null hypothesis is a unit root process without a structural change and whose opposing hypothesis is a stationary process with a structural change.

In addition, Zivot and Andrews (1992) proposed another test whose null hypothesis is a unit root process without any change in a drift term and whose alternative hypothesis is trend stationary process with a structural break. This proposed test can detect a time point of a structural change endogenously and its asymptotic distribution is constant regardless of the time points of structural changes.

Again Dickey *et al.* (1984) after the methodology proposed by Dickey and Fuller (1979) for the zero-frequency unit-root case, suggested the Dickey, Hasza and Fuller (DHF) test to test for seasonal unit root. The DHF test can only work for unit roots at all of the seasonal frequencies and has an alternative hypothesis which is restrictive, namely that, all the roots have the same modulus. In an attempt to solve these problem, Hylleberg*et al.*, (1990) propose a more general testing (HEGY's test) strategy that allows for unit roots at some (or even all) of the seasonal frequencies as well as the zero frequency. HEGY's methodology allows testing for unit roots at some seasonal frequencies without maintaining that unit roots are present at all seasonal frequencies.

Finally, Banerjee *et al.* (1992) also tested the null hypothesis of the presence of a unit root in the stochastic process governing Argentina's real gross domestic product (GDP). They developed three testing procedures that included; recursive, rolling and sequential by using annual and quarterly data. In all these testing procedures, they considered the possibility of a break in the deterministic trend as a possible characterization.

2.3.2 Traditional Time Series Methods

Time series as a stochastic process started in the mid-1920s (Gottman, 1981).Yule (1927) first developed an Autoregressive (AR) model when working on sunspot data and revealed that an AR process of order two (2) could reproduce intriguing patterns in a time series. Yule's 1927 work was consequently supplemented by Sluztky (1927 and 1937) who first constructed moving average (MA) models of independent and identically distributed (iid) shocks when studying white noise processes. Box and Jenkins (1970) developed the Autoregressive Moving average (ARMA) model and gave a full account of the Integrated Autoregressive Moving average (ARIMA) model.

Furthermore, Mann and Wald (1943) came out with a theorem to estimate the AR (p) parameters by the least squares method. Quenouille (1947) presented a simple test for AR (p) models and later extended to MA models. Also, Anderson (1971) developed a procedure to estimate the order of the AR model as well as the AR parameter.

In addition, a non-linear least squares technique procedure that led to a technique of approximated likelihood solution for ARMA (*p, q*) models was developed by Box and Jenkins (1976). Again, an exact likelihood method for estimating parameters of MA (*q*) models and for ARMA (*p, q*) models was developed by Newbold (1974). The Box-Pierce statistics was developed by Box and Pierce (1970) and modified by Ljung and Box (1978).

Next, Akaike (1974) proposed an information criterion to assist in the selection of an ARIMA model. A model with the smallest Akaike Information Criterion (AIC) is the best model to have minimum forecast mean square errors. On the information criterion, Schwarz (1978) argued that AIC was not consistent when probability approaches one, and proposed a Bayesian Information Criterion (BIC).

Also, an exact likelihood procedure to estimate parameters of an ARIMA model in State-Space form was developed by Harvey and Phillips (1979). The State-Space models are also called Structural Time Series (STS) models. Many researchers have pointed out the advantages of the State-Space form over the ARIMA models (Durbin and Koopman, 2001). A time series might be characterised with trend, seasonal cycle and calendar variations, together with the effects of explanatory variables and interventions. These components can be processed separately and for different purposes for a State-Space model. On the contrary, the Box-Jenkins ARIMA model is a black-box model, which solely depends on the data without knowledge of the system structure that produces the data. The second advantage is the recursive nature of the State-Space model that obviously allows change of the system overtime, while ARIMA models are homogenous through time, based on the stationary assumption.

Again, Granger and Joyeux (1980) and Hosking (1981) proposed an Autoregressive Fractionally Integrated Moving average (ARFIMA) model to study a long memory time series. The autocorrelation function in an ARFIMA (*p, d, q*) model decays at a hyperbolic rate for non-zero *d* which is slower than the usual geometric rate of a stationary ARMA (*p, q*) model.

Engle (1982) also made a significant contribution in the area of time series analysis by introducing the Autoregressive Conditional Heteroscedasticity (ARCH) model, to model changing volatility. The non-linear term is the variance of the disturbance. Bollerslev (1986) made an extension of the ARCH model to the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model.

An ARMA-ARCH model, in which an ARMA model is used to model mean behaviour and an ARCH model to model the residuals of the ARMA model was proposed by Weiss (1984). The quasi-maximum-likelihood method was used to estimate model parameters

Furthermore, in the field of multivariate time series, Hillmer and Tiao (1979) made a remarkable contribution where they developed an exact likelihood technique for Vector Autoregressive Moving average (VARMA) model. Harvey and Peters (1984) further proposed a state-space method to estimate parameters of VARMA (*p, q*) models.

In addition, Engle and Granger (1987) proposed a cointegrated multivariate time series and Error Correction Models (ECM) where the cointegrated concept captures the phenomenon of univariate non-stationary time series moving together. The ECM procedure involves two steps. The first step involves modeling the long term relationship between endogenous and exogenous variables. The variables involved have to comply with two constraints; non-stationary and being stationary after first differencing. In the second step, the dynamic short term process is modeled and only stationary variables enter the regression equation.

Finally, a Vector Autoregressive Fractionally Integrated Moving Average (VARFIMA) was proposed by Robinson and Yajima (2002) to take care of multivariate time series cointegration problems.

2.4 Conclusion

This chapter focused on reviewing of literature that is relevant to the study. It can be observed that several approaches have been used to model and forecast price volatility of cereals in various parts of the world. However little work has been done on price volatility of cereals in the Northern region of Ghana. This research therefore employed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) regression model to forecast the price volatility of the three major cereals in the northern region of Ghana.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter deals with the data and statistical techniques that were employed in order to achieve the objectives of the study. There are eight main headings under this chapter namely; source of data, unit root test, ARCH (*m*) model, GARCH (*m, s*) model, VAR (*p*) model, criteria for model selection and model diagnostics.

3.1 Source of Data

In order to achieve the objectives of this study, secondary data on monthly prices of Rice, Maize and Millet were obtained from the Ministry of Food and Agriculture, Northern Regional office. The data ranges from January 2000 to December 2013. The data were analysed using R, STATA, Minitab and Gretl statistical software.

3.2 Unit Root Test

Stationarity is an essential aspect of time series analysis. There are two main types of stationarity in time series analysis; strong stationarity and weak stationarity. A series is said to be strongly stationary if its mean, variance, auto covariance and all other higher moments at any lag say *k*, remain constant over time. On the other hand, a time series is said to be weakly stationary or second order stationary if its first and second moments (mean and auto covariance) are independent of time. This can be expressed mathematically as:

$$E(X_t) = \mu_x \qquad (3.1)$$

$$Cov(X_t, X_{t+k}) = \gamma_y(k) \quad (3.2)$$

Where μ is constant and γ_k is independent of time. However for practical applications, the assumption of strong stationarity is not always appropriate and so weak stationarity is always considered for the analysis of time series data.

Several approaches have been developed to test for the stationarity or otherwise of a time series data which include both graphical and quantitative approaches. The graphical approach is done through visual inspection of the nature of the Autocorrelation function (ACF) plots. The series will be stationary if the ACF decay rapidly after few lags. However if the ACF exhibits a strong and slow decaying pattern after several lags, then it presupposes that the series is non-stationary. In this study, two quantitative approaches were employed in addition to the ACF approach to test for unit root. These two quantitative methods include; the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The presence of a unit root indicates that the time series is not stationary and that differencing will make it stationarity.

3.2.1 Augmented Dickey Fuller (ADF) Test

The ADF test is an extension of the Dickey-Fuller (DF) test which was developed to deal with serial correlations in the time series. De Jong *et al*. (1992) recommended the ADF test as the overall best test for a unit root in the presence of auto correlated errors, mainly because it does not suffer size distortions under over parametizations, extreme autocorrelation, and increased sampling frequency. The ADF test assumes that the time series follows a random walk. Consider the first order Autoregressive AR (1) process given below

$$Y_t = \phi Y_{t-1} + \varepsilon_t \quad (3.3)$$

Where Y_t is the variable of interest, $t$is time index, ϕ is the model coefficient and ε_t denotes an independently and identically distributed error term with a zero-mean and constant variance. A unit root is present if$\phi = 1$. In this case, equation (3.3) becomes a random walk model without drift and hence a non-stationary process. The test is based on the regression of the observed variable Y_t on its one-period lagged valueY_{t-1}, sometimes including an intercept and a time trend. ϕ is then estimated to see whether it would be equal to one or not. Equation (3.3) can be rewritten as:

$$\Delta Y_t = (\phi - 1)Y_{t-1} + \varepsilon_t = \delta Y_{t-1} + \varepsilon_t \qquad (3.4)$$

Where Δ is the difference operator, implying that $\Delta Y_t = Y_t - Y_{t-1}$ and$\delta = \phi - 1$. From equation (3.4) we test the null hypothesis $\delta = 0$ against the alternative $\delta \neq 0$. If $\delta = 0$, it implies that $\phi = 1$, which confirms the presence of a unit root in the series.

The null hypothesis may be rejected or not, depending on the critical values of the DF test and the calculated value of the test statistic. The DF test is plagued by the presence of serial correlations in the residuals which often leads to biases in drawing conclusions. Due to this major defect, the ADF test includes enough lagged dependent variables in the Autoregressive (AR) model to rid the residuals of serial correlation. The model therefore becomes:

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \cdots + \gamma_{p-1} \Delta Y_{t-p+1} + \varepsilon_t \qquad (3.5)$$

Whereα is a constant, $\beta$is the coefficient on time trend series, $\gamma_1 \Delta Y_{t-1} + \cdots + \gamma_{p-1} \Delta Y_{t-p+1}$ is the sum of the lagged values of the dependent variable ΔY_t and p is the lag order of the AR process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modelling a random walk and using constraint $\beta = 0$ corresponds to modelling a random walk with drift. By including lags of the order p, the ADF formulation allows for higher-order AR processes. The ADF test is concerned with the value of the parameter δ. If $\delta = 0$, it presupposes that the series contains unit root and hence non-stationary.

The test statistic for the ADF test is given by

$$F_{\tau} = \frac{\hat{\delta}}{SE(\hat{\delta})} \qquad (3.6)$$

Where$\hat{\delta}$ is the least square estimate and$SE(\hat{\delta})$ is the standard error estimate of $\hat{\delta}$. If the calculated value of the test statistic is greater than the critical value, we reject the null hypothesis of $\delta = 0$.

3.2.2 Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test

The KPSS test has a null hypothesis of stationarity of a series around either the mean or a linear trend; and the alternative assumes that the series is non-stationary due to the presence of a unit root. There are three components in the KPSS test model. These include: deterministic trend, a random walk, and a stationary error term. One important assumption made in the test is that, if there is no linear trend term, the point of departure is a data generating process of the form

$$Y_t = r_t + \varepsilon_t \qquad (3.7)$$

Wherer_t denotes a random walk process, $r_t = r_{t-1} + u_t$, ε_t is an error term, and u_t denotes an error term of the random walk equation. It is assumed that u_t is a series of identically distributed independent random variables with expected value equal to zero and constant variance$\hat{\sigma}_u^2$. The null hypothesis of stationarity is equivalent to the assumption that the variance σ_u^2 of the random walk process r_t equals zero while the alternative assumes that the variance is greater than zero. The hypotheses are therefore stated as:

$$H_0: \sigma_u^2 = 0$$

$$H_A: \sigma_u^2 > 0.$$

If H_0 is true, then $Y_t$is composed of a constant and the stationary processε_t; which means Y_t is also stationary. The test statistic of the KPSS test is given by;

$$KPSS = \sum_{t=1}^{T} \frac{S_t^2}{\sigma_\infty^2} \qquad (3.8)$$

Where T denotes the number of observations,$S_t = \sum_{i=1}^{t} \varepsilon_i$, for$t = 1,2, \dots, T$, ε_i denotes estimated errors from a regression of Y_t on a constant and time and are computed as: $\varepsilon_t = Y_t - \bar{Y}$and $\hat{\sigma}_\infty^2$ is an estimator of the long-run variance of the ε_t process given as:

$$\sigma_\infty^2 = \lim_{T\to\infty} T^{-1} Var(\textstyle\sum_{t=1}^{T} \varepsilon_t) (3.9) \text{or}$$

$$\sigma^2 = limT^{-1}E[S_T^2] \qquad (3.10)$$

The decision rule is to reject the null hypothesis of stationarity if the computed value of the test statistic is greater than the critical value at a given level of significance.

3.3 Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model is a generalization of the ARMA model that is defined to include an integrated (I) component to handle time series data that are non-stationary in nature. In practice many time series data show non-stationary behaviour and such data are made stationary by applying finite differencing of the data points. In terms of backshift operator, the ARIMA (*p, d, q*) model is given as:

$$\phi(B)(1 - B)^d Y_t = \theta(B)\varepsilon_t \qquad (3.16)$$

where p, d and q are integers greater than or equal to zero and denote the order of the autoregressive, integrated, and moving average parts of the model respectively.The integer $d$controls the level of differencing. Generally,$d = 1$ is enough in most cases.

3.3.1 Autoregressive Model of Order p (AR (p)):

A time series $Y_t$is said to be an autoregressive process of order p, if it is a weighted linear sum of the past p values plus a random shock. The general AR model of order p is given by:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t \quad (3.11)$$

where the Y's and ε_t are respectively the original series and random error at timeperiod t, ϕ_i are the AR parameters to be estimated with $i = 1, 2, \ldots, p$ and p is the order of the AR model. Thus the value at time $t$depends linearly on the last p values and the model looks like a regression model; hence the term autoregression. Using the backward shift operator B such that;

$BY_{t-1} = BY_t$and $B^2 Y_t = Y_{t-2}$, the AR(p) model may be written more succinctly in the form

$$\phi(B) Y_t = \varepsilon_t \qquad (3.12)$$

Where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$is a polynomial in B of order p. The AR (p) time series is said to be stationary if the roots of the polynomial:

$m^p - \phi_1 m^{p-1} - \phi_2 m^{p-2} - \ldots - \phi_p$are less than one in absolute terms.

3.3.2 Moving Average Model of Order q (MA (q)):

A time series $Y_t$is said to be a moving average process of order q if it is a weighted linear sum of the last q random shocks. That is, the current values of the series depend on its past shocks. The general MA model is given by:

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \qquad (3.13)$$

where q is the order of the model, θ_j are the model parameters to be estimated and $j = 1,2, \ldots, q$. The random shocks are assumed to be a white noise process, that is a sequence of independent and identically distributed (i.i.d) random variables with zero mean and a constant variance σ^2. Regardless of the values of the weights, an MA process is always stationary. The MA (q) model can be expressed in terms of the backshift operator as:

$$Y_t = \theta(B)\varepsilon_t \qquad (3.14)$$

Where $\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q$ is a polynomial in B of order q. Generally, the random shocks are assumed to follow the typical normal distribution. The MA(q) process is invertible if the characteristic roots of the polynomial $m^q + \theta_1 m^{q-1} + \theta_2 m^{q-2} + \cdots + \theta_q = 0$ are less than one in absolute terms.

3.3.3 Autoregressive Moving Average (ARMA) Model

Autoregressive (AR) and Moving Average (MA) models can be effectively combined together to form a general and useful class of time series models, known as the ARMA (p, q) models, where p and q are the orders of the AR and MA processes respectively (Box *et al.*, 1994). Generally, an ARMA (p, q) model is given as:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \qquad (3.15)$$

where ϕ_i and θ_j are parameters of the autoregressive and moving average components respectively, i= 1, 2, …, p and j= 1, 2, …, q.

We can also express the ARMA (p, q) model as: $\phi(B)Y_t = \theta(B)\varepsilon_t$, where $\phi(B)$ and $\theta(B)$ are polynomials in B of finite order p and q respectively. The ARMA (p, q) process is stationary if the roots of the polynomial in the AR component are less than one in absolute terms. On the other hand, the process is invertible provided that the absolute values of the roots of the polynomial in the MA component are less than one.

3.4 Autoregressive Conditional Heteroskedasticity, ARCH (m) Model

An ARCH process is a mechanism that includes past variance in the explanation of future variances (Engle, 2004). The ARCH model was developed by Engle (1982) and it provides a systematic framework for volatility modelling. ARCH models specifically take the dependence of the conditional second moments in consideration when modelling. Let $\{x_t\}$ be the mean-corrected return, ε_t be the Gaussian white noise with zero mean and unit variance and I_t be the information set at time t given by $I_t = \{x_1, x_2, \ldots, x_{t-1}\}$. Then the ARCH ($m$) model is specified as:

$$x_t = \sigma_t \varepsilon_t \quad (3.17)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2 \quad (3.18)$$

where $\alpha_0 > 0$ and $\alpha_i \geq 0, \; i = 1, \ldots, m.$

and

$$E\,(x_t | I_t) = E\,[\,E\,(\,x_t |\, I_t)\,] = E\,[\sigma_t E\,(\varepsilon_t)] = 0 \quad (3.19)$$

$$Var(x_t | I_t) = E\,(x_t^2) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-i}^2 \quad (3.20)$$

and the error term, ε_t is such that

$$E\,(\,x_t |\, I_t) = 0 \quad (3.21)$$

and

$$Var\ (x_t|I_t) = 1 \qquad (3.22)$$

From equations (3.21) and (3.22), it can be seen that the error term ε_t is a conditional standardised martingale difference. A stochastic series $\{x_t\}$ is said to be a martingaledifference if its expectation with respect to past values of another stochastic series $\{y_i\}$ is zero (Amos, 2010).That is

$$E\ (x_{t+i}|y_i, y_{i-1}) = 0\ for\ i = 1,2, \dots \qquad (3.23)$$

From the structure of the model, it can be seen that the dependence of the present volatility $\{x_t\}$ is a simple quadratic function of its lagged values. The coefficients $\alpha_i, i = 0, \dots, m$ can consistently be estimated by using $\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2$. To ensure that the conditional variance σ_t^2 is always positive for all t, isrequired that $\alpha_0 > 0$ and $\alpha_i \geq 0,\ i = 1, \dots, m$. From equations (3.17) and (3.18) itfollows that large past squared values $\{x_{t-i}^2\},\ i = 1, \dots, m$ follows a large conditionalvarianceσ_t^2 for the present volatility $\{x_t\}$. Consequently, $\{x_t\}$ tends to assume a largevalue in absolute value. Hence under the ARCH framework, large shocks tend to befollowed by another large shock.

3.4.1 Estimation of the ARCH (m)

There are three likelihood functions that are commonly used in ARCH (m) estimation depending on the distributional assumption made on the error termε_t. The three common distributions are the normal distribution, standardized student-t distribution which is a heavy tailed distribution and the generalised error distribution (GED). Using the maximum likelihood estimation technique, the parameters can be estimated using

$$f(x_1, \dots, x_t|\theta) = f\ (x_t|x_{t-1})\ f\ (x_{t-1}|x_{t-2}) \dots f\ (x_{m+1}|x_m)\ f\ (x_1, \dots, x_m|\ \theta) =$$

$$\prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-x_t^2}{2\sigma_t^2}\right) f\,(x_1, \dots, x_m \mid \theta\,). \quad (3.24)$$

Where$\theta = (\alpha_0, \alpha_1, \dots, \alpha_m)'$ *and* $f\,(x_1, \dots, x_m \mid \theta\,)$is the joint probability density function of$x_1, \dots, x_m$. Since the exact form of $f\,(x_1, \dots, x_m \mid \theta\,)$ is complicated anddifficult to obtain, it is commonly dropped from the prior likelihood function, especially when the sample size is sufficiently large. Rather it is practically easier to condition on the first $x_1, \dots, x_m$ since they are usually known and equal to its observed values. This results in the conditional likelihood function being:

$$f(x_1, \dots, x_t | \theta\,;\, x_1, \dots, x_m) = \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-x_t^2}{2\sigma_t^2}\right) \quad (3.25)$$

Whereσ_t^2 can be evaluated recursively. Under the normality assumption, the estimates, $\widehat{\alpha_0}\widehat{\alpha_1}, \dots, \widehat{\alpha_m}$are obtained by maximising the prior likelihood function called the conditional maximum likelihood estimates (MLE) (Tsay, 2002).

Maximising the conditional likelihood function can be difficult to handle. An equivalent way which is easier to handle is to maximise the logarithm of the conditional likelihood function. Accordingly, the conditional log likelihood function is given as:

$$\ell(x_{m+1}, \dots, x_t | \theta\,;\, x_1, \dots, x_m) = \sum_{t=m+1}^{T} \left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma_t^2 - \frac{x_t^2}{2\sigma_t^2}\right)$$

$$= -\sum_{t=m+1}^{T} \left(\frac{1}{2}\ln \sigma_t^2 + \frac{x_t^2}{2\sigma_t^2}\right) + K \quad (3.26)$$

where $K = -\frac{(T-m)}{2}\ln 2\pi$.

Since the first term $\frac{1}{2}\ln 2\pi$ does not involve any parameter and hence its exclusion has no effect on the estimation process. Again $\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2$ can be evaluated recursively.

3.5 The Generalized Autoregressive Conditional Heteroscedasticity Model

The Generalized ARCH (GARCH) model was developed by Bollerslev (1986) as an extension of the ARCH model. The principle of parsimony may be violated when a model has a large number of parameters resulting in difficulties in using the model to adequately describe the data. In particular, although the ARCH model is simple, it may require many parameters as there might be a need for a large value of lag q and hence the principle of parsimony would be violated in such a case. GARCH model may contain fewer parameters when compared to an ARCH model. Thus a GARCH model may be preferred to an ARCH model using the principle of parsimony. Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset, u_t, the conditional mean of x_t. Then x_t follows a GARCH (m, s) model if

$$x_t = \sigma_t \varepsilon_t \qquad (3.27)$$

$$\sigma_t^2 = \widehat{\alpha_0} + \sum_{i=1}^{m} \widehat{\alpha_1} \sigma_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \qquad (3.28)$$

where $\{\varepsilon_t\}$ is a sequence of independent, identically distributed random variables with mean zero and unit variance and the parameters of the model are α_i , $i = 0, \dots, m$ and $\beta_j, j = 1, \dots, s$ suchthat $\alpha_i \geq 0$ and $\beta_j \geq 0$; ($\sum_{i=1}^{v}(\alpha_i + \beta_i) < 1$) where $v = \max(m, s)$ and $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$. The constraints on $\alpha_i + \beta_i$ implies that the unconditional variance of x_t is finite, whereas its conditional variance σ_t^2 evolves over time. From equations (3.27) and (3.28), it is seen that the GARCH (m, s) model employs the same equation as (3.17) for the mean corrected return x_t as in the ARCH (m) but the equation for the volatility includes s

new terms. Therefore equations (3.27) and (3.28) reduces to a pure ARCH (m) model if $s = 0$. Thus the GARCH model generalizes the ARCH model by introducing values of $\sigma_{t-1}^2,\ \sigma_{t-2}^2 \cdots$. The parameters α_i and β_i are respectively referred to as the ARCH and GARCH parameters. The GARCH (m, s) model can be stated differently. Let $\eta_t = x_t^2 - \sigma_t^2$ so that $\sigma_t^2 = x_t^2 - \eta_t$. By substituting$\sigma_{t-i}^2 = x_{t-i}^2 - \eta_{t-i}(i = 0, \ldots, m)$ into equation (3.28), the GARCH (m, s) can be written as

$$x_t^2 = \alpha_0 + \sum_{i=1}^{v}(\alpha_i + \beta_i)x_{t-i}^2 + \eta_t - \sum_{j=1}^{s}\beta_j\eta_{t-i} \qquad (3.29)$$

Where$v$= max (m, s),$\alpha_1 = 0$ for $i > m, \beta_i = 0\ for\ j > s$

Thus the equation of σ_t^2 has an Autoregressive Moving Average, ARMA (m, s) representation and it can be seen that$\{\eta_t\}$ is a martingale difference series.However, the $\{\eta_t\}$ is not an independent, identically distributed random sequence. In order to find the GARCH (m, s) process, we solve for α_0 in the equation (3.29) by letting the variance of x_t be σ_t^2 . This yields

$$\alpha_0 = \sigma_t^2\left(1 - \sum_{i=1}^{m}\alpha_i - \sum_{j=1}^{s}\beta_j\right) \qquad (3.30)$$

And substituting the value of α_0 as given by equation (3.29) into equation (3.30) gives

$$x_t^2 = \sigma_t^2\left[1 - \sum_{i,j=1}^{v}(\alpha_i + \beta_i)\right] + \left[\sum_{i,j=1}^{v}(\alpha_i + \beta_i)\right]x_{t-1}^2 - \sum_{j=1}^{s}\beta_j\eta_{t-i} + \eta_t$$

$$= \sigma_t^2 + \sum_{i,j=1}^{v}(\alpha_i + \beta_i)\left(x_{t-i}^2 - \sigma_t^2\right) - \sum_{j=1}^{s}\beta_j\eta_{t-i} + \eta_t \qquad (3.31)$$

Therefore

$$x_t^2 - \sigma_t^2 = \sum_{i,j=1}^{v} (\alpha_i + \beta_i)(x_{t-i}^2 - \sigma_t^2) - \sum_{j=1}^{s} \beta_j \eta_{t-i} + \eta_t \qquad (3.32)$$

Multiplying both sides of equation (3.32) by $(x_{t-k}^2 - \sigma_k^2)$ results in

$$(x_{t-k}^2 - \sigma_k^2)(x_{t-i}^2 - \sigma_t^2) = \sum_{i,j=1}^{v} (\alpha_i + \beta_i)(x_{t-i}^2 - \sigma_t^2)(x_{t-k}^2 - \sigma_k^2) - \sum_{j=1}^{s} \beta_j \eta_{t-i} +$$

$$\eta_{t-i}(x_{t-k}^2 - \sigma_k^2) + \eta_t(x_{t-k}^2 - \sigma_k^2) \qquad (3.33)$$

And taking expectations of equation (3.33), we have

$$E[(x_{t-k}^2 - \sigma_k^2)(x_{t-i}^2 - \sigma_t^2)] = E\left[\sum_{i,j=1}^{v} (\alpha_i + \beta_i)(x_{t-i}^2 - \sigma_t^2)(x_{t-k}^2 - \sigma_k^2)\right] - E\left[\sum_{j=1}^{s} \beta_j \eta_{t-i}\right] +$$

$$E[\eta_{t-i}(x_{t-k}^2 - \sigma_k^2) + \eta_t(x_{t-k}^2 - \sigma_k^2)] \qquad (3.34)$$

But $E[\eta_t(x_{t-k}^2 - \sigma_k^2)] = E(x_{t-k}^2 - \sigma_t^2)E(\eta_t|x_t) = 0$since η_t is a martingaledifference and also

$$E[\beta_j \eta_{t-i}(x_{t-k}^2 - \sigma_t^2)] = E[(x_{t-k}^2 - \sigma_t^2) E(\eta_{t-J}|x_{t-J})] = 0 \; for \; k < j \qquad (3.35)$$

Thus the auto covariance of the squared returns for the GARCH (m, s) model is given by

$$cov(x_t^2, x_{t-k+i}^2) = E\left[\sum_{i,j=1}^{v} (\alpha_i + \beta_i)(x_{t-i}^2 - \sigma_t^2)(x_{t-k}^2 - \sigma_k^2)\right]$$

$$= \sum\nolimits_{i,j=1}^{v} (\alpha_i + \beta_i) \; cov(x_t^2, x_{t-k+i}^2) \qquad (3.36)$$

Dividing both sides of equation (3.36) by σ_t^2 gives the autocorrelation function at lag k as

$$\rho_k = \sum_{i,j=1}^{v} (\alpha_i + \beta_i) \rho_{k-i}, for \; k \geq (m+1) \qquad (3.37)$$

This result is analogous to the Yule-Walker equations for an AR process. Hence the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the squared returns in a GARCH process has the same pattern as those of an ARMA process. The ACF and PACF are useful in determining the orders *m* and *s* of the GARCH (*m*, *s*) process. Also, the ACF is used in checking model accuracy; in which case, the ACF's of the residuals indicates the presence of a white noise if the model is adequate. The parameters $\alpha_0, \alpha_1, \dots, \alpha_m; \beta_1, \beta_2, \dots, \beta_s$ affect the autocorrelation but given the $\rho_k, \cdots, \rho_{m+1-v}$, the autocorrelation at higher lags are determined uniquely by the expression in equation (3.35) (Bollerslev, 1986) as cited in Ngailo (2011). Denoting the v^{th} partial autocorrelation for x_t^2 by ϕvv then

$$\rho_k = \sum_{i,j=1}^{v} \phi_{vv}\rho_{k-i}, k = 1, \dots, v = \max(m, s) \qquad (3.38)$$

It can be seen from equation (3.38) that, there are cut offs after lag *m* for an ARCH (*m*) process such that $\phi \neq 0$ for $k \leq m$ and $\phi = 0$ for $k > m$ and it is similar to the AR (*m*) process and decays exponentially (Bollerslev, 1986). To understand the theory and concepts of the GARCH model, we would focus on the special case of the GARCH (1, 1) model.

3.5.1 GARCH (1, 1) Model

The GARCH (1, 1) model is a particular case of the GARCH (*m*, *s*) model where the orders *m* and *s* are both equal to one (i.e. *m* = *s* = 1). Let $\{x_t\}$ be the mean corrected return, ε_t be a gaussian white noise with mean zero and unit variance. If I_t is the information set available at time *t* given by $I_t = \{x_1, x_2, \dots, x_{t-1};\ \sigma_t^2, \sigma_t^2, \dots, \sigma_{t-1}^2\}$, then the process $\{x_t\}$ follows a GARCH (1, 1)model if

$$x_t = \sigma_t \varepsilon_t \qquad (3.39)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3.40)$$

Whereα_0 , α_1 and β_1 are the parameters of the model such that $\alpha_0 \geq 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and ($\alpha_i +$ $\beta_i < 1$). The constraints on the parameters are to ensure that the conditional variance $\sigma_t^2$is positive. Clearly from (3.39) and (3.40), it is clear that large past mean corrected return x_{t-1}^2 or past conditional variance σ_{t-1}^2 give rise to large values of σ_t^2 (Tsay,2002). It can be seen that $\{x_t\}$ is a martingale difference as the conditional mean is zero (i.e. $E(x_t | I_t) = 0$). Taking $\eta_t = x_t^2 - \sigma_t^2$so that $\sigma_t^2 = x_t^2 - \eta_t$, the GARCH (1,1) can be represented differently. By substituting $\sigma_{t-i}^2 = x_{t-i}^2 - \eta_{t-i}$, into equation (3.40), the GARCH (1, 1) can be written as

$$x_t^2 = \alpha_0 + (\alpha_1 + \beta_1) x_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1}$$

$$= \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 (x_{t-1}^2 - \eta_{t-1}) + \eta_t \qquad (3.41)$$

Again it can be seen that $\{\eta_t\}$ is a martingale difference series as = 0 (i.e. $E(x_t | I_t) = 0$)

$E(\eta_t) = 0$ and $(\eta_t, \eta_{t-j}$) for $j \geq 1$) and $\{\eta_t\}$ is an uncorrelated sequence. From (3.41)

$$E(x_t^2) = \sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1) E(x_{t-1}^2) \qquad (3.42)$$

This implies

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1) E(\sigma_t^2 \varepsilon_t^2) \qquad (3.43)$$

thus

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_t^2 E(\varepsilon_t^2) \qquad (3.44)$$

which yields

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_t^2, \qquad (3.45)$$

Since

$$E(\varepsilon_t^2) = var\ (\varepsilon_t^2) = 1 \tag{3.46}$$

$$\sigma_t^2 = \frac{\alpha_0}{[\,1 - (\alpha_1 + \beta_1)]}, \tag{3.47}$$

Provided $|\alpha_1 + \beta_1| < 1$

3.5.2 Estimation of GARCH (m, s) model

Once the orders m and s have been identified, the parameters $\alpha_0, \alpha_1, \ldots, \alpha_m$; $\beta_1, \beta_2, \ldots, \beta_s$ of the GARCH (m, s) model can then be estimated. The maximum likelihood estimation is used to estimate the parameters of the model. The initial values of both the squared returns and past conditional variances are needed in estimating the parameters of the model. Bollerslev (1986) and Tsay (2002) suggest that the unconditional variance given in equation (3.28) or the past sample variance of the returns may be used as initial values. Therefore assuming $x_1, x_2, \ldots, x_m$; $\sigma_t^2, \sigma_t^2, \ldots, \sigma_s^2$ are known, the conditional log-likelihood is given by

$$\ell(x_{m+1}, \ldots, x_t; \sigma_{s+1}^2, \ldots, \sigma_t^2 | \theta\ ;\ x_1 x_2, \ldots, x_m; \sigma_1^2, \sigma_2^2, \ldots, \sigma_s^2)$$

$$= \sum_{t=v+1}^{T} \left(-\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_t^2 - \frac{x_t^2}{2\sigma_t^2} \right) \tag{3.48}$$

Where $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_m; \beta_0, \beta_1, \ldots, \beta_s)$ and $v = \max(m, s)$

It follows that the conditional maximum likelihood estimates are obtained by maximizing the conditional log-likelihood function given by equation (3.48)

3.5.3 Volatility and Half-life volatility Determination

The estimation of the volatility of a time series depends on its conditional variance. The squared of the conditional variance gives the conditional volatility. The conditional variance can be written

as:

$$\frac{\alpha_0}{[\,1 - (\alpha_i + \beta_i)]} \qquad (3.49)$$

The half-life volatility measures the time required for the volatility to move half way back towards its unconditional mean (Engle and Patton, 2001). The half-life was estimated using the relation;

$$\tau = \frac{\log((\alpha + \beta)/2)}{\log(\alpha + \beta)} \qquad (3.50)$$

Whereα and β are the coefficients of the conditional variance equation,

3.6. Vector Autoregressive (VAR) Modelling

A vector auto regression (VAR) model is a mechanism that is used to link multiple stationary time series variables together and it is an extension of the univariateautoregression (AR) model to dynamic multiple time series. The VAR model is useful for describing the dynamic behavior and relationship between economic and financial time series, for forecasting the series and for structural analysis. The VAR model fits a time series regression of each dependent variable on its lag values and on the lag values of other dependent variables considered. Forecast from VAR models are quite flexible because they can be made conditional on the potential future paths of specified time series variables in the model. If the variables are not individually covariance stationary, it can be differenced to make it stationary and a VAR model then fitted to the transformed series.

A VAR process consists of a set of k −endogenous time series variables $R_t = (r_{1t}, r_{2t}, \dots, r_{kt})'$ $for\ k = 1, \dots \dots, K$. A VAR model of order p denoted as VAR (p) is given by;

$$R_t = v + A_1 r_{t-1} + \cdots \dots \dots \dots + A_p r_{t-p} + u_t, \quad t = 0, 1, \dots, T \quad (3.51)$$

where $R_t = (r_{1t}, \dots, r_{kt})'$ is a $(k \times 1)$ random vector of the rates, $A_i, i = 1, \dots \dots, p$ is a fixed $(K \times K)$parameter (coefficient) matrice, $v = (v_1, \dots \dots, v_k)'$ is a fixed $(K \times 1)$ vector of intercept allowing for the possibility of a zero mean and$u_t = (u_{1t}, \dots, u_{kt})'$ is a K −dimensional white noise series or innovation process with time invariant positive definite covariance matrix and zero mean. It is assumed that u_t has a multivariate normal distribution.

3.6.1 Lag Order Selection

In fitting a VAR (p) model, one important step is the determination of the optimal lag of the VAR process. Lag order determinations enable us to ensure that the model chosen will reflect the observed process as precisely as possible with a small error term. In this study, we consider three lag order selection methods which differ by the severity of the penalty imposed for parameter profligacy and hence in the parsimony of the model selected. This study used the Akaike Information Criterion (Akaike, 1974), the Schwarz Bayesian Information Criterion (Schwarz, 1978) and the Hannan-Quinn Information Criterion (Hanan and Quinn, 1979) to determine the optimum lag order in fitting the VAR (p) model that describe the relationship between the set of time series variables. These criteria are given by;

$$AIC = ln\left|\widehat{\Sigma_u(p)}\right| + \frac{2}{T} pK^2 \quad (3.52)$$

$$HQIC = ln\left|\widehat{\sum_{u}(p)}\right| + \frac{2\, ln\{ln(T)\}}{T}pK^2 \qquad (3.53)$$

$$SBIC = ln\left|\widehat{\sum_{u}(p)}\right| + \frac{ln(T)}{T}pK^2 \qquad (3.54)$$

where T denotes the number of observations in the data, $p$assigns the lag order, $\widehat{\sum_u(p)} = T^{-1}\sum_{t=1}^{T}\widehat{u_t}\widehat{u_t}'$ is the residual covariance matrix without a degree of freedom corrected from the model and K is the number of parameters in the statistical model. For all the criteria, p is chosen so that the value of the criterion is minimized. The first part of these criteria measures the goodness of fit of the statistical model to the data whiles the second part is the penalty term of the criteria which penalizes a candidate model for the number of parameters used. Based on this penalty term, the SBIC and HQIC are consistent estimators and tend to select models with fewer parameters when the sample size is large than does the AIC. The lag order with the least values of these criteria is the optimum number of lags to be used.

3.6.2 Stability Condition of a VAR (*p*) Model

Statistical inference using a VAR (*p*) model depends crucially on the stability of the model parameters over time. Given sufficient starting values, a stable VAR (*p*) process generates stationary time series with time invariant means, variances and covariance structure. The stability is determined by evaluating the reverse characteristic polynomial equation of the VAR (*p*) model given as;

$$det(I_k - A_1 z^1 - \cdots - A_p z^p) \neq 0, \; for\ |Z| \leq 1 \qquad (3.55)$$

The VAR (*p*) process is stable if the reverse characteristic polynomial has no root in and on the complex unit circle (thus the process is stable if $|z| > 1$) (Lutkepohl, 2005). If the solution of the reverse characteristic polynomial has a root $z = 1$, then either some or all the variables in the VAR (p) process are integrated of order one ($I(1)$). In practice, if the eigenvalues of the parameter matrix, A_{i} are less than one (1) in modulus, then the VAR (p) is stable (which is $|A_i| < 1$ in univariate case). The stability of the VAR (p) model enables us to write the VAR (p) process as an invertible moving average process from which further inference such as Impulse Response Analysis can be made.

3.7 Criterion for Model Selection

In order to obtain the most adequate model that best describes a time series data, it is important for model selection criteria to be carried out. This is because there is the possibility of two or more models to compete in the selection of the best model. The Akaike Information Criterion (AIC), the Akaike Information Criterion corrected (AICc) and the Bayesian Information Criterion (BIC) are the model selection criteria that were employed in this study to select the most adequate model. The information criteria include a penalty that is an increasing function of the number of parameters. The penalty discourages over fitting, that is, increasing the number of parameters almost always improves the goodness of fit. The best model is the one with the smallest AIC, AICc or BIC values, given a set of candidate models. The AIC, AICc, and BIC are generally given by;

$$\text{AIC} = 2k - 2In(L) \tag{3.56}$$

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{n-k-1} \tag{3.57}$$

$$\text{BIC} = \log(\sigma_e^2) + \frac{k}{n}\log(n) \tag{3.58}$$

where;

k represents the number of parameters in the model

L denotes the maximised value of the likelihood function

n is the number of observations in the data

$\sigma_e^2$is the error variance

3.8 Model Diagnostics

To use the fitted models for statistical inference, it is essential to diagnose the model to determine whether the model best fit the series. This involves checking whether or not the residuals of the model fitted are white noise series; thus whether they are free from serial correlation and conditional heteroscedasticity. This study employed both univariate and multivariate model diagnostics techniques such as the univariate and multivariate Ljung-Box, the univariate and multivariate ARCH-LM as well as the CUSUM tests model diagnostics techniques.

3.8.1 UnivariateLjung-Box Test

The study employed the univariateLjung and Box (1978) test to test jointly whether or not several autocorrelations (r_l)of the residuals of the individual VAR (p) models fitted were zero. It is based on the assumption that the residuals contain no serial correlation (no autocorrelation) up to a given lag m. The univariate Ljung-Box statistic is given by:

$$Q(m) = T(T+2)\sum_{l=1}^{m}\frac{r_l^2}{T-l} \qquad (3.59)$$

Where $r_l$represents the residual sample autocorrelation at lag$l$, $T$is the size of the series, $m$is the number of time lags included in the test. $Q(m)$has an approximately chi-square distribution with

m degrees of freedom. We fail to reject H_0 and conclude at α-level of significance that, the residuals are free from serial correlation when the$p$_value is greater than the significance level.

3.8.2 Univariate ARCH-LM Test

For a fitted model to adequately fit a series, the variance of the models' residuals must be constant over time. The univariate ARCH-LM test proposed by Engle (1982) was used in this research to check for the presence or absence of conditional heteroscedasticity in the residuals of the individual equations of the model fitted. If there exist no ARCH-effect, it implies that the residuals of the model are homoscedastic and have constant variance. This statistic uses the linear regression model;

$$u_t^2 = a_0 + a_1 u_{t-1}^2 + \cdots + a_m u_{t-m}^2 + e_t \qquad (3.60)$$

$t = m + 1, \ldots\ldots, T$

Where e_t is the error term, T is the sample size and m is a positive integer. The ARCH-LM statistic tests the hypothesis that;

$H_0: a_1 = \cdots = a_m = 0$ (no ARCH − effect) against

$H_0: a_1 \neq \cdots \neq a_m \neq 0$ (ARCH − effect exist)

The ARCH-LM test statistic is calculated as;

$$LM = TR^2 \qquad (3.61)$$

whereR^2 is the coefficient of determination for the auxiliary regression. The decision rule is to reject H_0 and conclude that there is conditional heteroscedasticity (ARCH-effect) in the residuals of the model if $LM > \chi^2\,(m)$, or if the$p - value < \alpha$, where $m$is the lag order of ARCH-effect and α is the significance level chosen.

3.8.3 Multivariate Ljung-Box Test

In addition to testing for the accuracy of the individual equations of the fitted model, it is important to test for the accuracy of the overall VAR (p) process. This research therefore employed the multivariate Ljung-Box test to check for the presence or absence of serial correlation among the residuals of the overall VAR (p) model. The test is designed to test the hypothesis;

$H_0: R_m = (R_1, \dots\dots R_m) = 0$ (no serial correlation) against

$H_1: R_m \neq 0$ (there exist serial correlation in the residuals)

The multivariate Ljung-Box test is given by;

$$Q_m = T^2 \sum_{i=1}^{m} (T-i)^{-1} tr\left(\hat{c}_i' \hat{c}_0^{-1} \hat{c}_i \hat{c}_0^{-1}\right) \tag{3.62}$$

Where $\hat{c}_i = {}^1/_T \sum_{t=i+1}^{T} \hat{u}_t \hat{u}_{t-i}'$. If $T \rightarrow$ infinity, then$\frac{T}{T^2 (T-i)^{-1}} \rightarrow 1$. For large T and m, $Q_m \sim \chi^2(k^2(h-p))$. We fail to reject H_0 and conclude that there is no serial correlation in the residuals of the model when $Q_m < \chi^2(k^2(h-p))$ or the *p-value* of the statistic is greater than the chosen α-level.

3.8.4 Multivariate ARCH-LM Test

The multivariate ARCH-LM test was also used to test for conditional heteroscedasticity on the residuals of the overall VAR (p) model. The multivariate ARCH-LM test is based on the regression model below;

$$vech\,(u_t u_t') = a_0 + a_1 vech(u_{t-1} u_{t-1}') + \cdots + a_p vech\left(u_{t-p} u_{t-p}'\right) + e_t \tag{3.63}$$

wheree_t assigns a spherical error process, $vech$ is the column-stacking operator for symmetric matrices that stacks the columns from the main diagonal on downward.

$a_0$is ${}^{1}/_{2}\ k(k+1)$-dimension and $a_j{'s}$ are ${}^{1}/_{2}\ k(k+1) \times {}^{1}/_{2}\ k(k+1)$ coefficient matrices$(j = 1, \dots, p)$. The multivariate ARCH-LM statistic tests the pair of hypothesis;

$H_0 \ = \ a_1 = \cdots = \ a_p = 0$ (No ARCH-effect) against

$H_1 \ = \ a_1 \neq 0 \ or \dots or \ a_p \neq 0$ (ARCH-Effect exist)

If all the a_j matrices are zero, there is no ARCH effect in the residuals of the model. The LM statistic can be determined by replacing all unknown $u_t{'s}$ by estimated residuals from a VAR (*p*) model and estimating the parameters in the resulting auxiliary model by OLS. Denoting the residuals covariance matrix estimator by $\widehat{\Sigma_{vech}}$ and the corresponding matrix obtained for $q = 0$ by Σ_0^{-1}, the ARCH-LM statistic is given as;

$$LM_{ARCH\ (p)} = {}^{1}/_{2}\, TK(K+1) - Ttr\left(\widehat{\Sigma_{vech}}\Sigma_0^{-1}\right) \qquad (3.64)$$

The statistic has asymptotic $\chi^2\left(\frac{pK^2(K+1)^2}{4}\right)$ distribution, where *p*the lag is order of the process, k is the number of parameters and T is the size of the series.

3.8.5 Cumulative Sum (CUSUM) Test

The cumulative sum test by Brown *et al.*, (1975) was used to test for stability of the fitted model over time. The focus of this test is the maximal excursion (from zero) of the random walk defined by the cumulative sum of adjusted (-1, +1) digits in the sequence. The purpose of the test is to determine whether the cumulative sum of the partial sequences occurring in the tested sequence is too large or too small relative to the expected behavior of that cumulative sum for random sequences. The test statistic is defined as;

$$\text{CUSUM}_{\varphi} = \sum_{t=k+1}^{\varphi} \frac{\hat{q}_t^{(r)}}{\hat{\sigma}_q} \qquad (3.65)$$

Where $\hat{q}_t^{(r)}$ are the recursive residuals and $\hat{\sigma}_q$ is the standard error of the regression fitted to all T sample points and $\varphi = K + 1, \ldots, T$. For a structural unstable (random) model, the CUSUM wanders off too far from the zero line. A test with a significance level of 5% is obtained by rejecting stability if CUSUM_τ crosses the lines $\pm 0.948[\sqrt{T-K} + 2(\varphi - K)/\sqrt{T-K}]$ (Ploberger *et al*., 1989). The CUSUM test is designed basically to detect a non-zero mean of the recursive residuals due to shift in the model parameters.

3.8.6 Granger Causality Test

The idea behind Granger causality is that, if a time series variable x affects another z, then, x should help improve the prediction of variable z. A stationary time series variable x_t is Granger causal for another stationary time series variable z_t, if past values of x_t have additional power in predicting z_t after controlling for past values of z_t (Gelper and Croux, 2007). If the innovation to z_t and the innovation to x_t are correlated, then there exist instantaneous causality. Causality may be classified as unidirectional, bilateral or independent (Gujurati, 2003). Mathematically, the process x_t is said to Granger cause z_t if;

$$\textstyle\sum_z(h/\Omega_t) \leq \sum_z(h/\Omega_t \backslash \{x_s |\, s \leq t\}) \text{ for at least } h = 1, 2, \ldots \ldots \quad (3.66)$$

Where Ω_t is the information set containing all the relevant information in the universe available up to including $t z_t(h/\Omega_t)$ is the optimal $h-$step forecast of the process z_t at origin t base on the information in Ω_t. $\sum_z(h/\Omega_t)$ is the forecast Mean Square Error (MSE) and $(\Omega_t \backslash \{x_s |\, s \leq t\})$ is the set containing all relevant information in the universe except the information of past and present values of the x_t process. This implies that, with respect to x_t, the variance of the optimal linear predictor of z_{t+h} based on x_t is smaller than the variance of the optimal linear predictor of z_{t+h} based on $z_{t,}, z_{t-1}, \ldots ..$ alone (Lukepohl, 2005).

3.8.7 Impulse Response Function (IRF) Analysis

The Granger causality test introduced is quite useful to infer whether a time series variable helps predict another one. However, these analyses fall short of quantifying the impact of the impulse time series variable on the response variable over time. The impulse response analysis is used to investigate these kinds of dynamic interactions between the endogenous time series variables and is based upon the Wold's moving average representation of a VAR (*p*) process. IRF enables us to determine the response of one time series variable to an impulse or a shock in another time series variable in the system that involves a number of further variables as well. If there is a reaction of one time series variable to an impulse in another variable, then the latter is causal for the former. However, the effect of a unit shock in any of the variables dies away quite rapidly due to stability of the system. The Wold representation is based on the orthogonal errors η_t given by;

$$R_t = \mu + \Theta_0\eta_t + \Theta_1\eta_{t-1} + \Theta_2\eta_{t-2} + \ldots \qquad (3.67)$$

where$\Theta_0$is a lower triangular matrix. The impulse responses to the orthogonal shocks η_{jt} are;

$$\frac{\partial R_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial R_{i,t}}{\partial \eta_{j,t-s}} = \Theta_{ij}^{s} \quad i,j = 1,2,\ldots,k, s > 0 \qquad (3.68)$$

whereΘ_{ij}^{s} is the $(i,j)th$ element of Θ_0. For$k$ variables there are k^2 possible IRF.

3.8.8 Forecast Error Variance Decomposition (FEVD) Analysis

The forecast error variance decomposition tells us the proportion of the movements in a sequence due to its "own" shocks versus shocks to the other variable. The FEVD was used in this research to determine the contribution of the j^{th} variable to the *h*-step forecast error variance of the i^{th}variable. If the j^{th} variable shocks explain none of the forecast error variance of the i^{th}variable at all forecast horizons, then the i^{th}sequence is exogenous. Also, if the j^{th} variable shocks could explain all of the forecast error variance in the i^{th} sequence at all forecast horizons, then the i^{th} variable would be entirely endogenous. The FEVD is given as;

$$FEVD_{i,j}(h) = \frac{\sigma_{\eta_j}^2 \sum_{s=0}^{h-1} (\Theta_{ij}^s)^2}{\sigma_{\eta_1}^2 \sum_{s=0}^{h-1} (\Theta_{i1}^s)^2 + \dots + \sigma_{\eta_k}^2 \sum_{s=0}^{h-1} (\Theta_{ik}^s)^2} \quad i,j = 1, 2, \dots, k \tag{3.69}$$

where $\sigma_{\eta_j}^2$ is the variance of η_{jt}. A VAR (p) process with k variables will have $k^2 FEVD_{i,j}(h)$ values.

3.9 Conclusion

The chapter carefully presented statistical techniques that were employed in this study in a comprehensive manner.

CHAPTER FOUR

ANALYSIS AND DISCUSSION OF RESULTS

4.0 Introduction

This chapter deals with the analysis and discussion of the results obtained from the study. The chapter is sub-divided into three main headings namely; preliminary analysis, further analysis and discussion of results.

4.1 Preliminary Analysis

This section explains the descriptive statistics of the data on the returns of Rice, Maize and Millet. The maximum and minimum values of the Rice returns for the entire study period were 4.575 and -4.549 respectively. For the Maize returns, the maximum and minimum values were 2.400 and -2.885 respectively. Also, the maximum value for the Millet returns was 4.408 and that of the minimum was -4.438. Moreover, the average returns of Rice, Maize and Millet were 0.016, 0.001 and 0.020 respectively. The coefficients of variation for the returns of Rice, Maize and Millet were 3194.66, 33193.89 and 2869.75 respectively indicating that there is a greater variability among the returns of Maize compared to that of Rice and Millet. Furthermore, the returns of Rice, Maize and Millet for the entire period were all found to be negatively skewed and platykurtic in nature as shown in Table 4.1.

Table 4.1: Descriptive Statistics for the returns of Rice, Maize and Millet

Variable	Mean	CV	Minimum	Maximum	Skewness	Kurtosis
Rice	0.016	3194.660	-4.549	4.575	-0.020	70.080
Maize	0.001	33193.890	-2.885	2.400	-1.470	19.370
Millet	0.020	2069.750	-4.438	4.400	-0.090	46.090

Table 4.2 displays the monthly descriptive statistics for the returns of rice. It was revealed from Table 4.2 that the maximum returns of rice occurred in May, while the minimum value was recorded in April. Also, the coefficient of variation (CV) revealed that the largest variability occurred in August and the highest average returns occurred in May. Again, it was observed that Rice returns were positively skewed for the months of March, May, June, July, September and December while it was negatively skewed for the month of April, August, October and November. Moreover, for the Rice returns, the months of March, May, June, July, September and December were found to be platykurtic while that of April, August, October and November were leptokurtic in nature.

Table 4.2: Monthly descriptive statistics of Rice returns

Month	Mean	CV	Minimum	Maximum	Skewness	Kurtosis
March	0.047	299.110	-0.092	0.344	1.400	0.880
April	-0.309	-396.240	-4.549	0.098	-3.690	13.700
May	0.349	352.580	-0.395	4.575	3.600	13.250
June	0.123	107.810	0.000	0.526	2.310	6.810
July	-0.010	-754.920	-0.120	0.122	0.100	-0.930
August	0.005	3406.740	-0.344	0.271	-0.160	0.780
September	0.079	158.360	-0.154	0.356	0.300	1.170
October	0.010	1476.310	-0.319	0.240	-1.150	1.830
November	-0.038	-460.370	-0.453	0.197	-0.860	1.030
December	-0.049	-243.860	-0.309	0.239	0.360	3.700

Studies on the monthly returns of maize revealed that the maximum and minimum returns occurred in March and October respectively. Also, the highest average returns occurred in December and the coefficient of variation (CV) revealed that the largest variability in the returns occurred in November. Moreover, the Maize returns were found to be positively skewed for the months of May, June, August, March and December but negatively skewed for the months of, April, July, September, October and November. Again, it was observed that the Maize returns were platykurtic in nature for all the months except March, April, July, September and October as shown in Tables 4.3.

Table 4.3: Monthly descriptive statistics of Maize returns

Month	Mean	CV	Minimum	Maximum	Skewness	Kurtosis
March	0.026	-676.360	-2.028	2.400	0.270	12.870
April	0.040	311.870	-0.282	0.258	-1.120	3.000
May	0.104	76.780	0.124	0.271	0.780	-0.310
June	0.054	250.030	-0.219	0.310	0.180	0.540
July	-0.005	-2909.540	-0.384	0.263	-0.820	1.770
August	0.064	838.210	-0.437	1.848	3.180	11.120
September	-0.126	-609.270	-2.065	1.809	-0.010	6.020
October	-0.210	-370.710	-2.885	0.274	-3.580	13.140
November	-0.804	3464.500	-2.202	0.358	-3.510	6.040
December	0.108	99.770	-0.030	0.364	1.290	1.660

The monthly descriptive statistics for the returns of Millet is shown in Table 4.4. It was revealed from Table 4.4 that the highest average returns of Milletoccurred in June. Also, in terms of the minimum and maximum returns of millet, the month of May recorded the minimum returns and the month of June had the maximum returns.The coefficient of variation (CV) revealed that the largest variability in the returns occurred in October. Again, the months of March, April, June, August and December were observed to be positively skewed whiles May, July, September, October and November were observed to be negatively skewed for the Millet returns. Finally, the months of March, April, June, August and December were all platykurtic in nature whereas the rest were leptokurtic for the returns of Millet as shown in Table 4.4.

Table 4.4: Monthly descriptive statistics of Millet returns

Month	Mean	CV	Minimum	Maximum	Skewness	Kurtosis
March	0.017	489.400	-0.107	0.156	0.330	-1.160
April	0.046	172.180	-0.050	0.242	1.510	1.950
May	-0.239	-505.950	-4.438	0.175	-3.730	13.920
June	0.416	280.180	-0.114	4.408	3.580	13.080
July	-0.027	-987.310	-0.826	0.276	-2.240	5.980
August	0.217	294.280	-0.302	3.329	3.160	10.830
September	-0.146	-415.650	-2.172	0.302	-3.250	11.230
October	0.008	1412.470	-0.291	0.199	-1.000	3.420
November	0.025	436.860	-0.218	0.212	-0.340	0.780
December	-0.805	-131.800	-0.262	0.133	0.370	-0.070

Figure 4.1 displays the time series plots for the returns of the cereals. From the plot it was evident that the returns of the cereals fluctuates about a fixed point. This is an indication of stationarity in the returns of the cereals.

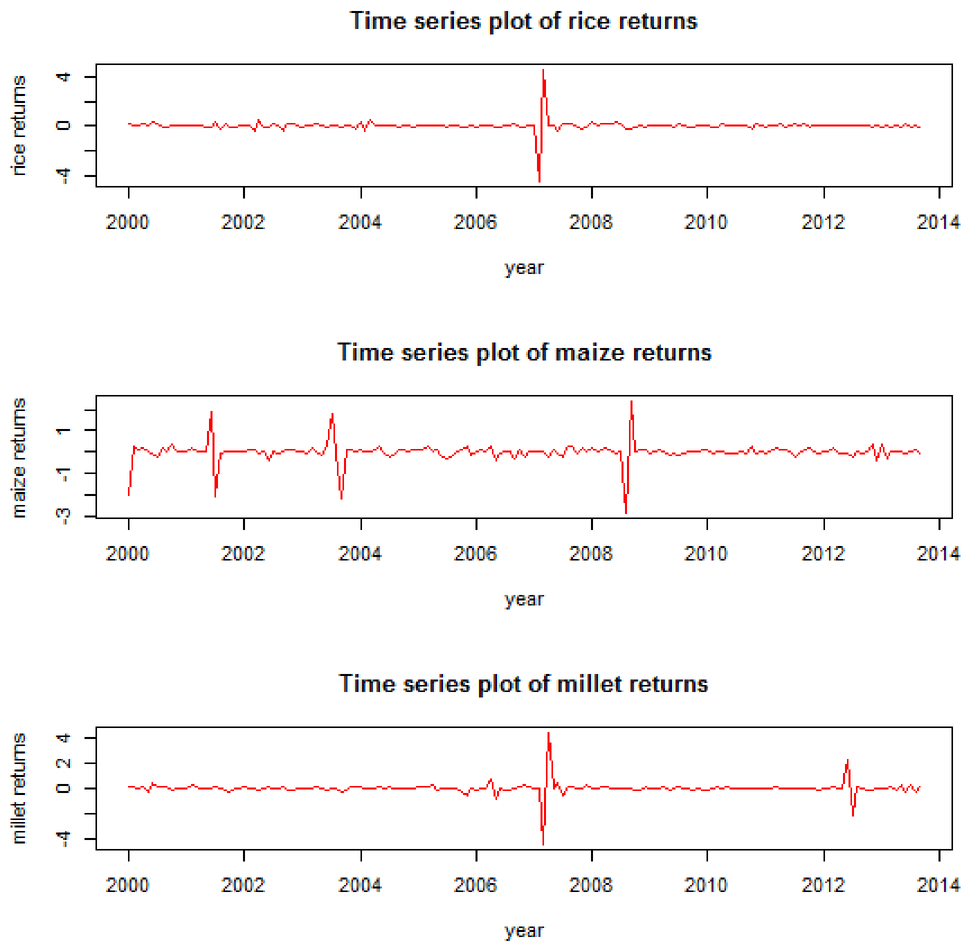

Figure 4.1: Time series plot of the returns of Rice, Maize and Millet

4.2 Further Analysis

4.2.1 Test for Unit Roots

A visual inspection of the ACF plot of the returns of Rice showed a rapid decay in the series suggesting stationarity of the series. The PACF plot also revealed very dominant significant spikes at lags 1, 2 and 3 as shown in Figures 4.2.

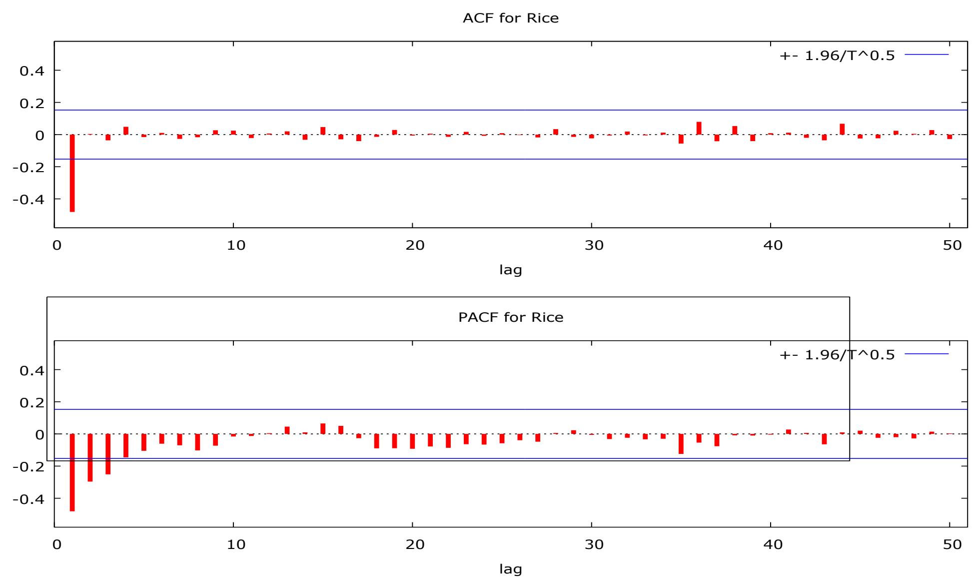

Figure 4.2: ACF and PACF plots of Rice returns

Also, Figure 4.3 shows the ACF and the PACF plots of the returns of maize. A visual observation of these plots indicates that both the ACF and the PACF decays fast and dies out after lag 1 and lag 3 respectively suggesting that the series is stationary.

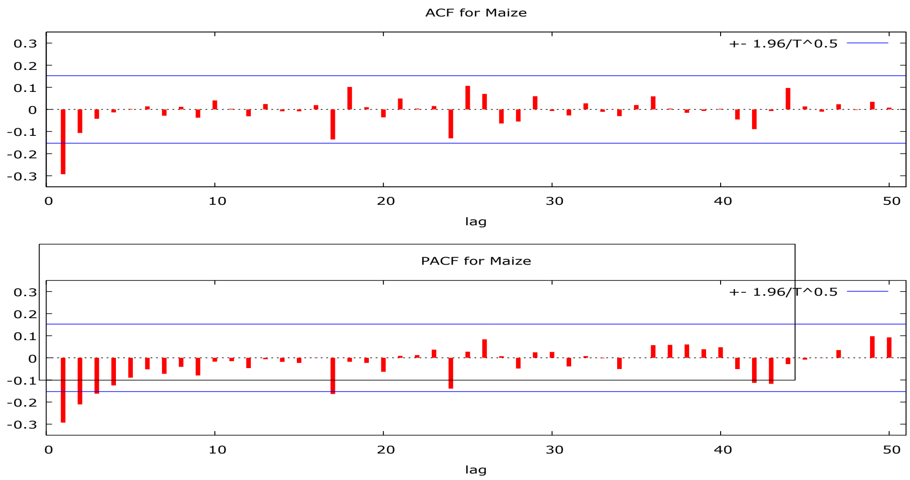

Figure 4.3: ACF and PACF plots of the returns of Maize

The ACF and the PACF of the returns of millet is shown in Figure 4.4 below. The ACF dies out after lag 1 and that of the PACF dies out after lag 4. The rapid decay in both plots indicates that the series is stationary.

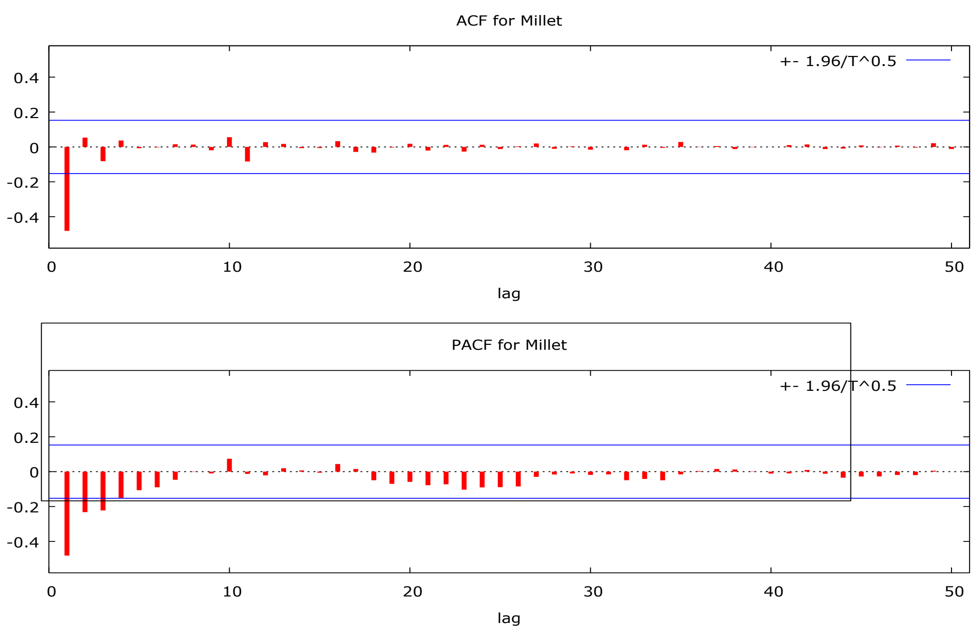

Figure 4.4: ACF and PACF plots of the returns of Millet

The KPSS and ADF test were carried out to further confirm the stationarity of the three series. From Tables 4.5, the KPSS test revealed that the returns of the three cereals were all stationary.

Table 4.5: KPSS test for the returns of Rice, Maize and Millet

Cereal	Test Statistic	Critical Value
Rice	0.026	0.464
Maize	0.051	0.464
Millet	0.032	0.464

Furthermore, an ADF test was performed with only a constant term and a constant with a trend. This affirmed the absence of unit root in the three series since in all cases, the *p*-values were less than the 0.05 level of significance as illustrated in Tables 4.6.

Table 4.6: ADF test for the returns of Rice, Maize and Millet

Cereal	Constant		Constant + Trend	
	Test Statistic	***P*-value**	Test Statistic	***P*-value**
Rice	-10.203	0.000	-10.172	0.000
Maize	-9.713	0.000	-9.693	0.000
Millet	-9.909	0.000	-9.883	0.000

4.2.2 Fitting an ARIMA Model for Maize returns

The various tentative models identified for Maize returns are shown in Table 4.7. Among these possible models ARIMA (1, 0, 1) was chosen as the appropriate model that fit the data well because it has the minimum values of AIC, AICc and BIC compared to other models.

Table 4.7: Tentative ARIMA models for Maize

Model	AIC	BIC	AICc
ARIMA(1, 0, 1)	183.460*	192.770*	183.600*
ARIMA(1, 0, 2)	185.450	197.880	185.700
ARIMA(1, 0, 3)	187.450	202.980	187.830

*: Means best based on the selection criteria

Table 4.8 displays the parameter estimates of the ARIMA (1, 0, 1) model. Observing the *p*-values of the parameters of the model, it can be seen that both the Autoregressive and Moving Average components were highly significant at the 5% level. The model appears to be the best model among the proposed models.

Table 4.8: Estimates of parameters for ARIMA (1, 0, 1)

Variable	Coefficient	Standard Error	*Z*-Statistic	*P*-Value
ϕ	0.334	0.095	3.529	0.000*
θ	-0.890	0.047	-19.120	0.000*

The estimated ARIMA (1, 0, 1) model for the returns of maize is given by;

$$Y_t = 0.334Y_t + \varepsilon_t - 0.890\varepsilon_{t-1} \qquad (4.1)$$

To ensure that, the fitted ARIMA (1, 0, 1) model is adequate, both the Ljung-Box and ARCH-LM tests were performed. The Ljung-Box test as shown in Table 4.9revealed that, the residuals of the model were free from serial correlation at lags 12, 24, 36, and 48 since the *p*-values of test statistic exceeds the 5% significance level at all these lags. This indicates that the mean of the residuals of the model were finite. Further, the ARCH-LM test also shown in Table 4.9 revealed that, the residuals of the model was free from conditional heteroscedasticity, since the ARCH-LM test fails to reject the null hypothesis of no ARCH effect in the residuals of the equation at the 5% significance level. This shows that the residuals of the models were uncorrelated, thus have zero mean and have a constant variance over time; hence are white noise series.

Table 4.9: Ljung-Box Test and ARCH-LM Test of ARIMA (1, 0, 1) Model for Maize returns

		Ljung-Box Test		ARCH-LM Test	
Model	**Lag**	**Test-Statistic**	***P*-Value**	**Test-Statistic**	***P*-Value**
ARIMA(1, 0, 1)	12	1.301	1.000	6.751	0.874
ARIMA(1, 0, 1)	24	9.800	0.995	7.984	0.999
ARIMA(1, 0, 1)	36	16.489	0.998	12.713	1.000
ARIMA(1, 0, 1)	48	26.650	0.995	6.470	1.000

4.2.3 Fitting an ARIMA Model for Rice returns

Taking the lower significant lags of both the ACF and PACF, a number of possible ARIMA models were identified for the returns of Rice. Comparing the AIC, AICc and BIC values of the various candidate models shown in Table 4.10, ARIMA (1, 0, 1) emerged as the best model.

Table 4.10: Tentative ARIMA models for Rice

Model	AIC	BIC	AICc
ARIMA(1, 0 ,1)	185.140*	194.640*	185.290*
ARIMA(1, 0, 2)	187.060	199.490	187.310
ARIMA(1, 0, 3)	188.980	204.510	189.360

*: Means best based on the selection criteria

Table 4.11 displays the parameter estimates of the selected model. From Table 4.11, it was observed that the *p*-value of the Moving Average (MA) component was highly significant but that of the Autoregressive (AR) component was not at the 5% level of significance. Therefore the AR component was dropped and a reduced model, ARIMA (0, 0, 1) was then fitted. The parameter estimate of the reduced model is shown in Table 4.12. The reduced model was therefore considered as the best model since it has the least values of AIC, AICc and BIC compared to that of the full model.

Table 4.11: Estimates of parameters for ARIMA (1, 0, 1) for Rice returns

Variable	Coefficient	Standard Error	*Z*-Statistic	*P*-Value
ϕ	-0.004	0.103	-0.036	0.9713
θ	-0.755	0.069	-11.023	0.000*

Table 4.12: Estimates of parameters for ARIMA (0, 0, 1) for Rice returns

Variable	Coefficient	Standard Error	*Z*-Statistic	*P*-Value
θ	-0.757	0.052	-14.65	0.000
AIC = 183.15	AICc = 183.22	BIC = 189.36		

The estimated ARMA (0, 0, 1) model for the returns of rice is given by;

$$Y_t = \varepsilon_t - 0.757\varepsilon_{t-1} \quad (4.2)$$

To justify the adequacy of the reduced model, both the Ljung-Box test and ARCH-LM test were performed. The Ljung-Box test result as shown in Table 4.13revealed that, the residuals of the reduced model was free from serial correlation at all the lags since the *p*-values of the test statistic exceeds the 5% significance level. This indicates that the mean of the residuals of the reduced model were finite. Also the ARCH-LM test result shown in Table 4.13, failed to reject the null hypothesis of no ARCH effect in the residuals of the reduced model at the 5% significance level.

Table 4.13: Ljung-Box Test and ARCH-LM Test of ARIMA (0, 0, 1) Model for Rice returns

		Ljung-Box Test		ARCH-LM Test	
Model	**Lag**	**Test-Statistic**	***P*-Value**	**Test-Statistic**	***P*-Value**
ARIMA(0, 0, 1)	12	1.993	0.999	0.535	1.000
ARIMA(0, 0, 1)	24	10.844	0.990	0.593	1.000
ARIMA(0, 0, 1)	36	14.428	1.000	0.720	1.000
ARIMA(0, 0, 1)	48	18.558	1.000	1.006	1.000

4.2.4 Fitting an ARIMA Model for Millet returns

The lower significant lags of the ACF and PACF plots of the Millet returns were used to fit tentative ARIMA models shown in Table 4.14. It was observed that ARIMA (1, 0, 1) had the least AIC, AICc and BIC values and hence was considered as the best model.

Table 4.14: Tentative ARIMA models for Millet returns

Model	AIC	BIC	AICc
ARIMA(1 ,0, 1)	223.800*	233.120*	233.950*
ARIMA(1, 0, 2)	225.500	237.920	225.750
ARIMA(1, 0, 3)	227.320	242.850	227.700
ARIMA(1, 0, 4)	227.390	246.020	227.920

*: Means best based on the selection criteria

It was observed from Table 4.15 that the *p*-value of the Moving Average components was highly significant at the 5% level of significance but that of the Autoregressive component was not. Therefore the AR component was dropped and a reduced model, ARIMA (0, 0, 1) was then fitted. The parameter estimate of the reduced model is shown in Table 4.16. The reduced model was therefore considered as the best model since it has the least values of AIC, AICc and BIC compared to that of the full model.

Table 4.15: Estimates of parameters for ARIMA (1, 0, 1) for Millet returns

Variable	Coefficient	Standard Error	*Z*-Statistic	*P*-Value
ϕ	-0.009	0.114	-0.081	0.935
θ	-0.680	0.084	-8.120	0.000*

Table 4.16: Estimates of parameters for ARIMA (0, 0, 1) for Millet returns

Variable	Coefficient	Standard Error	*Z*-Statistic	*P*-Value
θ	-0.685	0.057	-11.990	0.000

AIC = 221.8 AICc = 221.88 BIC = 228.02

The estimated MA (0, 0, 1) model for the returns of millet is given by;

$$Y_t = \varepsilon_t - 0.685\varepsilon_{t-1} \quad (4.3)$$

The Ljung-Box test and ARCH-LM test were carried out to determine the adequacy of the reduced model. The Ljung-Box statistic in Table 4.17 clearly shows that the *p*-values of the test statistic exceed the 5% level of significance for all lag orders which implies that there is no significant departure from white noise for the residuals. Further the ARCH-LM test results in Table 4.17 showed that there is no ARCH effect in the residuals of the selected model.

Table 4.17:Ljung-Box Test and ARCH-LM Test of ARIMA (0, 0, 1) Model for Millet returns

		Ljung-Box Test		ARCH-LM Test	
Model	**Lag**	**Test-Statistic**	***P*-Value**	**Test-Statistic**	***P*-Value**
ARIMA(0, 0, 1)	12	2.920	0.996	1.847	1.000
ARIMA(0, 0, 1)	24	10.628	0.991	1.840	1.000
ARIMA(0, 0, 1)	36	13.431	1.000	1.994	1.000
ARIMA(0, 0, 1)	48	15.570	1.000	2.488	1.000

4.2.5 Fitting an ARIMA-GARCH Model to the Cereals

In this section we developed ARMA (*p*, *q*) – GARCH (*m*, *s*) models for the Cereals. Before developing the models, we identified various GARCH models for the Cereals. Table 4.18

displays the tentative GARCH model for Rice. From the results, GARCH (1, 0) appears to be the best since it has the least AIC and BIC values.

Table 4.18: Tentative GARCH models for Rice

Model	AIC	BIC
GARCH (1, 0)	-1.080*	-1.004*
GARCH (1, 2)	-1.070	-0.976
GARCH (1, 3)	-1.058	-0.945

*: Means best based on the selection criteria

The GARCH (1, 0) model was then concatenated with ARIMA (1, 0, 1) model to form a composite model for the rice returns. The estimates for ARIMA (0, 0, 1)-GARCH (1, 0) model is shown in Table 4.19. Clearly from Table4.19, all the parameters were significant for both the mean equation and the variance equation at the 5% level of significance. Also, the conditional volatility as well as the half-life volatility of the returns of rice were computed in Table 4.19. The ARIMA (0, 0, 1)-GARCH (1, 0) model was diagnosed using Ljung-Box test and the ARCH-LM test at lag 20. Clearly, the tests results indicates that there was no serial correlation in the residuals of the model. The calculated ARCH-LM test statistic revealed that there was no ARCH effect, justifying the adequacy of the ARIMA (0, 0, 1)-GARCH (1, 0) model.

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Table 4.19: Estimate of the parameters of ARMA (0, 1)-GARCH (1, 0) model for rice

Mean Equation				
Parameters	**Estimates**	**Std. Error**	**T-statistic**	**Probability**
μ	0.013	0.005	2.695	0.007
θ	-0.322	0.079	-4.083	0.000
Variance Equation				
Parameters	**Estimates**	**Std. Error**	**T-statistic**	**Probability**
ω	0.016	0.007	0.218	0.027
α_1	0.735	0.488	0.048	0.041
Ljung -Box t-statistic		3.725	ARCH-LM	0.147
Probability		1.000	Probability	1.000
Volatility		0.004	Half-life volatility	3.251

TheARIMA (0, 0, 1) – GARCH (1, 0) model for the returns of rice is given by;

$$r_t = 0.0134 - 0.3221 r_{t-1} + a_t, \quad (4.4)$$

$$\sigma_t^2 = 0.0161 + 0.7345 a_{t-1}^2 \quad (4.5)$$

Also, a number of possible GARCH models were identified for the returns of Millet. Comparing the AIC and BIC values of the various competing models shown in Table 4.20, GARCH (1, 0) emerged as the best model.

Table 4.20: Tentative GARCH models for Millet

Model	AIC	BIC
GARCH (1, 0)	-0.947*	-0.855*
GARCH (1, 2)	-0.941	-0.846
GARCH (1, 3)	-0.930	-0.834

*: Means best based on the selection criteria

A composite model was developed for the ARIMA (0, 0,1) and GARCH (1, 0) model for the millet returns.Table 4.21 displays the parameter estimates of the ARIMA (0, 0, 1)-GARCH (1, 0) model for millet returns. Observing the *p*-values of the parameters of both the mean equation and the variance equation, it can be seen that all the parameters were highly significant at the 5% level. For adequacy of the fitted model, the Ljung-Box and ARCH-LM test were performed on the residuals of the model at lag 20 to check the presence of ARCH effect in the residuals. The results revealed that both the mean equation and the variance equation has no serial correlation in the residuals of the model. The ARCH-LM test statistic also revealed that there was no ARCH effect, affirming the adequacy of the ARMA (0, 1) – GARCH (1, 0) model. The conditional volatility and the half-life volatility of the returns were also calculated as shown in Table 4.21.

Table 4.21: Estimate of the parameters of ARMA (0, 1)-GARCH (1, 0) model for millet

Parameters	**Estimates**	**Std. Error**	**T-statistic**	**Probability**
μ	0.020	0.007	2.643	0.008
θ	-0.064	0.040	-1.614	0.016
Variance Equation				
Parameters	**Estimates**	**Std. Error**	**T-statistic**	**Probability**
ω	0.023	0.011	2.024	0.043
α_1	0.675	0.472	2.119	0.000
Ljung -Box t-statistic		9.178	ARCH-LM	2.790
Probability		0.981	Probability	1.000
Volatility		0.005	Half-life volatility	2.764

TheARIMA (0, 0, 1) – GARCH (1, 0) model for the returns of millet is given by;

$$r_t = 0.0194 - 0.0642 r_{t-1} + a_t, \quad (4.6)$$

$$\sigma_t^2 = 0.0232 + 0.6751 a_{t-1}^2 \quad (4.7)$$

Further, Table 4.22 shows the tentative GARCH model for the returns of Maize. From the results, GARCH (1, 0) has the least AIC and BIC and for that matter it was considered to be the best model.

Table 4.22: Tentative GARCH models for Maize

Model	AIC	BIC
GARCH (1, 0)	-0.417*	-0.342*
GARCH (1, 2)	-0.367	-0.272
GARCH (1, 3)	-0.401	-0.288

*: Means best based on the selection criteria

The GARCH (1, 0) model was then combined with ARIMA (1, 0, 1) model to form a composite model for the maize returns. Table 4.23 shows the parameter estimates of ARIMA (1, 0, 1) – GARCH (1, 0) model for maize returns. From the results all the parameters were significant for both the mean equation and the variance equation at the 5% level of significance.The adequacy of the fitted model was investigated using theLjung-Box and ARCH-LM test at lag 20. The results revealed that the residuals of the ARIMA (1, 0, 1) – GARCH (1, 0) model were free from serial correlation and conditional heteroscedasticity as the *p*-value of the test statistic was insignificant at the 5% significance level. The conditional volatility as well as the half-life volatility of the returns of maize were computed as shown in Table 4.23.

Table 4.23: Estimate of the parameters of ARMA (1, 1)-GARCH (1, 0) for maize returns

Mean Equation				
Parameters	**Estimates**	**Std. Error**	**T-statistic**	**Probability**
μ	0.014	0.006	2.355	0.019
ϕ_1	-0.110	0.009	-1.141	0.000
θ_1	0.110	0.031	3.668	0.000
Variance Equation				
Parameters	**Estimates**	**Std.Error**	**T-statistic**	**Probability**
ω	0.045	0.008	5.313	0.000
α_1	0.198	0.137	1.446	0.000
Ljung -Box t-statistic		3.677	ARCH-LM	0.287
Probability		1.000	Probability	1.000
Volatility		0.003	Half-life volatility	1.428

TheARIMA (1, 0, 1) – GARCH (1, 0) model for the returns of maize is given by;

$$r_t = 0.014 - 0.110 r_{t-1} + 0.110 a_{t-1} + a_t, \quad (4.8)$$

$$\sigma_t^2 = 0.045 + 0.198 a_{t-1}^2 \quad (4.9)$$

4.2.6 Fitting a VAR Model

The dynamic relationship among the returns of rice, maize and millet were studied by fitting Vector Autoregressive (VAR) model. To determine the optimal maximum lag order, *p* to be included in fitting the VAR model, three lag order selection criteria were used. The results shown in Table 4.24, revealed that, the AIC selected lag three (3) but BIC and HQIC selected lag two (2).

Table 4.24: lag Order Selection for Fitting VAR Model

Lag	AIC	BIC	HQIC
1	2.995	3.172	3.067
2	2.759	3.112*	2.902*
3	2.717*	3.247	2.932
4	2.790	3.497	3.077
5	2.856	3.740	3.215
6	2.934	3.995	3.365
7	3.011	4.248	3.514
8	3.083	4.496	3.657
9	3.109	4.700	3.755
10	3.121	4.888	3.838

*: Means best based on model selection criteria

Both VAR (2) and VAR (3) models were fitted to the series, and the Likelihood Ratio Test (LRT) was used to select the best model for investigating the dynamic relationship. From Table 4.25, the significant likelihood ratio test statistic revealed that the VAR (3) was best for modeling the dynamic relationship.

Table 4.25: Model Selection Criteria

Model	AIC	BIC	HQIC
VAR (2)	2.672	3.015*	2.811*
VAR (3)	2.632*	3.146	2.841
Likelihood Ratio Test Statistic = 24.48 *P*-value =0.004**			

*: Means best based on model selection criteria

**: significant at the 5% significance level

A VAR (3) model was then fitted to examine the dynamic relationship among the returns of rice, maize and millet.The results in Table 4.26 revealed that, lag 1and 2 of rice returns were useful predictors of itself at the 5% significance level. Also, lag 3 of millet returns was a useful predictor of rice returns. However, lag 3 of rice and lag 1, 2 and 3of maize were not statistically significant at the 5% significance level in predicting the returns of rice. Lag 1, 2 and 3 of maize returns were statistically useful predictors of itself. While lag 1, 2 and 3 of both rice and millet were not useful predictors of the returns of maize. It was also seen that, lag 1and 2 of millet returns were useful predictors of itself at the 5% significance level. Whereas lag 1, 2 and 3 of rice returns were statistically significant at the 5% significance level in predicting the returns of millet, lag 3 of millet and lag 1, 2 and 3 of maize were not statistically significant at the 5% significance level in predicting the returns of millet.

Table 4.26: Parameter estimates of VAR (3) Model

Equation	Variable	Coefficient	Std. Error	*t*- ratio	*P*-value
Rice	Rice.L1	-0.731	0.079	-9.274	0.000*
	Rice.L2	-0.498	0.146	-3.406	0.001*
	Rice.L3	-2.209	0.139	-1.504	0.135
	Maize.L1	0.056	0.082	0.683	0.496
	Maize.L2	0.069	0.086	0.801	0.424
	Maize.L3	0.098	0.077	1.286	0.201
	Millet.L1	-0.011	0.125	-0.088	0.930
	Millet.L2	-0.141	0.130	-1.088	0.278
	Millet.L3	-0.149	0.071	-2.087	0.039*
Maize	Rice.L1	-0.069	0.077	-0.905	0.367
	Rice.L2	-0.082	0.143	-0.578	0.564
	Rice.L3	-0.063	0.135	-0.466	0.642
	Maize.L1	0.442	0.080	-5.548	0.000*
	Maize.L2	0.314	0.084	-3.762	0.000*
	Maize.L3	0.180	0.075	-0.420	0.017*
	Millet.L1	0.032	0.122	0.261	0.794
	Millet.L2	0.006	0.126	0.046	0.963
	Millet.L3	-0.036	0.069	-0.516	0.607
Millet	Rice.L1	0.926	0.050	18.655	0.000*
	Rice.L2	0.579	0.092	6.285	0.000*
	Rice.L3	0.254	0.088	2.902	0.004*
	Maize.L1	0.053	0.052	1.033	0.303
	Maize.L2	0.013	0.054	-0.242	0.809
	Maize.L3	0.010	0.048	0.206	0.837
	Millet.L1	-0.597	0.079	-7.569	0.000*
	Millet.L2	-0.240	0.082	-2.935	0.003*
	Millet.L3	-0.031	0.045	-0.701	0.485
AIC = 2.632	BIC = 3.146	HQIC = 2.841	Log-Likelihood = -186.148		

*:Means significant at the 5% significance level

The estimated VAR (3) model without an intercept is given by;

$$\begin{bmatrix} Rice_t \\ Maize_t \\ Millet_t \end{bmatrix} = \begin{bmatrix} -0.731 & 0.056 & -0.011 \\ -0.069 & 0.442 & 0.032 \\ 0.926 & 0.053 & -0.597 \end{bmatrix} \begin{bmatrix} Rice_{t-1} \\ Maize_{t-1} \\ Millet_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} -0.498 & 0.069 & -0.141 \\ -0.082 & 0.314 & 0.006 \\ 0.579 & 0.013 & -0.240 \end{bmatrix} \begin{bmatrix} Rice_{t-2} \\ Maize_{t-2} \\ Millet_{t-2} \end{bmatrix}$$

$$+ \begin{bmatrix} -2.209 & 0.098 & -0.149 \\ -0.063 & 0.180 & -0.036 \\ 0.254 & 0.010 & -0.031 \end{bmatrix} \begin{bmatrix} Rice_{t-3} \\ Maize_{t-3} \\ Millet_{t-3} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \tag{4.10}$$

Table 4.27 shows additional information about each individual equation. It was clear that each individual time series model fitted for the returns of rice, maize and millet was statistically significant at the 5% significance level as indicated by the *F*-statistic.

Table 4.27: Test for Significance of the Equations of the VAR (3) Model

Equation	*F*-statistic	*P*-value
Rice	9.885	0.000*
Maize	3.933	0.000*
Millet	63.315	0.000*

*: Means significant at the 5% significance level

The stability of the VAR (3) model was also investigated. From Table 4.28, the results revealed that the parameters of the VAR (3) model were structurally stable over time as all the eigen-values of the parameters have modulus less than one (1). This affirms that the series used in fitting the VAR model were weakly stationary as required in fitting a VAR model.

Table 4.28: VAR (3) Model Stability test

Eigen values	Modulus
0.1600388 + 0.6341251*i*	0.654008
0.1600388 - 0.6341251*i*	0.654008
-0.55128 + 0.2715227*i*	0.614519
-0.55128 - 0.2715227*i*	0.614519
0.3553569 + 0.6125116*i*	0.613542
0.3553568 - 0.6125116*i*	0.613542
-0.2651747 + 0.4600057*i*	0.530964
-0.2651747 – 0.4600057*i*	0.530964
-0.5278848	0.527885

The stability of the VAR (3) model was further investigated using the CUSUM test. From Figure 4.5, the CUSUM plot of the residuals of each model falls within the 95% confidence limit indicating that, their individual residual mean are not significantly different from zero and have constant variance. This affirms that the parameters of each model were structurally stable over time. This clearly shows that, the VAR (3) model fitted provides an adequate representation of the short run relationship among the returns of rice, maize and millet.

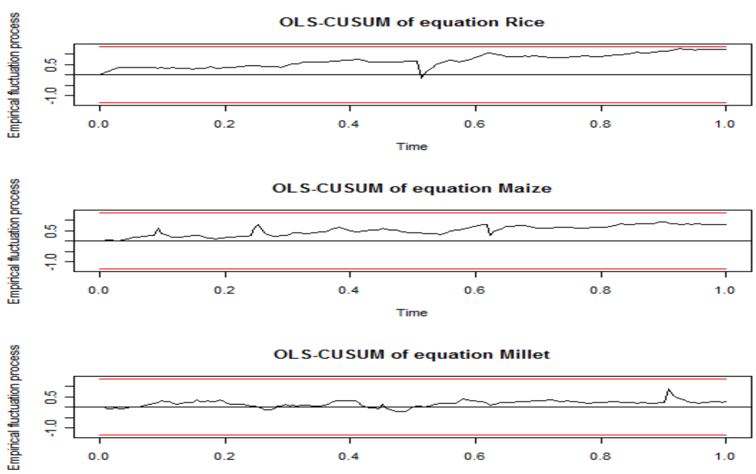

Figure 4.5: CUSUM Plots of the Individual Equations of the VAR (3) Model

To ensure that the fitted VAR (3) model is adequate, both univariate and multivariate model diagnostic tests were performed. The univariateLjung-Box test and ARCH-LM test as shown in Table 4.29 revealed that the individual equations of the VAR (3) model were free from serial correlation and conditional heteroscedasticity at lag 12, 24, 36 and 48 respectively since the *p*-values of all the test statistics were insignificant at the 5% significance level. This implies that the residuals of the models were uncorrelated, thus have zero mean and constant variance over time; hence are white noise series.

Table 4.29: UnivariateLjung-Box Test and ARCH-LM Test

	Ljung Box-Test			ARCH-LM Test	
Equation	**Lag**	**Test-Statistic**	***P*-value**	**Test-Statistic**	*P*-value
Rice	12	7.257	0.840	0.819	1.000
	24	14.856	0.925	0.828	1.000
	36	18.511	0.993	0.936	1.000
	48	21.512	1.000	1.205	1.000
Maize	12	7.914	0.792	5.808	0.925
	24	11.061	0.989	5.514	1.000
	36	19.565	0.988	6.992	1.000
	48	24.322	0.998	5.609	1.000
Millet	12	3.674	0.989	2.625	0.998
	24	7.353	1.000	4.779	1.000
	36	12.886	1.000	6.574	1.000
	48	18.391	1.000	10.528	1.000

The adequacy of the overall VAR (3) model was also investigated using the multivariate Ljung-Box and ARCH-LM test as shown in Table 4.30. The results revealed that the residuals of the

VAR (3) model were free from serial correlation and conditional heteroscedasticity as the *p*-values of the entire test statistic were insignificant at the 5% significance level. This implies that the residuals of the model were uncorrelated and have constant variance.

Table 4.30: Multivariate Ljung-Box Test and ARCH-LM Test of VAR (3) Model

	Ljung Box-Test			**ARCH-LM Test**	
Equation	**Lag**	**Test-Statistic**	***P*-value**	**Test-Statistic**	***P*-value**
	12	55.963	0.985	389.722	0.929
VAR (3)	24	159.345	0.943	828.000	0.806
	36	209.767	1.000	756.000	1.000
	48	33.873	0.999	684.000	1.000

4.2.6.1Causality Analysis

After the diagnostic tests revealed that the VAR (3) model was adequate, the VAR (3) model was used to investigate the Granger causality between the returns of the cereals. Results of the Granger causality test as shown in Table 4.31 revealed that, the returns of millet Granger-cause the returns of rice but the returns of maize, and its linear combination with millet do not Granger-cause the returns of rice. This is seen from an insignificant chi-square statistic obtained for the individual returns of maize as well as their combination at the 5% significance level. This implies that, there is a relationship between the rice returns and millet returns indicating that the maize returns cannot improve the prediction in the returns of rice. The results also showed that, the returns of rice and millet, individually and their linear combination, do not Granger-cause the returns of maize as the chi-square statistic obtained for the individual returns as well as their combination is insignificant at the 5% significance level. This suggest that, there is no relationship between the maize returns and these variables and that returns of these variables cannot improve the prediction of price in the returns of maize. Also, the returns of rice Granger-cause the returns of millet whiles millet returns does not Granger-cause rice returns, indicating a unidirectional causality between these two cereals. The maize returns alone does not Granger-cause millet returns. However, a linear combination of the returns of rice and maize, together Granger-cause millet returns. This implies that, past price of rice and the past prices of the linear combination of the returns of rice and maize can improve future prediction of the price of millet but past price of maize alone cannot improve prediction of rice returns.

Table 4.31: Granger Causality Test

Equations	Excluded	Chi-Squared	Df	*P*-value
Rice	Maize	1.980	3	0.577
	Millet	5.033	3	0.016**
	All	6.965	6	0.324
Maize	Rice	0.987	3	0.804
	Millet	0.374	3	0.945
	All	1.141	6	0.980
Millet	Rice	368.840	3	0.000**
	Maize	1.662	3	0.645
	All	369.090	6	0.000**

**Means significant at 5% significance level.

4.2.6.2 Impulse Response Function (IRF) Analysis

The impulse response function explains how the returns of rice, maize and millet in the model interact with each other following a shock in the VAR (3) model. When the response variable was rice returns,the rice returns showed a negative reaction in the first period and then a positive reaction after the second period until a stable response was obtained after period ten. The maize returns caused a negative shock in the first period, a positive shock in the second period, negative shock in the third period, a positive shock in the fourth period, a negative shock in period five and a positive shock in period seven until a stable response was obtained after the eleventh period. The rice returns reacted positively to a shock in the millet returns in the first period followed with a negative response in the second period and a positive shock from the third period to the sixth period and then a stable response for the rest of the periods.

For the returns of maize as a response variable, a shock in the returns of rice cause a positive reaction in the returns of maize in the first period, a negative reaction in second period, a positive reaction in the third period, a negative reaction in the fourth period, a positive reaction in the fifth period and a negative reaction from the sixth period to the ninth period and then a stable response after period eleven. In the first period, the maize returns showed a positive response to a shock in its own values, the second period showed a negative response up to the fifth period and then a stable response to its own shocks onwards. Also, the returns of maize showed a negative reaction at the first, fourth and seventh period and then a positive reaction at the third, sixth and tenth period and stabilizes after period twelfth to a shock in the returns of millet.

When the response variable was millet returns, a shock in the rice returns caused a negative reaction of millet returns in the first two periods, a positive reaction in the third period, a negative reaction in the fifth period, a positive reaction in the sixth period until a stable response was obtained after the seventh period. Maize returns showed a positive reaction in the first period, a negative response at the second period, a positive reaction between the third and fourth periods, a negative reaction at the fifth period and a continues positive reaction from period six to period fifteen with a stable response for the rest of the periods. Millet returns showed a negative response to a shock in its own values at the first period and both negative and positive reactions between period two and period seven and then a stable response to its own shocks onwards.

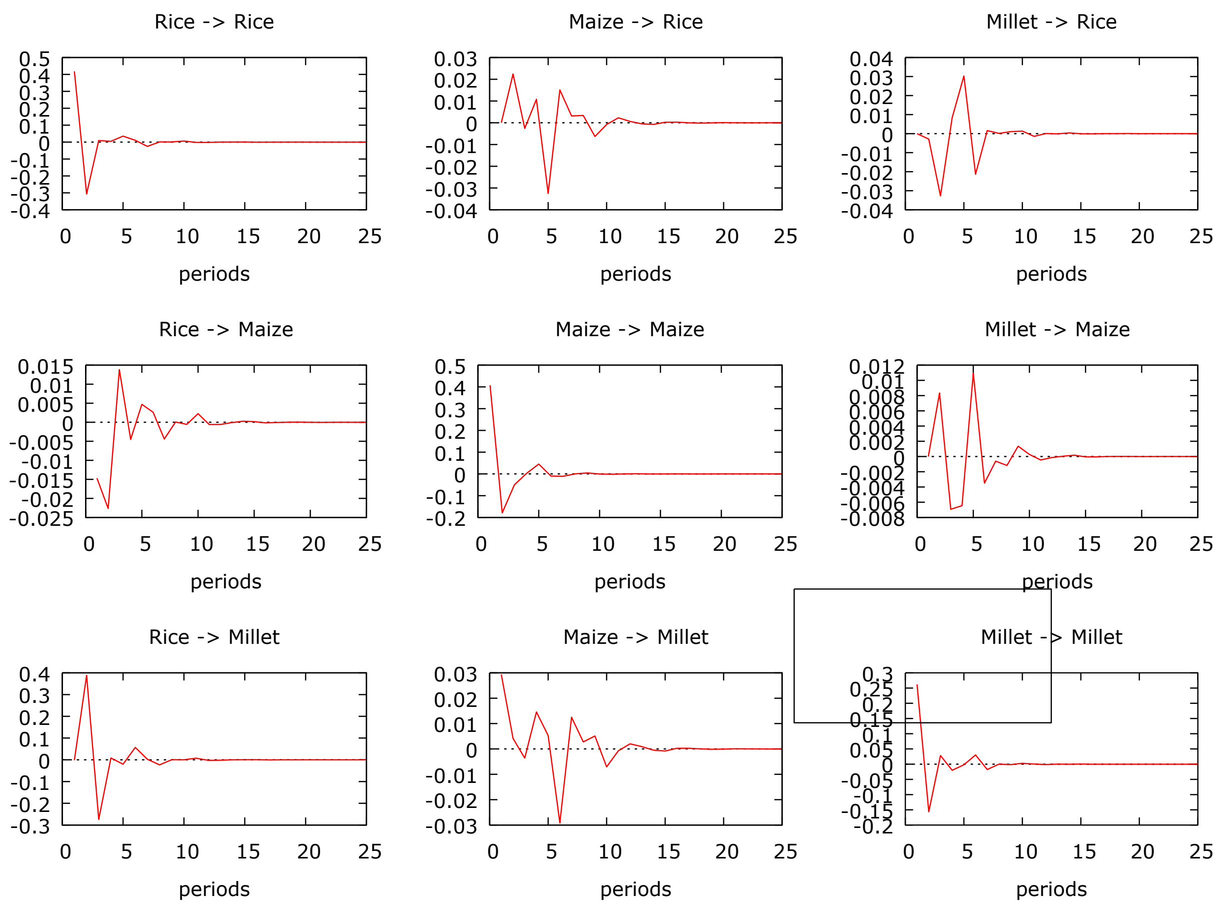

Figure 4.6: Plot of Impulse Response Analysis

4.2.6.3 Forecast Error Variance Decomposition (FEVD) Analysis

The variance decomposition was used to determine the proportion of forecast error variance of a returns that is explained by itself and by the other endogenous returns in the study. Table 4.32 gives the variance error decomposition of the returns of rice. It was realised that, much of the forecast variance in the returns of rice have been explained by innovations in the returns of rice itself. For instance, in the tenth period, about 98.4% of the error variance in the returns of rice is explained by innovations in the returns of rice, whiles only about 0.2%, and 1.4% of its error variance have been explained by the returns of maize and millet respectively. This affirms the results of the VAR (3) model and Granger causality that the returns of maize and millet do not contribute significantly to improve future predictions of the returns of rice.

Table 4.32: Forecast Error Variance Decomposition for rice

Period	Std. Error	Rice	Maize	Millet
1	0.419	100.000	0.000	0.000
2	0.519	99.810	0.186	0.003
3	0.521	99.414	0.188	0.398
4	0.521	99.345	0.231	0.424
5	0.524	98.633	0.614	0.753
6	0.525	98.389	0.695	0.916
7	0.525	98.389	0.697	0.915
8	0.525	98.385	0.701	0.915
9	0.525	98.370	0.715	0.915
10	0.525	98.369	0.215	1.415

From the forecast error variance decomposition of the returns of maize in Table 4.33, the returns of maize contributes most in forecasting the uncertainty of the maize. At period ten, about 99.4% of the error variance in the returns of maize have been explained by innovations in the returns of maize, whiles 0.5% and 0.1% of the error variance explained by innovations in the returns of rice and millet respectively. The results of maize variance decomposition also agrees with views of the Granger causality test and the estimated VAR (3) model which revealed that, the past prices of rice and millet are not the most influencing determinant of the price of maize.

Table 4.33: Forecast Error Variance Decomposition for maize

Period	Std. Error	Rice	Maize	Millet
1	0.408	0.130	99.870	0.000
2	0.446	0.366	99.599	0.035
3	0.449	0.456	99.486	0.058
4	0.449	0.466	99.455	0.079
5	0.452	0.471	99.392	0.137
6	0.452	0.475	99.383	0.143
7	0.452	0.484	99.373	0.143
8	0.452	0.484	99.373	0.144
9	0.452	0.484	99.372	0.145
10	0.452	0.486	99.369	0.145

Finally, Table 4.34 displays the forecast error variance decomposition of the millet. From the results, rice returns contributes most in forecasting the uncertainty of millet. For instance, at period ten, about 70.2% of the error variance in the returns of the millet have been explained by innovations in the returns of rice, whiles about 29.2% of the error variance explained by innovations in the millet itself and 0.7% by the maize returns. The results of the millet variance decomposition also agrees with views of the Granger causality test and the estimated VAR (3) model which revealed that, the past price of rice is the most influencing determinant of the price of millet. It also confirms the unidirectional relationship between the returns of rice and that of millet.

Table 4.34: Forecast Error Variance Decomposition for millet

Period	Std. Error	Rice	Maize	Millet
1	0.264	0.006	1.240	98.754
2	0.495	61.571	0.359	38.070
3	0.566	70.404	0.279	29.3173
4	0.567	70.277	0.344	29.380
5	0.567	70.308	0.352	29.340
6	0.572	70.221	0.607	29.172
7	0.572	70.123	0.654	29.223
8	0.572	70.170	0.655	29.175
9	0.572	70.164	0.663	29.173
10	0.572	70.152	0.678	29.170

4.3 Discussion of Results

The descriptive statistics of the returns of rice, maize and millet revealed that the returns of the cereals were platykurtic in nature compared to the normal distribution.The platykurtic nature of the returns of these cereals indicates that the returns are widely distributed around the mean, hence low volatilities in these returns over time. The monthly distribution of the returnsof rice and millet clearly shows that the highest returns were recorded in the month of May and June respectively.This is usually the beginning of the rainy season in the region where the prices of the major staples are usually high.It was also shown that the maximum returns of maize was recorded in March. This could be attributed to the fact that among the cereals, maize is the most consumed staple within the region. Most of the local dishes are prepared using maize. Example of these local dishes are; ‘Twouzaafi’, ‘Banku’, ‘Kenkey’, ‘pouridge’ just to mention a few. Also, majority of the poultry farmers depend on maize as a source of feed for their birds. Again, traders usually buy it and store with the intention of making profit in the near future. Due to the huge demand for this cereal, its usually shoots up even at the middle of the dry season.

The unit root test conducted, showed that all the ACF of the cereals decays fast indicating that each of the series is in statistical equilibrium. The KPSS test and the ADF test were performed to further confirm the stationarity of each series. The results of both the KPSS test and the ADF test revealed that the mean, variance and covariance of each of the series were constant.

Forecasting is an essential component in time series analysis; hence it was important to forecast the returns of the three cereals. This will serve as a guiding tool to the Ministry of Food and Agriculture, individuals and farmers in taking decisions regarding the future prices of these cereals. Three forecasting models were developed for the returns of rice, maize and millet to help in the prediction of the returns. These were ARIMA (0, 0, 1) for the returns of rice, ARIMA (1, 0, 1) for the returns of maize and ARIMA (0, 0, 1) for the returns of millet. The diagnostic checks carried out on these models proved that the models were all adequate for predicting the returns of rice, maize and millet in the Northern region of Ghana.

Also, a number of ARIMA (*p, d, q*)-GARCH (*m, s*) models were fitted to the cereals to help in calculating the volatility of the cereals. These were ARMA (0, 1)-GARCH (1, 0) for the returns of rice, ARIMA (1, 0, 1)-GARCH (1, 0) for the returns of maize and ARIMA (0, 0, 1)-GARCH (1, 0) for the returns of millet. Clearly, the diagnostic checks carried out on these models proved that the models were all adequate for predicting the volatility of the returns of rice, maize and millet in the Northern region of Ghana. The volatility of the returns of rice, maize and millet were 0.003, 0.005 and 0.004 respectively. This means that the returns of millet was more volatile compared to that of rice and maize. The half-life volatility of the returns of the cereals were also examined. The half-life volatility measures the time required for the volatility to move half way back towards its unconditional mean (Engle and Patton, 2001). The estimated half-life volatility for the returns of rice and millet was approximately three months and that of maize was approximately one month. This means that any shocks to this volatility takes approximately 3 months for rice and millet returns and a month for the returns of maize to return half-way back without any further shocks to this volatility.

VAR model was fitted to the series. Three model selection criteria were used to determine the appropriate order of the VAR model. From the results, the AIC selected lag 3 while the BIC and HQIC selected lag 2. Both VAR (3) and VAR (2) model were fitted to the datasets and the Likelihood Ratio Test used to select the best model. The results of the LRT revealed that the VAR (3) model was the better choice to the datasets, and the parameters of the VAR (3) model were estimated. From the results, it was observed that a dynamic relationship exist between the returns of rice, maize and millet. The lag 1, 2 and 3 of rice were useful in predicting millet returns. Lag 1 and 2 of rice were useful predictors of itself. It was also seen from the results that lag 3 of millet was a useful predictor of the returns of rice. The significance of the lag values of both the returns of rice and millet in predicting each other affirms the existence of a dynamic relationship between the two cereals. Again, maize was only a useful predictor of itself but not statistically significant at the 5% significance level in predicting the returns of rice and millet at all the lags. This implies that there is a weak relationship between maize returns and the returns of rice and millet. In order to make inference with the model, a number of diagnostic techniques were performed on the model to determine the adequacy of the model. Both the univariate and multivariate Ljung-Box test revealed that the model was free from serial correlation whiles the univariate and multivariate ARCH-LM test also revealed that the model was free from conditional heteroscedasticity. The stability of the model parameters were also investigated using the eigenvalues and CUSUM test. Both test revealed that the model parameters were structurally stable, indicating that the residuals of the individual models in the VAR (3) model have zero mean and constant variance.

The Granger causality test was employed to examine the nature of the relationship between the returns of rice, maize and millet. The results revealed that the returns of rice granger cause the returns of millet and vice versa, confirming the bilateral relationship between the rice returns and millet returns. This implies that if the previous values of rice returns are known, then future values of millet returns can be predicted and vice versa.

Furthermore, an impulse response analysis was employed to examine how the cereals in the VAR (3) model will interact following a shock in the VAR (3) model. The results revealed that there was a relationship among the returns of these cereals. The FEVD further confirms the existence of a relationship among the returns of the cereals. For instance in the tenth period, about 29.2% of the forecast uncertainty in millet returns was explained by innovations in the returns of millet, whiles about 70.2% of the forecast uncertainties in millet returns have been explained by the returns of rice. Again, in the tenth period there turns of rice explained about 98.4% of the forecast uncertainty in rice returns whiles the returns of millet explained about 1.4% of the forecast uncertainty in rice returns.

4.4 Conclusion

This chapter dealt with the analysis and discussion of the results obtained.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter dealt with the conclusion and recommendations of the study.

5.1 Conclusion

In this study, themonthly returns of rice, maize and millet from March 2000 to December 2013 were studied. Before fitting models to determine the volatility of these cereals, the monthly characteristics of each series were investigated. The investigation revealed that the three series were all stationary. The three models developed for modelling the volatility of the three cereals were all adequate based on the diagnostic techniques used in this study. The results also revealed that the returns of millet was more volatile than that of rice and maize.

A multivariate time series model was also fitted to examine the dynamic relationship between the returns of these three cereals. The diagnostic tests revealed that the VAR (3) model fitted to investigate the dynamic relationship between the returns of rice, maize and millet was adequate. The VAR (3) model was then used to make inference about the relationship between the returns of these cereals. The Granger causality test revealed a bilateral relationship between the returns of rice and that of millet whiles the returns of maize is independent of the returns of rice and millet. The impulse response analysis and forecast error variance decomposition analysis both affirm that there exist a dynamic relationship between the returns of the three cereals. There is however the need for continuous monitoring of the performance of these models, review of the returns of these cereals and necessary adjustments are required to make the use of these models more realistic.

5.2 Recommendations

Based on the outcomes of this research, it is recommended that;

i. The Ministry of Food and Agriculture as well the Savannah Accelerated Development Authority should pay attention to these major cereals to ensure that food is secured since they are the major staples in the region. The government should also support and encourage farmers to take advantage of the irrigation dams to embark on dry season farming of these cereals.

ii. It is also recommended that further studies should be carried out on the returns of these cereals over time to appropriately model the relationship between the returns.

iii. MoFA and community based NGO's should educate farmers on modern storage and processing technologies to prolong the shelf life and add value to these cereals, in order to stablise prices.

iv. Government should subsidies farm inputs such as fertilizer to farmers to encourage them to produce more of these cereals locally. It should also expand the agriculture credit system to give farmers more credit and loans to expand their farms.

v. Government should also absorb oil prices from the world market to ease the burden on farmers in order to increase their production.

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