UNIVERSITY FOR DEVELOPMENT STUDIES, TAMALE

MARKOV CHAIN ANALYSES OF PRODUCTION OUTPUTS OF SELECTED CEREALS IN GHANA

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THESIS SUBMITTED TO THE DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS, FACULTY OF AGRIBUSINESS AND APPLIED ECONOMICS, UNIVERSITY FOR DEVELOPMENT STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF PHILOSOPHY DEGREE IN AGRICULTURAL ECONOMICS

JUNE, 2019

DECLARATION

Student

I hereby declare that, this thesis is the result of my own work and that it has previously not been submitted for the award of any other degree in this University or elsewhere.

Candidate's Signature:…………………… Date:…………………

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Supervisors

We hereby declare that the preparation and presentation of the thesis was duly supervised in accordance with the guidelines on supervision of thesis laid down by the University for Development Studies.

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ABSTRACT

The increasing demand for maize and rice in Ghana has attracted the attention of government and other stakeholders. For instance, the Government of Ghana's Planting for Food and Jobs initiative aims to increase the outputs of maize and rice by some 30% and 49% respectively in order to make the country food secure. Although previous studies have researched on the production outputs of these cereals in Ghana, such studies have mostly been based on parametric methods. To improve understanding of the output trends for agricultural policy directives, this study modelled the annual production outputs of maize and rice in Ghana using the non-parametric (Markov chain process) approach. The annual production outputs of maize and rice from 1960 to 2018 sourced from index mundi were classified into three states (low, stable and high). The result showed that, at equilibrium both maize and rice have 50.4% and 63.2% chances respectively of being in a high state. It was found that it would take approximately 2 years for the production outputs of these cereals to return to high state after leaving it and the expected length the production output of maize and rice stays in the high state were approximately 2 years and 3 years respectively. Also, the average production output of maize and rice in the long-run were 837,786 metric tons and 126,027 metric tons respectively. Five years forecast of these cereals revealed that maize production will be increasing whiles rice production will exhibit a fluctuating pattern. The study concludes that the production of maize and rice in Ghana is characterised by high variability index, which implies that the production outputs of these cereals are largely unstable.

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DEDICATION

This work is dedicated to my husband, children and the entire family.

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LIST OF ACRONYMS

ADF	Augmented-Dickey Fuller
AIC	Akaike Information Criterion
ARIMA	Autoregressive Integrated Moving Average
BIC	Bayesian Information Criterion
GDP	Gross Domestic Product
HQIC	Hannan-Quinn Information Criterion
MAD	Mean Absolute Deviation
MAPE	Mean Absolute Percentage Error
MSD	Mean Squared Deviation
MT	Metric Tons
PC	Production Cycle
PFJ	Planting for Food and Jobs
PP	Phillips-Perron
VAR	Vector Autoregressive
VEC	Vector Error Correction

CHAPTER ONE

INTRODUCTION

1.0 Background of the Study

In developing countries, economic development heavily depends on the agricultural sector. The sector contributes 33.5% to employment and 19.7% to the Gross Domestic Product (GDP) of Ghana (World Bank 2019; GSS, 2019). Thus, governments of developing countries are implementing policies to boost agricultural production in order to provide food for their people and curb increases in food prices due to low production. Experiences from the green revolution in Asia and Latin America revealed that agriculture is imperative in early development process and is a relevant force for poverty reduction worldwide (Christiaensen *et al*., 2010).

The need to increase food production is an important step to ensuring food security. Thus, there is the need to plant more food crops for human and livestock consumption. Among the crops grown worldwide for human and livestock consumption, cereal crops form the majority. They are grown in large quantities and serve as a source of energy food than any other type of crop. Cereals are staple crops of great socio-economic importance, which do not only provide food but also play relevant role in agricultural gross domestic product (Celik, 2016). The demand for agricultural products in particular cereals have increased due to the rise in the world's population. Cereal crops such as wheat, rice and maize provide about two-third of all the energy in human diet (Kotra and Shaik, 2016).

Recently, the government of Ghana, through its initiative to increase food production in the country, launched the Planting for Food and Jobs (PFJ) program with the aim of encouraging farmers to increase their productivity. The program support farmers to easily access both inputs and output markets as well as create avenue for employment in the agricultural value chain. One of the primary goals of the program is to increase cereal production and establish 750,000 direct and indirect employments in the country. It is estimated that under this initiative, maize production is expected to increase by 30%, rice by 49%, soya bean by 25% and sorghum by 28% (Mabe *et al*., 2018). The government's initiative to increase the production of cereals in Ghana is not only imperative for food security but also open new avenues to increase income generation. This would help to reduce prices of cereals in the country and hence lower food inflation as well as reduce the importation of these cereals into the country.

In order to achieve the goal of the PFJ or any government intervention in the cereal sector, knowledge about the production dynamics of the cereal crops in the country is necessary. This would play a key role in decision making and provide the necessary information regarding the production of the cereals to stakeholders. Hence, this study seeks to investigate and forecast the production dynamics of maize and rice in Ghana using Markov chain analysis.

1.1 Problem Statement

Over the years, food insecurity has been a major concern for many developing countries especially in the Sub-Saharan Africa. This can partly be attributed to the over dependence on rain-fed agriculture and low rate of farmers' adoption of modern farming technologies among others. Although Ghana has been classified as generally food secured, there are some pockets of food insecurity which is distributed across all the regions of the country (Darfour and Rosentrater, 2016).

The major cereal crops in Ghana are maize, rice, millet and sorghum. The performances of these crops over the years have not been impressive. According to the deficit/surplus analysis of major food crops in Ghana, the country could not meet its demand for maize, rice and millet from domestic production from 2011 to 2017 (Table 1.1).

Table 1.1: Deficit/surplus of selected staple crops (MT)

Crops	**Deficit/Surplus (MT)**						
	2011	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
aize	90,359	230,070	45,784	21,069	-61,007	-68,537	87,012
ice (Milled)	-354,205	-540,280	-503,875	-479,515	-608,602	-577,977	-580,300
illet	35,762	26,775	2,814	-88	-5,904	-3,197	-1,508
orghum	125,497	114,035	91,210	90,115	85,692	58,214	96,677
assava	6,169,042	6,221,456	7,151,811	7,431,688	7,681,605	8,130,414	8,969,099
am	1,930,112	2,072,344	2,355,910	2,314,737	2,266,545	2,413,745	2,982,777
ocoyam	240,663	170,353	141,199	152,304	93,602	144,209	159,663
antain	969,579	825,877	882,737	924,762	937,416	999,816	1,181,490
roundnuts	133,121	116,630	50,773	59,137	32,723	43,543	46,516
owpea	79,115	60,215	38,194	35,759	30,004	33,879	34,966
oyabean	90,134	77,133	65,012	66,163	63,880	65,117	87,004

Source: MoFA, 2017

According to MoFA (2017), regardless of the attempt to initiate early-maturing crop diversification and institute sustainable water management practices, cereal production is still futile to resolute climatic conditions. Maize and rice are the mainstay of the diet of many Ghanaians as well as poultry, livestock and the brewing industries. Increase in these cereals will improve Ghana's food security. The total amount of maize and rice consumed in 2017 are 1,925 Million metric tonnes and 1,100 Million metric tonnes respectively. These increased in 2018 by 13.16% and 11.61% respectively (USDA, 2018; Indexmundi, 2018). As a result, Ghana imports about 70% and 15% of rice and maize respectively to supplement consumption which contributes greatly to the country's increasing imports bills. Considering the contribution of cereal production to the agricultural sector and the economy of Ghana as a whole, this study finds it necessary to study the production dynamics of these cereals. The results of this analysis will serve as a guide to stakeholders and help them monitor and plan for policies relating to the production of maize and rice.

Although a number of researchers such as Nasiru and Sarpong (2012a and b); and Luguterah *et al*. (2013) have studied and predicted the production of some cereals in Ghana with parametric models like linear, non-linear or exponential functional forms respectively. However, these parametric methods which have been mostly used in agricultural economics literature enforces a specific distributional relationship or econometric assumptions such as normality, homogeneity of the variance and independent errors about the data in modelling. These econometric assumptions sometimes may not hold for a given dataset, making inferences with such parametric methods may be incorrect due to specification error (Featherstone and Kastens, 2000).

This study use a nonparametric technique (Markov Chain analysis) which only considers the characteristics of past behaviour of the dataset to model the production of maize and rice in Ghana. The merit of using nonparametric forecasting method is that it is free from specification error of the distributional assumptions and functional relationship about the data. Thus, nonparametric methods could result in more accurate and robust economic model (Featherstone and Kastens, 2000).

1.2 Research Questions

From the argument above, this research seeks to answer the following research questions.

i. What are the current trends of maize and rice production in Ghana?

ii. What are the various states of maize and rice production in Ghana?

iii. How long does it take for production of maize and rice to be in a particular state?

iv. How long does it take for the production of maize and rice to go back to its previous state from the current state?

v. How do the future production values of maize and rice for the various states look?

1.3 General Objective

The main objective of the study is to model the production outputs of maize and rice in Ghana as a Markov chain process to examine and explain directional movements of the production for agricultural policy directives.

1.4 Specific Objectives

The study seeks to achieve the following specific objectives.

i. Construct the production states of the rice and maize and determine their corresponding state probabilities.

ii. Estimate the expected length of each of the production states and the production cycle of rice and maize.

iii. Determine the expected recurrent times for each production state of rice and maize.

iv. To predict future production values for the various states within an equilibrium condition for rice and maize.

1.5 Justification of the Study

Forecasting of agricultural production outputs are expected to be relevant to farmers, governments and the agribusiness industries. Cereals production plays important role in determining a nation's food security. Thus, governments over the years have become the principal users of agricultural forecasts. Due to the increasing demand for maize and rice as well as the increasing importation of these cereals, government require forecasts of the production dynamics of these cereals to aid in formulation and execution of policies that provide technical and market support for the agricultural sector. Also, it will enable the country to ascertain quantities of cereals needed to fill the deficit in production for the subsequent agricultural years. Additionally, government's advance knowledge about cereals production trend contributes to the measurement and distribution of the national income.

Cereals production forecasts are imperative in informing stakeholders in the forecasting of prices of agricultural inputs, storage facilities, credit availability and levels of households food security. Food processing industries, poultry industries and others in the marketing chain require forecasts in cereals to aid in their value chain.

This study used a nonparametric model (Markov Chain) compared to other studies which relied on a parametric approach. The results of this approach are considered to be more robust than the former.

The results of this study can be used to forecast food availability, understand food price variations, plan for import and /or export of maize and rice, and monitor domestic stock in relation with population size. They may provide vital information to stakeholders when planning for the development of food processing industries and revamping the poultry industries in the country.

1.6 Organization of the Thesis

The thesis is organized into five chapters. Chapter one presents the background of the study, research questions, problem statement, objectives and justification of the study. Chapter two presents relevant literature on forecasting techniques that have been used over the years to model and forecast agricultural production outputs. In chapter three, the methodology of the study is presented. The results and discussion of the study are given in chapter four. Finally, the conclusions and recommendations of the study are also presented in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter presents literature relevant to the study. It is subdivided into five sections as: maize production in Ghana, rice production in Ghana, some policies implemented by past governments, relevance of forecasting in agriculture and forecasting techniques related to agriculture.

2.1 Maize Production in Ghana

Maize is one of the most essential cereal crops grown for food security purposes in Ghana. In 2018, a total of 2.263 million metric tonnes of maize was produced from 970,290 hectares which gave an average of 2.05 metric tonnes per hectare (MoFA, 2017; Indexmundi, 2018). In terms of maize production, Ghana (2.263 million metric tonnes) is ranked 36th in the world whiles United States of America (381.78 million metric tonnes) and China (254 million metric tonnes) are ranked first and second respectively (Indexmundi, 2018). Ghana is ranked 10th in Africa and 7th in Sub-Saharan Africa (Indexmundi, 2018).

In Ghana, majority of the maize producers are small holder farmers (averagely farming 0.43 hectares). Maize grows very well in the tropical rainforest, semi deciduous forest and guinea savanna areas of the country due to the rainfall pattern which enhances good yield. With regards to the area of cultivation of maize, the Brong Ahafo Region recorded the largest (224,436 hectares), followed by Eastern Region (162,500 hectares), Northern Region (143,240 hectares) and the Greater Accra Region was the least (3,276 hectares) in

2017. The Eastern Region had the highest average yield (2.84 metric tonnes per hectare), followed by Central Region (2.43 metric tonnes per hectare) and the Greater Accra Region had the least average yield (1.32 metric tonnes per hectare) in 2017. Also, in terms of production, Eastern Region (461, 513 metric tonnes) was the highest followed by the Brong Ahafo Region (440, 594 metric tonnes) and the Greater Accra Region (4,318 metric tonnes) recorded the least production for the year 2017. Table 2.1 displays the area cropped, average yield and production of maize for ten regions of Ghana for the year 2017 (MoFA, 2018).

Table 2.1: Area, average yield and production output of maize

Region	**Area Cropped**	**Average Yield**	**Production**
	(Hectares)	**(MT/Hectare)**	**(MT)**
Western	48,977	1.50	73,288
Central	81,885	2.43	199,193
Greater Accra	3,276	1.32	4,318
Volta	58,820	1.98	116,444
Eastern	162,500	2.84	461,513
Ashanti	127,410	1.71	217,391
Brong Ahafo	224,436	1.96	440,594
Northern	143,240	1.72	246,934
Upper East	52,688	1.82	95,677
Upper West	67,059	1.95	130,446

Source: MoFA, 2018 (Unpublished)

The socio-economic importance of maize cannot be overlooked as it cuts across different spheres of life (Oyewo, 2011). Due to its richness in vitamins, minerals, carbohydrates, fats, oils and proteins, it is mainly used domestically for preparing different kinds of food and drinks for human consumption as well as animal feeds. Humans and animals domestic consumption yearly are about 1,900,000 metric tonnes and 300,000 metric tonnes respectively (Indexmundi, 2018). Maize also plays a vital role in the Ghanaian industry for the production of starch, alcohol, glues, candles, chewing gums, beverages among others.

2.2 Rice Production in Ghana

Rice is one of the most important staple cereals per consumption in Ghana besides maize. With the total area harvest of 255,108 hectares, Ghana produced 510,000 metric tonnes of rice in the year 2018 and resulted in an average yield of 3.06 metric tonnes per hectare (MoFA, 2017; Indexmundi, 2018). This placed Ghana 42nd in the world with China as first with 146 million metric tonnes and India as second with 107.5 million metric tonnes. However, Ghana is ranked 13th in Africa and 3rd in Sub-Saharan Africa. The country's rice domestic consumption in 2018 was estimated as 1.25 million metric tonnes and imports 800, 000 metric tonnes annually (Indexmundi, 2018). This heavy dependence on imports can be attributed to domestic consumption exceeding production, urbanization and ease of cooking among others (Kranjac-Berisavljevic, 2000). Over the years, the need to increase the production output of rice in Ghana has been the primary goal of stakeholders. However, the production area has increased considerably with some increased output variations recorded at the same time for the last three years (MoFA, 2017).

A large proportion of both production area and output of rice comes from the Volta, Northern and Upper East Regions. The Volta, Northern and Upper East Regions have total cropped area of 55,001 hectares, 75,980 hectares and 43,350 hectares respectively. The Volta Region produced the largest production output of 270,676 metric tonnes, followed by the Northern Region (182,352 metric tonnes) and the Upper East Region (115,037 metric tonnes) for the year 2017. The region with the least production output in the year 2017 is the Central Region (3,287 metric tonnes). The Greater Accra Region had the highest average yield (7.01 metric tonnes per hectare) and the Western Region has the least average yield (1.34 metric tonnes per hectare) for the year 2017 (MoFA, 2018). Table 2.2 shows the area cropped, average yield and the production outputs for the ten regions of Ghana for the year 2017.

Table 2.2: Area, average yield and production output of rice

Region	**Area Cropped (Hectares)**	**Average Yield (MT/Hectares)**	**Production (MT)**
Western	25,885	1.34	34,797
Central	1,907	1.72	3,287
Greater Accra	3,472	7.01	24,337
Volta	55,001	4.92	270,676
Eastern	9,736	3.9	37,966
Ashanti	13,837	2.73	37,755
Brong Ahafo	4,678	1.45	6,763
Northern	75,980	2.4	182,352
Upper East	43,350	2.65	115,037
Upper West	5,497	1.57	8,636

Source: MoFA, 2018 (Unpublished)

2.3 Some Policies Implemented by Past Governments

Agriculture plays an important role in the economic development of Sub- Saharan African countries including Ghana. Governments over the years have introduced a variety of measures to improve productivity in this sector. Although some were successful, others were not. Some of these policies are; the establishment of co-operative and state farm during Dr. Kwame Nkrumah's regime. Twenty six state farms were established in 1962 which targeted crops like rubber, oil palm, cotton, coconuts, fibre plants, livestock and cocoa. District and regional tractor stations were also set up to help farmers rent them for plough as well as prices of cutlasses were subsidized for farmers. Food marketing board was also established during this regime to help regulate the markets and prices of farms produce (Jotie, 2019).

The National Liberation Council (NLC) in 1966 implemented programmes to rapidly increase the production of foods and crops like the construction of feeder roads, water conservation and irrigation dams, extension advices and agricultural credits (Jotie, 2019). The National Redemption Council (NRC) in 1972 also launched the big agricultural plan as known as "Operation Feed Yourself" which was intended to boost food production in some selected areas of the country. Under this policy, educational institutions had specific production target to achieve (Jotie, 2019).

During the Provisional National Defence Council (PNDC) regime in 1982, the government prioritized the production of maize, rice and fish farming among others with the aim of achieving a green revolution country. This was achieved by mobilizing communities to establish farms (Jotie, 2019).

During the 1990-2000, then government launched the Medium-Term Agricultural Development Programme (MTADP) with the aim of promoting market-oriented agriculture growth and reforming institutions in order to increase investments. This programme covered from 1991 to 2000 with an expected annual growth rate of 4%. Along the MTADP, programmes like the Agricultural Diversification Project, National Agricultural Research Project, National Agricultural Extension Project, Agricultural Sector Adjustment Credit, Accelerated Agricultural Growth and Development Strategy (AAGDS) and Agricultural Sector Investment Project (ASIP) among others were also initiated. In the year 1995, Ghana's long-term national development policy framework "Vision 2020" was also launched. This framework had a medium-term programme called the Coordinated Programme of Economic and Social Development Policies (CPESDP) which started from 1996 to 2000 with the focus of increasing agricultural productivity through science and technology as well as ensuring environmental quality. The Agricultural sector experienced an average growth rate of 3.9% from the 1996 to 2000 which was higher than the expected target of 3.8% (Dzanku and Aidam, 2013).

In the fourth republic (2001-2008), the first New Patriotic Party (NPP) government implemented the Fertilizer Subsidy program and the free cocoa spraying exercise in cocoa growing areas to reduce some of the financial burden of farmers. The government initiated the Ghana Poverty Reduction Strategy (GPRS I) which was to cover from 2003 to 2005. The aim was to increase productivity in agricultural sector by modernizing agriculture and improve infrastructure. GPRS II was also introduced at the end of 2005 to look at non-traditional export crops like pineapple, mangoes, cashew nuts and vegetables in order to reduce the Ghana's over dependence on especially Cocoa. Alongside GPRS I & II, Food and Agricultural Sector Development Policy (FASDEP I & II) were also introduced. These policies had similar objectives like the MTADP and GPRS I& II. Medium-Term Agricultural Sector Investment Plan (METASIP) was designed to help reduce poverty by

2015 and achieve a 6% annual sectorial growth rate set by the Comprehensive Africa Agriculture Development Programme (CAADP). Although, they achieved a recommendable agricultural growth rate of 5.4% but was still below the 6% CAADP target (Dzanku and Aidam, 2013).

From 2009- 2011, the government of the National Democratic Congress party (NDC) continued with their predecessor NPP's programs like the FASDEP II,METESIP and GSGDA and also constructed more roads to these cocoa growing areas as well as subsidized the prices of tractors (Dzanku and Aidam, 2013).

From the year 2016, the replica of the operation feed yourself "Planting for Food and Jobs" program was launched with the aim of making Ghana food sufficient, internationally competitive, reduce import bills and reduce the unemployment rate in the country. The program seeks to provide improved certified seeds to farmers, supply subsidized fertilizers, provide dedicated extension services using information communication technology and improve the marketing chain among others.

In conclusion, some of these policies have not been able to give the agricultural sector its befitting face lift possibly because of political leadership and the lack of involvement of the beneficiaries of these programmes in decision making among others (Jotie, 2019).

2.4 Relevance of Forecasting in Agriculture

Decision-making process in agriculture often need reliable crop response model. Specialist in agricultural management requires simple and accurate estimation technique to predict crop yields in the planning process (Paswan and Begum, 2013). Economic forecasting in agriculture has some common traits with business and macroeconomic forecasting.

Agriculture crop forecasting is a process of computing the most probable yield and production of a crop on the basis of known facts at the time of making the prediction. Crop production is very vital for agriculture related organizations, consultants, producers and other important stakeholders in the sector. Hence, accurate and timely forecast is necessary for marketing, storage and transportation decisions (Mishra *et al*., 2016).

Prediction of agricultural production and prices are meant to be useful for farmers, government and agribusiness industries. Because of the vital position of food production in a nation's food security, governments have become both main suppliers and users of agricultural forecasts. They require internal forecasts to implement policies that give technical and market support to the agricultural sector (Allen, 1994).

Agricultural predictions are useful to stakeholders in the sector. For instance, farmers may rarely make predictions, but they constitute the largest group of users. They need to decide on production and marketing decisions that may have financial consequences many months in the future. Processors of food and fibre, and others in the marketing chain require forecast to help in their purchasing and storing decisions. They too would probably be interested in the price forecasts but would be able to make greater use of forecasts of production in their decisions than farmers (Allen, 1994).

2.5 Forecasting Techniques related to Agriculture

Barrage of forecasting techniques have been employed in the literature over the past decades for predicting future values of phenomena. These techniques can be classified as parametric or non-parametric in nature with their own advantages and disadvantages. Even though the parametric approach has often been used mostly in agricultural economics literature, it involves econometric modelling and assumes a specific distributional relationship like normality, homogeneity of the variance and independent errors about the data to model. The major drawback of these techniques is the possibility of specification error which could result in incorrect inference. On the other hand, the non-parametric method makes no distributional assumptions and no functional relationship about the underlying distribution of the data. Hence free from specification error and could result in more accurate and robust economic models (Featherstone and Kastens, 2000). In this section, literature on models that have been used to forecast time series data over the years have been reviewed. The section is divided into two subsections and it includes: forecasting with parametric models and forecasting with non-parametric models.

2.5.1 Forecasting with parametric methods

A number of time series models have been employed to model and forecast time series observations. Awaab *et al*. (2018) predicted the production pattern of paddy rice in Ghana using annual data from 1961 to 2015. They identified ARIMA (0, 1, 2) model as the best for predicting future paddy rice production pattern. Five years forecast with their proposed ARIMA (0, 1, 2) model showed an increasing pattern in the paddy rice production.

Chaudhury and Jones (2014) studied crop yield in Ghana using models like simple exponential smoothing, double exponential smoothing, Damped-trend linear exponential smoothing and autoregression moving average. Data from 1992 to 2008 on maize yield in Bole, Damango, Salaga, Tamale and Yendi were used in the study. Among the models used, the results of the study showed that the autoregression moving average model provided the best fit for the maize yield data.

Hamjah (2014) used ARIMA model to predict the rice production of Bangladesh in Aus, Boro and Aman seasons for the country employing data from 1972 to 2006. The study revealed that the best model for Aus and Aman production was ARIMA (2, 1, 2), and for Boro production was ARIMA (1, 1, 3). The findings indicated that ARIMA model provides good forecasting for only short term analysis.

Thirunavukkarasu and Rajarathinam (2014) modelled and predicted future production of milled rice in India with time series observation from 1960 to 2014. They used the ARIMA and Brown exponential smoothing models for predicting the future behavioural pattern of the milled rice. They assessed the appropriateness of the models used for the prediction using R-square, the root mean square error, mean absolute percent error and Bayesian information criterion. They concluded that all the forecasting models were good since their R^2 values were above 95%.

Rahman *et al*. (2013) estimated the pattern of growth and also developed ARIMA model for forecasting production of chickpea, pigeon pea, and field pea pulse in Bangladesh using data from 1967 to 2011. They found that the appropriate models for the productions were ARIMA (1, 1, 3), ARIMA (1, 1, 1) and ARIMA (0, 1, 0) for field pea pulse, pigeon pea and chickpea respectively.

Muhammad and Abdullah (2013) used data from 1970 to 2010 to study the yearly paddy rice production of Kelantan using several time series models. They employed the Brown's double exponential smoothing, Brown's linear exponential smoothing, Winter's multiplicative exponential smoothing, damped trend and random walk models to identify the best model for forecasting the production data. The integrated forecast model of Holt's linear and damped trend exponential smoothing model was identified as the appropriate model. The forecast results from the selected model showed an increasing trend of total paddy rice production for the next five years.

Kumar and Mahto (2013) employed moving average, simple exponential smoothing and least squares methods to forecast juice production in India using data from 2000 to 2011. They evaluated the performance of the models by comparing their mean absolute percent error, mean absolute deviation and mean square error. The result of their study indicated that the least square method was more accurate than the other methods.

Verma *et al.* (2013) used the ARIMA model to study the production of sugarcane crop from 1960 to 2010 in Karnal, Kurukshetra and Ambala districts of Haryana. The result indicated that ARIMA (0, 1, 1) model were best for Kurukshetra district for the pre-harvest crop yield prediction.

Tripathi *et al.* (2013) investigated the trend in productivity, production and area of pearl millet in India using data from 1950 to 2010. They predicted the production and cultivated area of pearl millet with ARIMA model. The findings of their study showed that the compound trend models best describe trends in productivity, production and cultivated area of pearl millet. The area, production and yield of the pearl millet were estimated to be 8.67million hectares, 9.15million tonnes and 1083.12kg per hectares respectively in 2020.

Mishra *et al*. (2013) investigated pattern in production, productivity and area of onion in India with data from 1978 to 2009 and also forecasted the cultivated area and production using ARIMA. The findings of the study showed that the parametric cubic trend model was the best. The study forecasted that onion production would be 23.02 million tonnes in the year 2020.

Sudha *et al*. (2013) used data from 1970 to 2009 to investigate the growth trends of area, production and productivity of maize in India using linear, logarithmic, inverse, quadratic, cubic, compound, power and exponential growth functions. They concluded that the cubic function model was the best among all the candidate models used and hence the most appropriate for forecasting maize area, production and productivity. The result revealed that maize production was expected to reach 962.22 thousand tonnes and productivity increase to 17.39 tonnes per hectares in 2015.

Biswas and Bhattacharya (2013) used data from 1947 to 2008 and ARIMA model to predict the production and area of rice in West Bengal. The result of the study revealed that ARIMA (2, 1, 1) and ARIMA (2, 1, 3) were the appropriate models for predicting the rice production and area respectively.

Luguterah *et al*. (2013) used the vector autoregressive model of order one to investigate the dynamic relation between the growth rates of the production of three major cereals in Ghana from 1960 to 2012. The results of their study revealed that there was a bilateral relation between the growth rates of rice and millet. They further concluded that the growth rate in maize production cannot be used in predicting the growth rate of millet and rice.

Nasiru and Sarpong (2012a) predicted the production and consumption of maize in Ghana from 1960 to 2010 using the autoregressive integrated moving average (ARIMA) model. The results of their study indicated that ARIMA (2, 1, 1) and ARIMA (1, 1, 0) were suitable for predicting the production and consumption respectively. They indicated in their work that, the forecasted values for both the production and consumption of maize revealed an increasing pattern.

Nasiru and Sarpong (2012b) studied and forecasted milled rice production in Ghana from 1960 to 2010 using the Box-Jenkins methodology. Their results indicated that ARIMA (2, 1, 0) model was good for forecasting milled rice production in the country and ten years prediction with their model revealed an upward trend in production.

Harris *et al*. (2012) used ARIMA to model the pattern of Ghana's annual coffee production using data from 1990 to 2011. The results of their study revealed that in general, the total coffee production exhibit an upward and downward movement trend. They identified ARIMA (0, 3, 1) model as the best for predicting the annual coffee production and their forecast figures showed that, the annual coffee production will decrease continuously for the next five years.

Sivapathasundaram and Bogahawatte (2012) studied the present, future and past production patterns of paddy in Sri Lanka using data from 1952 to 2011 and established a model to determine long term pattern as well as predict changes in future production of paddy for the three leading years. They identified ARIMA (2, 1, 0) as the most appropriate model. They found that the overall three years production (2011 to 2013) had an increasing trend of 4.07, 4.12 and 4.22 million metric tonnes respectively.

Arunachalam and Balakrishnan (2012) used data from 1950 to 2010 to investigate the patterns in production, productivity and area of wheat crop in India using different nonlinear models. The findings revealed that none of the nonlinear model was appropriate for describing the trend in the area data. Patterns in productivity as well as of wheat crop production were best described using a sinusoidal model as indicated by the results of the study. The study also revealed that production and productivity showed an increasing trend with area of cultivation playing a major role.

Zakari and Ying (2012) used data from 1970 to 2011 to predict Niger grain production and harvested areas for sorghum and millet using ARIMA model. The results of their study revealed that the total grain and total harvested area would be 12677.9 thousand tonnes and 21317.4 thousand hectares in 2030 respectively. They concluded that total production of sorghum and millet would be 1574.8 tonnes and 4503 thousand tonnes respectively during the period.

In Nigeria, Badmus and Ariyo (2011) adopted the ARIMA model to predict the production of maize and its cultivated area with data from 1970 to 2006. The results of their study revealed that the forecast for the maize production for the year 2020 would be about 9952.72 thousand tonnes. They found that the area of the maize would be 9229.74 thousand hectares during the same period.

Awal and Siddique (2011) used data from 1972 to 2008 to study the growth in trend of rice production in Bangladesh for different seasons and developed ARIMA models for Aus, Aman and Boro rice production. Their result showed that ARIMA (4, 1, 4), ARIMA (2, 1, 1) and ARIMA (2, 2, 3) were the appropriate models for predicting Aus, Aman and Boro rice production respectively. They found that the calculated Aus, Aman and Boro rice production for the year 2008-09 were 1,735 thousand metric ton, 11,056 thousand metric ton and 13,538 thousand metric ton respectively. They concluded that the total rice production (Aus, Aman and Boro) would be 1,591 thousand metric ton, 11,612 thousand metric ton and 15,541 thousand metric ton in the year 2012-13 if the current trend is maintained.

Sharma (2010) modelled food grain production using data from 1950 to 2004 in India using ARIMA model. Using the Akaike information and Bayesian information criteria, ARIMA (0, 1, 1) was chosen as the appropriate model for the food grain production data. Six years forecast estimated food grain production to increase to 224.74Mt in 2010.

Dhakre and Sharma (2010) employed data from 1979 to 2005 to investigate the pattern of growth in production, productivity and area of maize in Nagaland, India using index numbers and compound growth rates by fitting exponential function. They concluded that among production, area and productivity of maize, production has the highest instability.

Nehru and Rajaram (2009) forecasted production of wheat in India using ARIMA approach. ARIMA (1, 1, 0) were discovered as the appropriate model based on the Akaike information and Bayesian information criteria. The forecasted estimates revealed that there will be steady increase from 2008-2009 to 2014-2015 in the wheat production values.

Sandika and Dushani (2009) used data from 1977 to 2008 to study the growth performance of the rice sector in Sri Lanka in order to identify the suitable model to forecast future trend of the sector. The cubic model was identified to be appropriate for predicting the total production and productivity of rice in the study.

From the literature reviewed on parametric models for forecasting, it was revealed that these methods require the data to fulfil econometric assumptions such as stationarity and normality among others. When the data does not meet these requirements, these parametric techniques are not suitable for forecasting.

2.5.2 Forecasting with Non-Parametric Models

This subsection presents researches based on non-parametric approach. Tettey *et al*. (2017) adopted two-state Markov chain to analyse the patterns of rainfall in five different geographical locations of Ghana using data from 1980 to 2010. They realized that the rainy or dry season trend using the monthly steady state vectors coincides with the monthly rainfall pattern.

Thirunavukkarasu (2015) modelled the production of barley crop using three-state, six-state and twelve-state Markov chains in India. The study made use of yearly data on production of barely crop from 1960 to 2013. The findings indicated that with an increase in the number of states, the state probability converges to the value that does not depend on the initial state. The study concluded that there is possibility of increase in barley production in future.

Jasinthan *et al.* (2015) also used both two-state and three-state Markov chains to model daily vegetable price movements in Jaffna in Sri Lanka. Data on daily market prices of vegetables from 2009 to 2013 were used in the study. They recommended that although they predicted only movement in price pattern, the same Markov framework can be employed to predict amount of movements in prices.

Matsumura *et al.* (2015) compared the fit of artificial neural network with multiple linear regression in forecasting maize yield in Jilin, China. A time series data from 1961 to 2004 was used. The cross-validation results indicated that the neural network model was better than the multiple linear regression.

Raheem *et al*. (2015) employed Markov chain with three states to examine pattern and distribution of rainfall in Uyo metropolis in Nigeria from 1995 to 2009. The findings of the study revealed that agricultural (planting) in Uyo metropolis should begin from pre-monsoon when the cycle of the weather is quite longer and the peak should start during the monsoon time when the likelihood of rainfall is the greatest.

Taru (2014) used Markov chain to model the probability distribution of cereal grain prices in North-eastern Nigeria. Time series data on cereal prices of maize, rice and sorghum from 2001 to 2010 were used for the analysis. The result of the study revealed that maize farmers would experience unfavourable price, rice farmers will receive better price for rice and sorghum farmers will not receive better prices in the long-run.

Zhu *et al.* (2012) developed a forecasting Markov model for prices of vegetable in China using vegetable prices for the year 2011. The results of the study revealed that the Markov chain model is a feasible technique for forecasting the future trends of the vegetable prices. They concluded that the commodity prices would fluctuate around the commodity value in the free market economy.

Ghodsi *et al.* (2012) used artificial neural network to predict wheat production in Iran using data from 1988 to 2006. Rainfall, guaranteed purchasing price, area under cultivation, subsidy, insured area, inventory, import, population and value added of agricultural crop were the input variables for the artificial neural network model. The findings of the study revealed that the artificial neural network model is suitable for predicting wheat production.

Rajarathinam and Vinoth (2011) used data from 1949 to 2009 to analyse the trends, rate of growth and jump points in production, area and productivity of mustard crop grown in Gujurat, India. The study employed non-parametric and parametric regression models. The findings revealed that the non-parametric regression model best describes the data.

Zhang *et al*. (2011) used Markov technique to study patterns in wetland trends in arid Yinchuan plain in China using wetland distributions from 1991 to 1999. They predicted the changing pattern in different types of wetland and the distribution area with the model. They found that the Markov model can forecast the changes in the wetland in future based on the current wetland management standard conditions, and serve as guidance for wetland system restoration and sustainable environmental development.

Garg and Singh (2010) employed Markov chain technique with three states to investigate the pattern of rainfall in India using data from 1961 to 2002. The findings of the study revealed that the likelihood of the week being dry, wet or rainy does not depend on the initial weather condition.

Okyere-Boakye (2009) used Markov technique to investigate the behaviour of rainfall and sunshine in three towns in Ghana, Kumasi, Tamale and Accra using data from 2003 to 2007. The result of his study revealed that Kumasi has the highest probability of rainfall followed by Tamale and then Accra. Also, it was shown that Tamale has the highest probability of duration of sunshine followed by Accra and then Kumasi.

Paulo and Pereira (2007) explained that modelling with Markov chain is vital in comprehending the random characteristics of drought and rainfall by analysing probabilities for each severity class times in southern Portugal. They used data from 1931 to 1999 and realized that the approach can be satisfactorily employed to predict movements among drought severity categories up to three months.

Okpachu (2006) used Markov chain technique in the study of the efficiency measurement in soybean marketing in Benue state, Nigeria. Sales of soybean for two successive years, 2003 and 2004 were classified into three size categories (physical states), and used for the analysis. He estimated the transitional probability matrix for the quantities of soybean sold by the soybean traders and computed the proportion of soybean traders expected to be in the different size categories at equilibrium. He classified the quantities sales into three size categories (physical states) as S_1 (1-1000 bags), S_2 (1001-2000 bags) and S_3 (greater than 2000 bags). The result showed that at equilibrium, 42% of the respondent will sell from 1-1000 bags, 47% will sell 1001-2000 bags while 11% will sell above 2000 bags. The study revealed that in the long-run, the market power will be concentrated in the hands of 89% of the respondents who sold 1-2000 bags.

Chandran and Prajneshu (2005) used non-parametric regression and autocorrelated errors to investigate the trend of India’s fish production employing data from 1971 to 2000. They compared the non-parametric regression with the ARIMA approach and the results indicated that the non-parametric regression performs better than the ARIMA.

Kottegoda *et al*. (2004) modelled daily rainfall data from 1996 to 1998 from central and northern part of Italy using Markov chain. The results of the study revealed that the Markov model fitted the daily rainfall observations in Italy well.

Steinemann (2003) employed Markov chain with six states of severity to categorise probabilities for drought state and period in drought state using data from 1931 to 2001 in United State of America. The results of the study were used to develop measures for early-activating of the drought plans at the basin scale.

Onu (2000) analysed the structure and performance of cotton marketing in Northern Nigeria using data from 1996 to 1998. He classified the purchasers on the basis of the quantity of cotton that was bought by each buyer and the categories were S_1 (0-20 tonnes), S_2 (21-80 tonnes), S_3 (81-101 tonnes) and S_4 (above 171 tonnes). The result revealed that 2% of the firms buy less than 20 tonnes, 12.49% would buy between 21-80 tonnes, 41.98% would buy between 81-101 tonnes and 43.53% would purchase above 170 tonnes. This measure of structural development reveals that the cotton market in Northern Nigeria during the period of study tended towards high concentration.

Lohani and Loganathan (1997) employed Markov chain to develop early warning system for management of drought in two climatic areas of Virginia in the United State of America. Annual rainfall data from 1895 to 1990 were used and the results indicated that the Markov chain can forecast the drought conditions very well.

Matis *et al*. (1989) employed Markov chain to predict cotton yield from pre-harvest crop data. They investigated the appropriateness of the Markov approach in forecasting crop yield. They separately analysed cotton yield observations from two important states, California and Texas using four year period (1981-1984). They estimated the probability distribution using Markov chain. They estimated four transition matrices each for California and Texas. They estimated the predicted yield distribution by multiplying consecutive transition matrices. For the yield forecast they employed the averages of the predicted yield distribution. The findings of the study showed that the Markov chain model predicts the cotton yield with minimal error for both California and Texas.

Gabriel and Neumann (1962) studied the behaviour of daily rainfall in Tel Aviv, Israel using Markov chain. Daily data on rainfall pattern in Tel Aviv for 27 rainy seasons from the year 1923 to 1950 were used in the study. They realized in their study that the Markov model fitted the daily rainfall observations well.

Judge and Swanson (1962) discussed the idea of Markov process and indicated the importance of it in agricultural economics where detailed time-ordered observations are available. They gave examples on the potential and past size distribution of a sample firms in central Illinois producing hog using data from 83 hog-producing firms from 1946 to 1958. They indicated that the Markov approach has been recommended as a means of classifying economic data and of predicting path of some variables in economics.

The literature reviewed on non-parametric models for forecasting revealed that, these methods do not require the data to fulfil econometric assumptions such as stationarity and normality among others. The robustness of these models makes inference more accurate.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter presents data and methods used to achieve aims of the study. The chapter is divided into four main sections, namely: data and source, theoretical framework for Markov chain, empirical framework for Markov chain and trend analysis.

3.1 Data and Source

The study used secondary data on production output of maize and rice grown in Ghana. The data on the production output of the cereals from the period 1960 to 2018 grown in Ghana were obtained from the official website (www.indexmundi.com) maintained by the United State of America department of agriculture and analysed using STATA 14, MINTAB 16 as well as R version 3.3.1 software.

3.2 Data Analyses

The study employed both descriptive and inferential statistics using Stata14 to analyse the data. Descriptive statistics such as means, skewness, kurtosis, coefficient of variation, minimum and maximum values were used to study the underlying basic characteristics of the production outputs.

The pattern of the production outputs of the maize and rice were investigated using linear, quadratic and exponential trend models. The best trend model for describing the pattern of the production outputs was identified using measures of accuracies such as the mean absolute percent error, mean square deviation and mean absolute error. The specific objectives of the study were achieved using Markov chain technique since the knowledge of today is required to predict tomorrow.

3.2 Theoretical Framework for Markov Chain

In this section, theories that are relevant for the implementation of the study are presented. The section is divided into eight subsections and includes: stochastic process, discrete-time Markov chain, state transition probability matrix, n-step state transition probability, limiting state probabilities, state transition diagram, classification of states and sojourn time.

3.2.1 Stochastic Process

A stochastic process is a collection of random variables, $\{X(t): t \in T\}$, where t represents time. That is, for any time t in the set T, a random number $X(t)$ is obtained. The stochastic process is referred to as a first order Markov process if the future knowledge about the process depends on only the current state and is not affected or altered with the additional information about the past states of the process. Hence, for any $t_0 < t_1 < ... < t_n$, the conditional cumulative distribution function of $X(t_n)$ for any given values of $X(t_0), X(t_1), ..., X(t_{n-1})$ depends only on $X(t_{n-1})$. That is

$$P\left[X(t_n) \le x_n \mid X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}, ..., X(t_0) = x_0\right]$$
$$= P\left[X(t_n) \le x_n \mid X(t_{n-1}) = x_{n-1}\right]. \qquad (3.1)$$

The Markov property of Equation (3.1) implies that the future state is independent of the past given the present state of the process (Ibe, 2009; 2014).

The random variable $X(t)$ referred to as stochastic process represents random events that have been observed in a particular state at a given time for a given subject. For instance, if we are interested in the production output of a given cereal over a period of time, it is worth noting that the output may be low, stable (no change) or high. Thus, the stochastic variable $X(t)$ will represent the production output of that cereal at time t. The possible random variables $X(t)$ of a stochastic process that are assumed are collected in a state space $\bar{S} = \{\bar{s}_1, \bar{s}_2, ..., \bar{s}_k\}$.

3.2.2 Discrete-Time Markov Chain

A discrete-time stochastic process $\{X_t, t = 0,1,2,...\}$, is referred to as a Markov chain if for all $i, j, t, ..., m$, we have:

$$P[X_t = j \mid X_{t-1} = i, X_{t-2} = \alpha, ..., X_0 = \theta] = P[X_t = j \mid X_{t-1} = i] = p_{ijt}. \qquad (3.2)$$

The quantity p_{ijt} denotes the state transition probability, which is the conditional probability that the process will transit to state j at time t given that it was in state i at the time $t-1$. Any Markov chain that follows this preceding rule is referred to as non-homogeneous Markov chain. However, in this study only homogeneous Markov chains are considered. That is, Markov chains in which $p_{ijt} = p_{ij}$. This implies that homogeneous Markov chains are independent of the time unit. Thus,

$$P[X_t = j \mid X_{t-1} = i, X_{t-2} = \alpha, ..., X_0 = \theta] = P[X_t = j \mid X_{t-1} = i] = p_{ij}. \qquad (3.3)$$

Hence, homogeneous state transition probability p_{ij} satisfies the following conditions:

i. $0 \le p_{ij} \le 1$

ii. $\sum_j p_{ij} = 1,\ i = 1,2,\dots,$

which implies that the states are mutually exclusive and collectively exhaustive (Ibe, 2009; 2014; Taylor and Karlin, 1998).

3.2.3 State Transition Probability Matrix

Given k states Markov chain, the transition probability matrix $\boldsymbol{P}$ is a $k \times k$ matrix such that p_{ij} is the entry in the i^{th} row and j^{th} column and is given as;

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}. \tag{3.4}$$

The transition probability matrix is a stochastic matrix with row i, $\sum_{j=1}^{k} p_{ij} = 1.$

3.2.4 The n-Step State Transition Probability

Suppose $p_{ij}(n)$ represents the conditional probability that the system will be in state j after exactly n movements, given that it is presently in state i. That is,

$$\begin{cases} p_{ij}(n) = P\left[X_{m+n} = j \mid X_m = i\right] \\ p_{ij}(0) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \\ p_{ij}(1) = p_{ij} \end{cases}. \tag{3.5}$$

Example: a two-step transition probability $p_{ij}(2)$, which is given as

$$p_{ij}(2) = P\left[X_{m+2} = j \mid X_m = i\right]. \tag{3.6}$$

Suppose $m = 0$, then

$$
\begin{aligned}
p_{ij}(2) &= P\left[X_2 = j \mid X_0 = i\right] \\
&= \sum_k P\left[X_2 = j, X_1 = k \mid X_0 = i\right] \\
&= \sum_k p_{kj} p_{ik} \\
&= \sum_k p_{ik} p_{kj}.
\end{aligned} \tag{3.7}
$$

The last equation implies that, the probability of transitioning from state i to state j is the probability that we first move from state i to an intermediate state k, then transit from state k to state j; the summation is taken over all possible intermediate states k (Ibe, 2009; 2014).

The $p_{ij}(n)$, is the ij^{th} entry in the matrix $\boldsymbol{P}^n$. That is,

$$
\boldsymbol{P}^n = \begin{bmatrix} p_{11}(n) & p_{12}(n) & \cdots & p_{1k}(n) \\ p_{21}(n) & p_{22}(n) & \cdots & p_{2k}(n) \\ \vdots & \vdots & \vdots & \vdots \\ p_{k1}(n) & p_{k2}(n) & \cdots & p_{kk}(n) \end{bmatrix}, \tag{3.8}
$$

where k, represents the number of states. For a one-step transition probability matrix, n is equal to one. Alternatively, the n-step transition probability matrix can be obtained by multiplying the transition probability matrix by itself n times.

3.2.5 Limiting-State Probabilities

The limiting-state probabilities for a class of Markov chains in which the limit exist is defined as follows:

$$
\lim_{n\to\infty} P\left[X(n) = j\right] = \pi_j, \quad n = 1, 2, \ldots, k. \tag{3.9}
$$

Since the n-step transition probability can be written in the form

$$p_{ij}(n) = \sum_k p_{ik}(n-1)\, p_{kj}, \tag{3.10}$$

If the limiting-state probabilities exist and do not depend on the initial state, we have

$$\begin{aligned} \lim_{n\to\infty} P\left[X(n) = j\right] = \pi_j & \\ &= \lim_{n\to\infty} \sum_k p_{ik}(n-1)\, p_{kj} \\ &= \sum_k \pi_k\, p_{kj}. \end{aligned} \tag{3.11}$$

Suppose the limiting-state probability vector is defined as $\boldsymbol{\pi} = \left[\pi_1, \pi_2, ..., \pi_k\right]$, then we have

$$\begin{cases} \pi_j = \sum_k \pi_k\, p_{kj} \\ \boldsymbol{\pi} = \boldsymbol{\pi} \boldsymbol{P} \\ \sum_j \pi_j = 1, \end{cases} \tag{3.12}$$

where the last equation is due to the law of total probability (Ibe, 2014).

3.2.6 State Transition Diagram

State transition diagram is a graph in which the states are represented by circles with directed arcs representing the transition between states (Ibe, 2009). The state transition probabilities are labeled on the appropriate arcs and Figure 3.1 illustrates a two-state transition diagram.

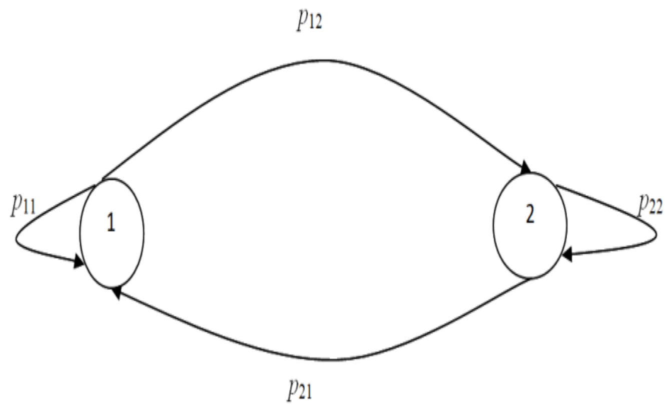

Figure 3.1: Two-state transition diagram

The corresponding transition probability matrix is given as,

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

3.2.7 Classification of States

A state j of a Markov process is describe as an accessible (or can be reached) state if starting from a certain state i, it is possible that the process will ever enter state j. Hence, for some $n > 0$, $p_{ij}(n) > 0$. If two states are accessible from each other, then they are said to communicate with each other.

A *transient (or non-recurrent)* state j is the one with a positive probability that the process will never return to it again after it leaves it. On the other hand, a recurrent (or persistent) state j is the state with probability one (1) that, the process will certainly return to it after it leaves it. A collection of recurrent states forms a single chain if every member of the set communicates with all other members of the set.

A state is referred to as an *absorbing state or trapping state* if it is not possible to leave that state once the process enters it. This means that, when the process enters an absorbing state, it remains there such that transitioning from that state to any other state is impossible (Ibe, 2009; 2014).

3.2.8 Sojourn Time

Let the process be in state i for which $p_{ii} > 0$ and we want to find the probability that the process remains in the state for exactly d time units. If the random variable D_i denotes the number of time unit that the process remains in the state before leaving the state, given that it enters the state, then the probability mass function of D_i is given by

$$\begin{aligned} p_{D_i}(d) &= P[D_i = d] \\ &= P[X_0 = i, X_1 = i, X_2 = i, ..., X_{d-1} = i, X_d \neq i] \\ &= p_{ii}^{d-1}(1 - p_{ii}), \end{aligned} \tag{3.13}$$

using the Markov chain rule. If the state of the process represents the members of an observation sequence, then p_{D_i} is the probability that the sequence remains unaltered exactly $d-1$ times before changing. Since D_i is a geometrically distributed random variable, the average sojourn time in state i is given by

$$E[D_i] = \frac{1}{1 - p_{ii}}. \tag{3.14}$$

If the state i is an absorbing state, then $p_{ii} = 1$ and $E[D_i] = \infty$, which is true because the process remains in the state indefinitely. For $p_{ii} \neq 1$, $E[D_i]$ is finite (Ibe, 2009).

3.3 Empirical Framework for Markov Chain

The use of Markov chain to address problems relating to planning in agriculture has been demonstrated by a number of researchers (Atobatele, 1986; Dittoh, 1985; Okpachu, 2006; Onu, 2000). In this section, the empirical framework of the study using Markov chain analysis is discussed.

3.3.1 Construction of States and Transition Matrices

The data collected on annual production output of the cereals (Maize and Rice) were model as three-state Markov chain process with state space $\overline{S} = \{\text{low, stable, high}\}$. In order to classify the production values into states, the growth rate was used. If the growth rate for a particular year is negative (less than zero) then the production of that particular year is classified as low, if the growth rate is zero then the production of the particular year is classified as stable (no change) and if the growth rate is positive (greater than zero) then the production of that particular year is classified as high.

Let PO_t denote the production output of the cereals at the t^{th} year. Then the change in production output, ΔPO_t, is a random variable defined as:

$$\Delta PO_t = PO_t - PO_{t-1}. \qquad (3.15)$$

Thus, each year is classified as having high, stable or low production output than the production output of the preceding year. The state of the system forms a trinary random variable given by:

$$\overline{S} = \begin{cases} \textbf{low} \text{ if, } \Delta PO_t < 0; \text{ that is decrease in production from } t-1 \text{ to } t \\ \textbf{stable} \text{ if , } \Delta PO_t = 0; \text{ that is no change in production from } t-1 \text{ to } t \\ \textbf{high} \text{ if , } \Delta PO_t > 0; \text{ that is increase in production from } t\text{-}1 \text{ to } t \end{cases} \qquad (3.16)$$

The observed frequency of being in a production state j after leaving production state i, $i, j = \{\text{low, stable, high}\}$ are presented in Table 3.1.

Table 3.1: Frequency of production being in state j preceded by production state i

		Current Year (j)			
		low (l)	stable (s)	high (h)	Total
Previous Year (i)	low (l)	n_{ll}	n_{ls}	n_{lh}	$n_{l.}$
	stable (s)	n_{sl}	n_{ss}	n_{sh}	$n_{s.}$
	high (h)	n_{hl}	n_{hs}	n_{hh}	$n_{h.}$

where;

n_{ll} is the number of low production preceded by low production,

n_{ls} is the number of stable production preceded by low production,

n_{lh} is the number of high production preceded by low production,

n_{sl} is the number of low production preceded by stable production,

n_{ss} is the number of stable production preceded by stable production,

n_{sh} is the number of high production preceded by stable production,

n_{hl} is the number of low production preceded by high production,

n_{hs} is the number of stable production preceded by high production,

n_{hh} is the number of high production preceded by high production,

$n_{l.} = n_{ll} + n_{ls} + n_{lh}$ is the total number of low production,

$n_{s.} = n_{sl} + n_{ss} + n_{sh}$ is the total number of stable production,

$n_{h.} = n_{hl} + n_{hs} + n_{hh}$ is the total number of high production.

The maximum likelihood estimators p_{ij} of $i, j = \{l,\ s,\ h\}$ are given by:

$$p_{ij} = \frac{n_{ij}}{\sum_{j=l}^{h} n_{ij}} \quad . \qquad (3.17)$$

The transition probability matrix is given in Table 3.2 and is defined by $\boldsymbol{P} = p_{ij} = p(j\,|\,i)$, where $i, j \in \overline{S}$

Table 3.2: Transition probability matrix

		Current Year (j)		
		low (l)	stable (s)	high (h)
Previous Year (i)	low (l)	p_{ll}	p_{ls}	p_{lh}
	stable (s)	p_{sl}	p_{ss}	p_{sh}
	high (h)	p_{hl}	p_{hs}	p_{hh}

where;

$p_{ll} = p(l\,|\,l)$ is the probability of a low production preceded by a low production,

$p_{ls} = p(s\,|\,l)$ is the probability of stable production preceded by low production,

$p_{lh} = p(h\,|\,l)$ is the probability of high production preceded by low production,

$p_{sl} = p(l\,|\,s)$ is the probability of low production preceded by stable production,

$p_{ss} = p(s\,|\,s)$ is the probability of stable production preceded by stable production,

$p_{sh} = p(h\,|\,s)$ is the probability of high production preceded by stable production,

$p_{hl} = p(h\,|\,l)$ is the probability of low production preceded by high production,

$p_{hs} = p(s\,|\,h)$ is the probability of stable production preceded by high production,

$p_{hh} = p(h\,|\,h)$ is the probability of high production preceded by high production.

The transition probabilities in Table 3.2 are subject to the condition that the sum of probabilities of each row is one. That is,

$$\begin{cases} p_{ll} + p_{ls} + p_{lh} = 1 \\ p_{sl} + p_{ss} + p_{sh} = 1 \\ p_{hl} + p_{hs} + p_{hh} = 1 \end{cases} . \qquad (3.18)$$

3.3.2 Estimation of Long-Run (Equilibrium) Probabilities

Suppose π_1, π_2 and π_3 are the probabilities of low, stable and high production outputs in the long-run. Then these values can be estimated by the matrix product

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} \begin{bmatrix} p_{ll} & p_{ls} & p_{lh} \\ p_{sl} & p_{ss} & p_{sh} \\ p_{hl} & p_{hs} & p_{hh} \end{bmatrix} . \qquad (3.20)$$

This finally gives the estimates of the long-run probabilities for each of the states as:

$$\begin{cases} \pi_1 = \pi_1 p_{ll} + \pi_2 p_{sl} + \pi_3 p_{hl} & \text{(for low)} \\ \pi_2 = \pi_1 p_{ls} + \pi_2 p_{ss} + \pi_3 p_{hs} & \text{(for stable)}, \\ \pi_3 = \pi_1 p_{lh} + \pi_2 p_{sh} + \pi_3 p_{hh} & \text{(for high)} \end{cases} \qquad (3.21)$$

subject to the condition that $\pi_1 + \pi_2 + \pi_3 = 1$.

Hence, the expected (or mean) recurrent times m_j are given by:

$$m_j = \frac{1}{\pi_j}, \; j = l, s, h. \qquad (3.22)$$

3.3.3 Expected Length of the Production States and Production Cycle

Low State: A low state of production output of length "*l*" is defined as a sequence of consecutive low production years preceded and followed by stable or high production years. The probability of "*l*", low years of production is given by:

$$p(l) = (p_{ll})^{l-1}(1 - p_{ll}). \qquad (3.23)$$

The expected length of low production years is given by:

$$E(L) = \frac{1}{(1 - p_{ll})}, \qquad (3.24)$$

where $1 - p_{ll}$ is the probability of a year being stable or high.

Stable State: A stable state of production output of length "*s*" is defined as a sequence of consecutive stable production years preceded and followed by low or high production years. The probability of "*s*", stable years of production is given by:

$$p(s) = (p_{ss})^{s-1}(1 - p_{ss}). \qquad (3.25)$$

The expected length of stable production years is given by:

$$E(S) = \frac{1}{(1 - p_{ss})}, \qquad (3.26)$$

where $1 - p_{ss}$ is the probability of a year being low or high.

High State: A high state of production output of length "*h*" is defined as the sequence of consecutive high production years preceded and followed by low or stable production years. The probability of "*h*", high years of production is given by:

$$p(h) = (p_{hh})^{h-1}(1 - p_{hh}). \qquad (3.27)$$

The expected length of stable production years is given by:

$$E(H) = \frac{1}{(1 - p_{hh})}, \qquad (3.28)$$

where $1 - p_{hh}$ is the probability of a year being low or stable.

Production Cycle: The production cycle (PC) is the period it takes for the production to go through all the three states and return to a particular state after leaving it. The PC is given by:

$$E(PC) = E(L) + E(S) + E(H), \tag{3.29}$$

where;

$E(PC)$ is the expected length of the PC, that is the number of years it will take the production process to be in each of the three states (low, stable, high) and return to a particular state after leaving the state,

$E(L)$ is the expected duration of low production years,

$E(S)$ is the expected duration of stable production years and,

$E(H)$ is the expected duration of high production years.

3.3.4 Forecasting

The transition matrix $\boldsymbol{P}$ enables each step of a Markov chain to be predicted from the previous step. Thus, the expected long-run production forecast is given by:

$$F_{\text{long}} = \boldsymbol{\pi}\boldsymbol{\mu}_0, \tag{3.30}$$

and that of the short-run production forecast is given by:

$$F_{\text{short}} = \boldsymbol{P}^n \boldsymbol{\mu}_0, \tag{3.31}$$

where F_{long} and F_{short} are the expected long-run and short-run production forecasts respectively, $\boldsymbol{\pi}$ is the vector of long-run probabilities and n is the number of steps. The vector $\boldsymbol{\mu}_0$ is an initial state vector which has dimensions $k \times 1$. The components of the initial state matrix can be numbers, probabilities or individual trial results. In this study, the components of the initial state matrix constitute average productions of the states (Okonta *et al*., 2017).

3.5 Trend Analysis Models

Many time series agricultural data exhibit trend. It is therefore necessary to investigate the type of trend characterizing the data. The trend is the slow long-run growth in the time series. The trend in a time series may be a linear function of time or a non-linear function of time (Nasiru, 2013). Three different trend models were used to study the pattern of the production outputs of maize and rice. If the trend in the time series is a linear function of time t, then

$$Y_t = \beta_0 + \beta_1 t + e_t, \qquad (3.32)$$

where Y_t are the observations of the time series (maize and rice), t is a time dummy $(t = 1, 2, ..., n-1, n)$ and e_t is a random error component.

Sometimes, the time series may exhibit a quadratic trend. If the trend is quadratic in nature, then

$$Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t. \qquad (3.33)$$

If the trend in the time series follows the exponential growth trend model, then

$$Y_t = \beta_0 \times \beta_1^t \times e_t. \qquad (3.34)$$

3.5.1 Measures of Accuracy

In order to identify the best trend model that describes the pattern of the production output for each of the cereals, three measures of accuracies were used, namely: the mean absolute percentage error (MAPE), mean absolute deviation (MAD) and mean squared deviation (MSD). These three measures of accuracies adopted are the most commonly used in literature for assessing the adequacy of trend models in most statistical software. Thus, the MINTAB 16 software used for the trend analyses permits the assessment of the performance of the trend models using these measures. The trend models with the smaller values of these measures of accuracies are considered as the best model.

The MAPE, measures the accuracy of the fitted time series observations. It expresses accuracy as a percentage.

$$MAPE = \frac{\sum_{t=1}^{n} \left| \left(Y_t - Y_t\right) \big/ Y_t \right|}{n} \times 100, \qquad (3.35)$$

where $Y_t \neq 0$, Y_t equals the fitted value, and n equals the number of observation (Amin *et al*., 2014).

The MAD, measures the accuracy of the fitted time series observations by expressing it in the same unit as the data (Amin *et al*., 2014). It is given by

$$MAD = \frac{\sum_{t=1}^{n} \left| Y_t - Y_t \right|}{n}. \qquad (3.36)$$

The MSD is also a measure of accuracy of the fitted time series values but it is more sensitive to an unusually large forecast error than MAD (Amin *et al*., 2014). It is given by

$$MSD = \frac{\sum_{t=1}^{n} \left(Y_t - Y_t\right)^2}{n}. \qquad (3.37)$$

3.6 Unit Root test

An imperative aspect of time series analysis is to ensure that the data is stationary. That is, the mean, variance and covariance of the time series data do not change with time. A time series that exhibit such traits is said to be weakly stationary (Nasiru, 2013). Both graphical and quantitative methods for testing for the stationarity of a time series data exist in literature. However, this study employed two quantitative techniques to investigate the stationarity of the data. The techniques used in this study are Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The two tests were employed to affirm the stationarity of the data. Although the ADF test has the ability to take care of serial correlation in the regression residuals it is not robust when there is heteroscedasticity. Thus, the PP test which is robust to general form of heteroscedasticity is employed as an additional test to ensure that the right decision on the stationarity of the data is arrived at.

3.6.1 Augmented-Dickey Fuller Test

The ADF test was developed by Dickey and Fuller (1979) to overcome drawbacks of the Dickey-Fuller test. It was developed based on the idea that the time series observation follows random walk. Given an autoregressive process of order one,

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \qquad (3.38)$$

where ε_t represents a white noise sequence with zero mean and constant variance. If $\phi = 1$, equation (3.38) is a non-stationary process. The fundamental principle of ADF test is to regress Y_t on its lagged value Y_{t-1} and find out if the estimated ϕ is statistically equal to one or not. By subtracting Y_{t-1} from both sides of equation (3.38), the equation can be written as

$$\Delta Y_t = \delta Y_{t-1} + \varepsilon_t, \qquad (3.39)$$

where $\delta = \phi - 1$ and $\Delta Y_t = Y_t - Y_{t-1}$. In practice instead of estimating equation (3.38), we rather estimate equation (3.39) and test for the null hypothesis of $\delta = 0$ against the alternative $\delta \neq 0$. If $\delta = 0$, then $\phi = 1$, meaning that the time series data have a unit root. The decision to reject the null hypothesis or not is based on the Dickey-Fuller critical values. The Dickey-Fuller test is based on the assumption that the error terms are uncorrelated. However, the errors of the Dickey-Fuller test usually show evidence of serial correlation. To avoid this problem, the ADF test includes the lags of the first difference series in the regression equation to make the error term white noise and therefore the regression equation becomes

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-i} + \varepsilon_t. \qquad (3.40)$$

If we include the intercept and time trend, equation (3.40) becomes

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-i} + \varepsilon_t, \qquad (3.41)$$

where α is a constant, β is the coefficient on time trend series, $\sum_{i=1}^{p} \gamma_i \Delta Y_{t-i}$ is the sum of the lagged values of the dependent variable ΔY_t. The test statistic for the ADF test is defined as

$$F = \frac{\delta}{SE(\delta)},$$

where $SE(\delta)$ is the standard error of the least square estimate of δ. The null hypothesis is rejected if the test statistic is greater than the critical value (Nasiru, 2013).

3.6.2 Phillips-Perron Test

Another useful unit root test is the Phillips-Perron (PP) test proposed by Phillips and Perron (1988). The PP test is a semi-parametric technique for testing for the existence of unit root in a time series data. This test checks for any autocorrelation and heteroscedasticity in the residual term ε_t nonparametrically. The PP test statistic, tests the null hypothesis of non-stationarity against the alternative of stationarity. The PP test involves estimating the models

$$Y_t = \beta + \varphi Y_{t-1} + \varepsilon_t. \tag{3.42}$$

The intercept can be excluded from the model and a time trend added to obtain

$$Y_t = \alpha t + \varphi Y_{t-1} + \varepsilon_t. \tag{3.43}$$

The PP test is made up of two test statistics known as the Phillips Z_ρ and Z_τ defined as

$$Z_\rho = n(\rho_n - 1) - \frac{1}{2}\frac{n^2\sigma^2}{s_n^2}(\lambda_n^2 - \gamma_{0,n}) \tag{3.44}$$

and

$$Z_\tau = \sqrt{\frac{\gamma_{0,n}}{\lambda_n^2}} \times \frac{\rho_n - 1}{\sigma} - \frac{1}{2}(\lambda_n^2 - \gamma_{0,n})\frac{n\sigma}{\lambda_n s_n}, \tag{3.45}$$

where

$\gamma_{j,n} = \frac{1}{n}\sum_{i=j+1}^{n} \varepsilon_i \varepsilon_{i-j}$, when $j = 0$, then $\gamma_{j,n}$ is a maximum likelihood estimate of the variance of the error terms, while for $j > 0$ is an estimate of the covariance between two error terms j periods apart.

$\lambda_n^2 = \gamma_{0,n} + 2\sum_{j=1}^{q}\left(1 - \frac{j}{q+1}\right)\gamma_{j,n}$, if there is no autocorrelation between the error terms, $\gamma_{j,n} = 0$ for $j > 0$, then $\lambda_n^2 = \gamma_{0,n}$.

$s_n^2 = \frac{1}{n-k}\sum_{i=1}^{n} \varepsilon_i^2$ is an unbiased estimator of the variance of the error term, k is the number of independent variables in the regression, q is the number of Newey-West lags to use in the calculation of λ_n^2 and σ.

3.7 Cointegration Test

The idea of cointegration can be linked to Engle and Granger (1987). Two or more variables are said to be cointegrated if they share a common stochastic trend in the long-run. The most common order of integration in time series is either zero or one (Brooks, 2008). This study used the Johansen's approach to investigate the existence or absence of cointegration in the production outputs of the cereals.

The Johansen method has the ability to detect several cointegration vectors in a system of variables (Johansen and Juselius, 1990, Kasa, 1992). The Johansen approach depends on a vector autoregressive (VAR) model. The simplest form of a VAR model, where k represents the lags to be included (Brooks, 2008) is given by

$$\mathbf{Y}_t = \boldsymbol{\beta}_1\mathbf{Y}_{t-1} + \boldsymbol{\beta}_2\mathbf{Y}_{t-2} + \ldots + \boldsymbol{\beta}_k\mathbf{Y}_{t-k} + \mathbf{U}_t, \tag{3.46}$$

where $\mathbf{Y}_t$ is a $p \times 1$ random vector, $\boldsymbol{\beta}_i, i = 1,....,k$ is a fixed $p \times p$ parameter matrices and $\boldsymbol{U}_t$ is a p-dimensional white noise series.

To use the Johansen test, the VAR model is transformed into a vector error correction (VEC) model by differencing. The VEC model is given by

$$\Delta \mathbf{Y}_t = \prod \mathbf{Y}_{t-k} + \boldsymbol{\Gamma}_1 \Delta \mathbf{Y}_{t-1} + \boldsymbol{\Gamma}_2 \Delta \mathbf{Y}_{t-2} + \ldots + \boldsymbol{\Gamma}_{k-1} \Delta \mathbf{Y}_{t-(k-1)} + \mathbf{U}_t \ , \qquad (3.47)$$

where there are g variables in the model and $k-1$ lags of the dependent variables. $\boldsymbol{\Gamma}$ is the coefficient matrix for every lagged variable and $\boldsymbol{\Pi}$ is the long-run coefficient matrix. For the Johansen approach, two tests are employed to check the existence of cointegration and the number of cointegrating vectors r (Enders, 2008):

- The trace test:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{g} \ln(1-\lambda_i). \qquad (3.48)$$

  The null hypothesis of r or less than r cointegrating vectors is tested against the alternative of more than r cointegrating vectors.

- The maximum eigenvalue test:

$$\lambda_{\max}(r, r+1) = -T \ln(1-\lambda_{r+1}). \qquad (3.49)$$

  The null hypothesis of exactly r cointegrating vectors is tested against the alternative of $r+1$ cointegrating vectors.

λ_i is the computed eigenvalue of order i from the $\boldsymbol{\Pi}$ matrix and T is the number of observations.

3.8 Granger Causality Test

Cointegration informs us whether the variables are cointegrated. But it is important to note that the variables can be related in the short-run even when they are not cointegrated. Granger causality test is used to investigate the short-run relation among variables. A variable Y_t Granger-cause another variable Z_t if the past values of Y_t has additional power in predicting Z_t after controlling for the past values of Z_t (Gelper and Croux, 2007). Causality can be described as unidirectional, bilateral or independent (Gujurati, 2003).

Consider the simple VAR model

$$\mathbf{Z}_t = \boldsymbol{\beta}_1\mathbf{Z}_{t-1} + \boldsymbol{\beta}_2\mathbf{Z}_{t-2} + \ldots + \boldsymbol{\beta}_k\mathbf{Z}_{t-k} + \boldsymbol{\alpha}_1\mathbf{Y}_{t-1} + \boldsymbol{\alpha}_2\mathbf{Y}_{t-2} + \ldots + \boldsymbol{\alpha}_p\mathbf{Y}_{t-p} + \mathbf{U}_t. \qquad (3.50)$$

If all $\boldsymbol{\alpha}$ coefficient on lagged values of $\mathbf{Y}_t$ are significant then Y_t Granger-cause Z_t.

3.9 Lag Order Selection

An important step in performing cointegration test or fitting VAR model is to find the correct lag order. This study employed three lag order selection techniques which differ by severity of the penalty they imposed for identifying the optimal lag. The study used Akaike Information Criterion (AIC) (Akaike, 1974), Bayesian Information Criterion (BIC) (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQIC) (Hannan and Quinn, 1979) to determine the optimal lag order. These criteria are given by

$$AIC = \ln\left|\Sigma_u(p)\right| + \frac{2}{T} pK^2,$$
$$BIC = \ln\left|\Sigma_u(p)\right| + \frac{\ln(T)}{T} pK^2,$$
$$HQIC = \ln\left|\Sigma_u(p)\right| + \frac{2\ln(\ln(T))}{T} pK^2 , \quad (3.51)$$

where T denotes the number of observations in the data, p is the lag order, $\Sigma_u(p) = T^{-1}\sum_{t=1}^{T}\varepsilon_t\varepsilon_t^{'}$ is the residual covariance matrix without degree of freedom corrected from the model and K denotes parameters in the model. The BIC and HQIC are consistent estimators and selects models with fewer parameters when the sample size is large than the AIC (Schwarz, 1978; Hannan-Quinn, 1979). The lag order with the least values of these criteria is the bests.

3.10 Impulse Response Function Analysis

The Granger causality test is useful when determining whether a time series variable helps in forecasting another variable. However, it is unable to quantify the impact of the impulse time series variable on the response variable overtime. The impulse response analysis is employed to assess these kinds of interactions between the dependent time series variables using Wold's decomposition. The Wold representation is based on the orthogonal errors η_t (Luguterah *et al*., 2013):

$$Y_t = \mu + \Theta_0\eta_t + \Theta_1\eta_{t-1} + \Theta_2\eta_{t-2} + \ldots, \quad (3.52)$$

where Θ_0 is a lower triangular matrix.

3.11 Forecast Error Variance Decomposition

It gives the proportion of the movement in a sequence due to its own shocks versus shocks to other variables. The forecast error variance decomposition was used in this study to investigate the contribution of the j^{th} variable to the $h-$step forecast error variance of the i^{th} variable. The forecast error variance decomposition is defined as:

$$FEVD_{ij}(h)=\frac{\sigma_{\eta_j}^2\sum_{s=0}^{h-1}(\Theta_{ij}^s)^2}{\sigma_{\eta_1}^2\sum_{s=0}^{h-1}(\Theta_{ij}^s)^2+...+\sigma_{\eta_n}^2\sum_{s=0}^{h-1}(\Theta_{ij}^s)^2},i,j=1,2,...,n, \qquad (3.53)$$

where $\sigma_{\eta_j}^2$ is the variance of η_{j_t} .

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

The results and discussion of the findings are presented here. The chapter is divided into six main sections, namely: preliminary data analysis, trend analyses, unit root test and cointegration, vector autoregressive and Granger causality test, Markov chain analysis of maize production and Markov chain analysis of rice production.

4.1 Descriptive statistics of Maize and Rice Production Outputs from 1960 to 2018

Table 4.1 displays the descriptive statistics of the production outputs of both maize and rice in thousand metric tonnes. The results revealed that the average production outputs of maize and rice were 823,400 metric tonnes and 124,700 metric tonnes respectively. This implies that the production output of maize was higher than that of rice over the entire period. The minimum production outputs were 172,000 metric tonnes and 21,000 metric tonnes for maize and rice respectively. The maximum production output for maize was 2,263,000 metric tonnes whiles that of rice was 510,000 metric tonnes. The minimum and maximum production outputs for maize occurred in the years 1983 and 2018 respectively. The minimum production output for rice occurred in the years 1960, 1962 and 1965 whiles the maximum production output occurred in the year 2018. The standard deviations for the production outputs for maize and rice were 553,100 metric tonnes and 119,200 metric tonnes respectively. To investigate the degree of variability, the coefficient of variation for the production outputs were estimated and the results as displayed in Table 4.1 revealed that, there was greater variability in the production outputs of rice compared to maize. This high variability index implies that the rice production output fluctuates widely overtime. This can partly be attributed to poor rainfall patterns, high importation of rice among others.

The coefficients of skewness and kurtosis were also estimated for the production outputs. From the results, both the production outputs of maize and rice were positively skewed with an estimated coefficient of skewness of 0.6 and 1.58 respectively. Thus, there is an evidence of lack of symmetry in the production outputs of the data. The excess kurtosis value for the production output of maize was -0.96 and that of rice was 1.84. This means that the distribution of the production output of maize is less peaked as compared to that of the normal curve and that of rice is more peaked than the normal curve. Looking at the coefficient of skewness and kurtosis, it can be inferred that the production outputs over the entire period were not normally distributed.

Table 4.1: Descriptive statistics for production outputs of maize and rice (000 MT)

Statistic	**Maize**	**Rice**
Mean	823.40	124.70
Standard Deviation	553.10	119.20
Coefficient of Variation (%)	67.13	95.61
Minimum	172.00	21.00
Maximum	2,263.00	510.00
Skewness	0.60	1.58
Excess Kurtosis	-0.96	1.84

Source: Author’s computation, 2019

To affirm the non-normality of the production outputs for the cereals, the Kolmogorov-Smirnov and Anderson-Darling tests for normality were performed and the results are shown in Table 4.2 revealed that the production outputs for the maize and rice were not normally distributed. This is because the *p*-values obtained for the tests were all less than the 0.05 significance level as shown in Table 4.2. It can thus be inferred that any econometric model that requires the data to be normally distributed may not be suitable for modelling the production outputs. Although some schools of thoughts may suggest transformation of the data to make it normal, this may destroy some of the underlying structure of the data (Luguterah *et al*., 2013).

Table 4.2: Normality tests for cereals

Cereals	Kolmogorov-Smirnov	Anderson-Darling
Maize	0.166*	2.262*
	(*p*-value=0.010)	(*p*-value=0.005)
Rice	0.192*	4.381*
	(*p*-value=0.010)	(*p*-value=0.005)

*: Means significant at the 0.05 significance level
Source: Author's computation, 2019

The Pearson correlation analysis was performed to investigate the nature of the relationship between the production output of maize and that of the rice. Figure 4.1 displays the scatter plot, trend line and the Pearson correlation coefficient. From the scatter diagram, it can be seen that there is a linear relationship between the two production outputs. The Pearson correlation coefficient of 0.908 affirms strong positive linear relationship between the production outputs of maize and rice. This linear relationship between the two production outputs is significant with a *p*-value of 0.000. This implies that as the production output of maize increases that of rice also increases.

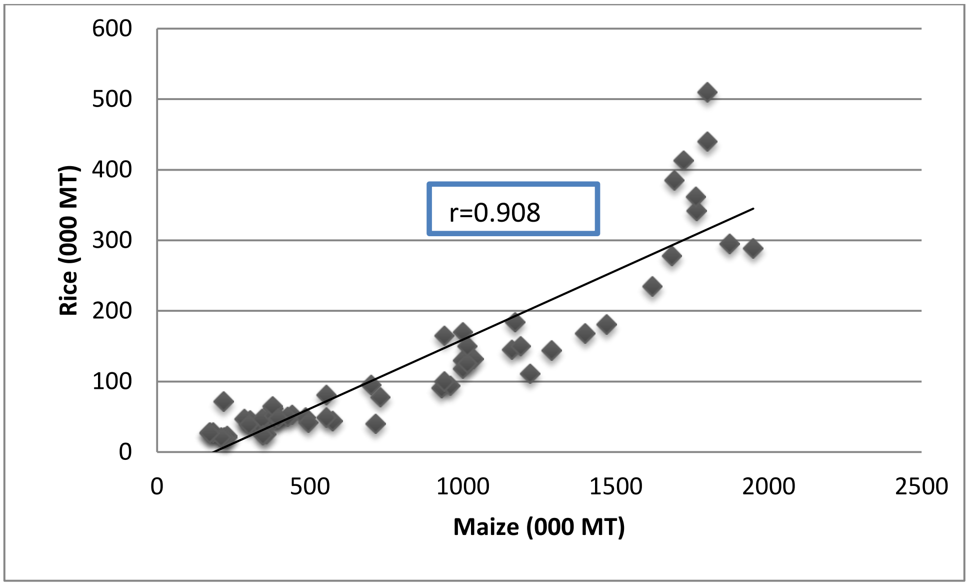

Figure 4.1: Scatter plot of production outputs of maize and rice

4.2 Production Outputs of maize and rice by states

The results in Table 4.3 revealed that the average production outputs of maize in the low, stable and high states were 699,000 metric tonnes, 1,204,000 metric tonnes and 899,000 metric tonnes respectively. The minimum production outputs of maize in the low, stable and high states were 172,000 metric tonnes, 1,000,000 metric tonnes and 209,000 metric tonnes respectively. Similarly, the maximum production outputs for maize in the low state, stable and high states were 1,764, 000 metric tonnes, 1,800,000 metric tonnes and 2,263,000 metric tonnes respectively. With regards to the variability of the production outputs of maize for the various states, it was observed that the stable state had less variability compared to the low and high states.

Also, the average production outputs of rice in the low, stable and high states were 83,000 metric tonnes, 37,500 metric tonnes and 153,600 metric tonnes respectively as displayed in Table 4.3. This implies that over the entire period there were higher production output values for rice than the low and stable production values. The minimum production outputs of rice in the low, stable and high states were 21,000 metric tonnes, 28,000 metric tonnes and 23,000 metric tonnes respectively. The maximum production outputs of rice in the low, stable and high states were 278,000 metric tonnes, 47,000 metric tonnes and 510,000 metric tonnes respectively. Comparing the variability of the production outputs of rice for the various states, it is can be seen from Table 4.3 that the stable state has less variability than the low and high states. This implies that, production outputs in the stable state were not far apart from the average production for the entire study period.

Table 4.3: Descriptive statistics for production states of maize and rice (000 MT)

Cereals	State	Mean	Standard Deviation	Coefficient of Variation (%)	Minimum	Maximum
Maize	Low	699	560	80.01	172	1,764
	Stable	1,204	398	33.03	1,000	1,800
	High	899	543	60.45	209	2,263
Rice	Low	83	64.20	77.39	21	278
	Stable	37.5	13.44	35.83	28	47
	High	153.6	135.40	88.15	23	510

Source: Author's computation, 2019

Further exploratory analysis of the production outputs of maize and rice were performed to determine the frequency of production outputs and their corresponding percentages for the various states of production for the cereals. From Table 4.4, 25 of the production output values for maize representing 43.1% were in low state, 4 of the production output values representing 6.9% were in the stable state and 29 of the production output values representing 50% were in the high state. Hence, more of the production output values for maize were in the high state over the entire period. For the production output values for rice, 19 of them constituting 32.8% were in low states, 2 of them constituting 3.4% were in stable state and 37 of them constituting 63.8% were in higher state. This implies that majority of the production output values were in higher state.

Table 4.4: Frequency for production output values

Cereals	State	Frequency	Percentage
Maize	Low	25	43.10
	Stable	4	6.90
	High	29	50.00
Rice	Low	19	32.80
	Stable	2	3.40
	High	37	63.80

Source: Author's computation, 2019

4.3 Time Series plot for maize production

Figure 4.2 shows the time series plot of the production outputs of maize from the period 1960 to 2018. It can be seen that the production outputs from the year 2000 to 2018 were higher than those from the year 1960 to 1999. From the plot, it was obvious that the production outputs fluctuate overtime. Hence, there is an evidence of cyclical fluctuation in the production outputs of the maize. This means that the production output of the maize is seen to increase in one period and after sometime it decreases. These fluctuations can partly be attributed to the unstable nature of government policies and poor climatic conditions over the years among others. Although the highest production output was recorded in the year 2012, the production output decreased after that until the year 2016 when it started to increase again.

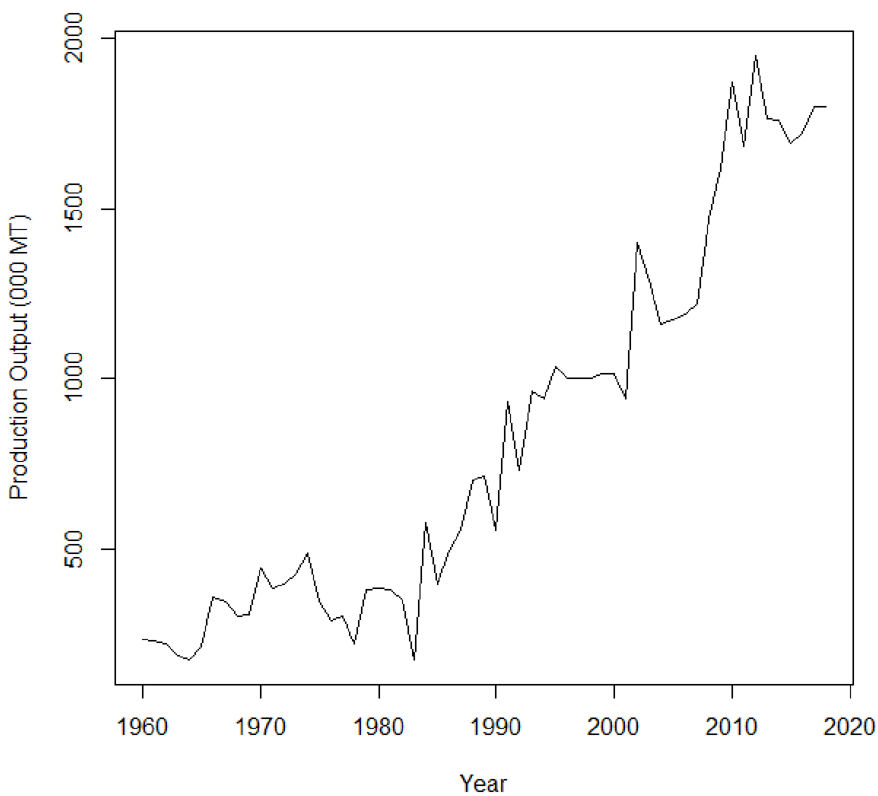

Figure 4.2: Time series plot of the production outputs of maize

4.4 Time Series Plot for Rice Production

The time series plot of the production outputs of rice is shown in Figure 4.3. The production outputs of the rice from the year 2010 to 2018 were higher than those from the year 1960 to 2009. This could be due to the fact that from the year 2000 onwards governments have implemented specific policies to boost the production of the cereal. There is also an evidence of cyclical fluctuations in the production outputs of the rice. However, from the year 2010 to 2018 there have been continued increase in the production outputs of the rice.

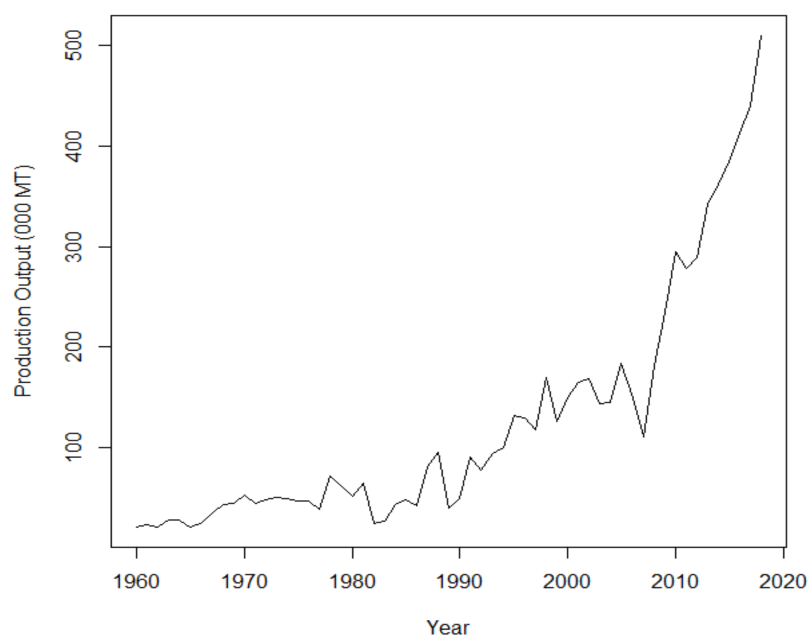

Figure 4.3: Time series plot of the production outputs of rice

4.2 Trend Analyses for Maize and Rice

In this section, trend analyses were carried out to investigate the nature of the trend characterising the production output values of the cereals. The linear, quadratic and exponential growth curve trend models were fitted to the production output values of the cereals and the best trend model identified using appropriate measures of accuracy. Table 4.5 shows the various trend models with their corresponding measures of accuracy fitted to the maize production output values. The results in Table 4.5, shows that the quadratic trend model provides the best fit to the maize production output values since it has the smallest values for the MAPE, MAD and MSD. This means that the pattern of growth for the maize production output values is quadratic in nature.

Table 4.5: Measures of accuracy for fitted trend models for maize production

Model	MAPE	MAD	MSD
Linear	36.10	159.80	34810.40
Quadratic	19.00*	103.60*	16532.00*
Exponential growth curve	19.40	112.80	20102.80

* Means best based on measure of accuracy
Source: Author's computation, 2019

Figure 4.4 displays the original production output values, the fitted trend line, the fitted trend equation, the forecasted values and the three measures of accuracy of the quadratic trend model fitted to the maize production output values. The plots for the linear and the exponential growth curve models for the maize production output values can be found in the appendix A. It can be seen from Figure 4.4 that the maize production output values show a general upward trend, though with some evident of downward and upward fluctuations over the entire period. Five years forecast with the quadratic trend model revealed an increasing pattern in the trend of the maize production output as shown in Figure 4.4. This finding concurs with that of Nasiru and Sarpong (2012a). The increasing pattern of the maize production output can be attributed to some government interventions over the years such as the fertilizer subsidy, certified seeds and intensified extension services. The quadratic trend model for the production output values for maize can be written as

$$Y_t = 233.5 - 1.01t + 0.5215t^2 \ . \qquad (4.1)$$

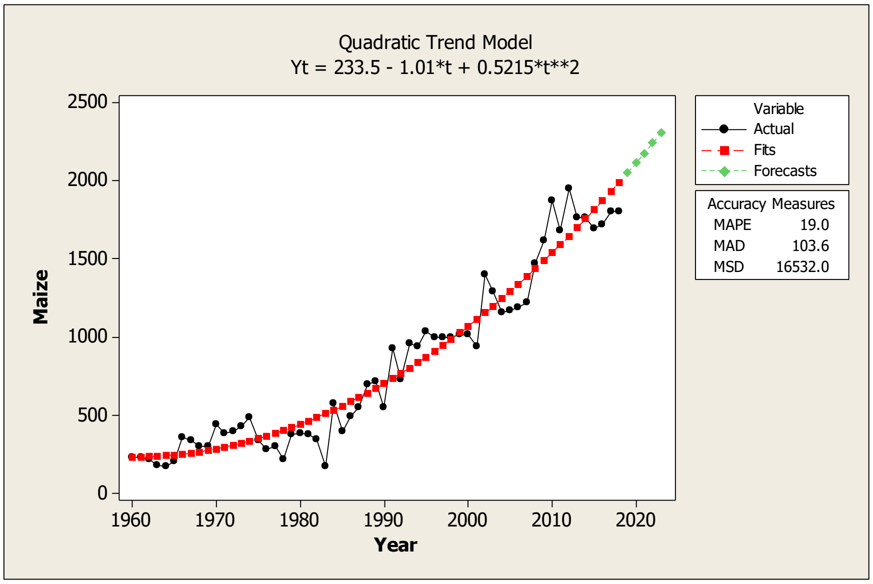

Figure 4.4: Trend analysis plot for maize production output values

The measures of accuracy for the linear, quadratic and exponential growth curve models fitted to the rice production output values are presented in Table 4.6. It can also be seen from Table 4.6 that, the MAPE and MAD selected the exponential growth curve as the best while the MSD selected the quadratic model. However, since majority of the measures of accuracy selected the exponential growth curve model, it was considered as the most suitable trend model for the production output values for the rice.

Table 4.6: Measures of accuracy for fitted trend models for rice production

Model	MAPE	MAD	MSD
Linear	76.29	50.54	3947.25
Quadratic	38.13	26.33	1218.82*
Exponential growth curve	25.72*	26.05*	1682.07

* Means best based on measure of accuracy
Source: Author's computation, 2019

The plot of the original production values, the fitted trend line, the fitted trend equation, forecasted values and the three measures of accuracy of the exponential growth curve trend model fitted to the rice production output values are shown in Figure 4.5. The plots for the linear and quadratic trend models can also be found in the appendix A. Although the production output values for the rice exhibits upward and downward movement over the entire period, it can be seen that the pattern of growth was generally increasing. Five years forecasted values for the rice production output indicated an increasing pattern as shown in Figure 4.5. This result is in agreement with the findings of Nasiru and Sarpong (2012b) on forecasting milled rice production in Ghana. This increasing pattern can be ascribed to government interventions like subsidizing fertilizer, provision of improved seeds among others to boost production of rice in the country as well reduce its importation. The exponential growth curve model for rice production output values is given by

$$Y_t = 19.3652 \times (1.05006^t). \qquad (4.2)$$

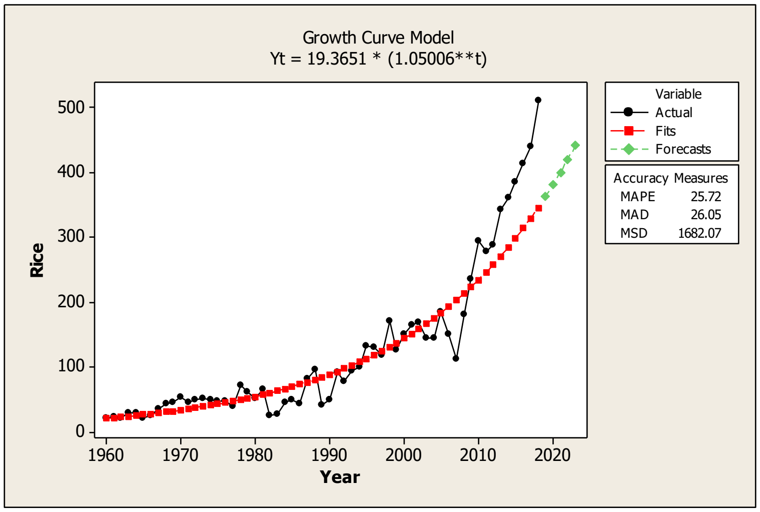

Figure 4.5: Trend analysis plot for rice production output values

4.3 Unit root and Cointegration Tests

In this section, the unit root and cointegration tests were carried out to examine the time series characteristics of the production outputs of maize and rice. The ADF and PP tests were performed to investigate whether the production outputs of the maize and rice were stationary. From the results in Table 4.7, the production outputs of the two cereals were not stationary as indicated by the ADF and PP tests. It implies that the means and variances of the production outputs of the maize and rice changes with time.

Table 4.7: ADF and PP tests for raw production output values

Cereals	ADF test		PP test	
	Test statistic	***p*-value**	**Test statistic**	***p*-value**
Maize	-1.9984	0.5754	-14.9260	0.2204
Rice	1.8007	0.9900	3.9603	0.9900

Source: Author's computation, 2019

In order to make the production outputs stationary, the production outputs were logarithmically transformed to stabilize the variance and first differenced to make the mean stationary. From Table 4.8, the ADF and PP tests revealed that the logarithmically transformed and first differenced series were stationary as both tests rejected the null hypothesis of non-stationarity. Hence, the transformed production outputs were integrated of order one. This implies that, the production outputs of the maize and rice have a similar stochastic process and have the tendency of co-movement in the long-run. Thus, cointegration test was carried out using the production outputs of the transformed data.

Table 4.8: ADF and PP tests for transformed production output values

Cereals	ADF test		PP test	
	Test statistic	***p*-value**	**Test statistic**	***p*-value**
Maize	-5.5872	0.0100*	-75.8270	0.0100*
Rice	-4.6830	0.0100*	-56.9600	0.0100*

*: means significant at 5%
Source: Author's computation, 2019

Using the transformed data, Johansen's cointegration test was undertaken to examine the long-run equilibrium relationship between the production outputs of the cereals. Before performing the cointegration test the appropriate lag order was determined and the results are shown in Table 4.9. The AIC selected lag 3 whiles the BIC and HQIC selected lag 2 as the optimal lag order for the test because these lags have the minimum values for the selection criteria. Hence, the optimal lag order of 2 was used to perform the test since more of the selection criteria chose it.

Table 4.9: Lag order selection for cointegration test

Lag	AIC	HQIC	BIC
1	0.4431	0.5310	0.6747
2	0.0836	0.2301*	0.4697*
3	0.0646*	0.2697	0.6051
4	0.0699	0.3336	0.7648
5	0.1920	0.5143	1.0414
6	0.0753	0.4562	1.0779
7	0.1392	0.5787	1.2975
8	0.2266	0.7247	1.5393
9	0.2488	0.8054	1.7160
10	0.2956	0.9108	1.9172

*: optimum lag selected
Source: Author's computation, 2019

The cointegration test result shown in Table 4.10 revealed that the maize and rice production outputs are not cointegrated. The Johansen's test fail to reject the null hypothesis of no cointegration (that is $r = 0$) when both the trace and maximum-eigenvalue statistics were used. This means that the production outputs of the cereals have no long-run equilibrium relationship. Thus, the maize production outputs and the rice production outputs drift apart in the long-run.

Table 4.10: Johansen's cointegration test

Rank	Trace statistic	5% Critical Value	Maximum-Eigenvalue statistic	5% Critical Value
$r = 0$	11.556*	15.4100	10.7538*	14.0700
$r \leq 1$	0.8023	3.7600	0.8023	3.7600

*: failing to reject the null hypothesis of no cointegration
Source: Author's computation, 2019

4.4 Vector Autoregressive and Granger Causality test

The cointegration test revealed that there is no long-run relationship between the production outputs of the cereals. Hence, the vector autoregressive (VAR) model and the granger causality test were employed to study the short-run dynamics between the production outputs of the cereals. Since the VAR model requires the data to be stationary, the logarithmically transformed first differenced production outputs were used in fitting the model. In order to fit the VAR model, we first need to identify the optimal lag order to be included in the analysis. The AIC selected an optimum lag of 2 whiles the BIC and HQIC selected lag 1.The result as presented in Table 4.11 revealed that the optimal lag order for fitting the VAR model is one (1) since two of the selection criteria chose one.

Table 4.11: VAR lag order selection

Lag	AIC	HQIC	BIC
1	0.0454	0.1043*	0.2013*
2	0.0389*	0.1567	0.3507
3	0.1565	0.3333	0.6243
4	0.2237	0.4594	0.8474
5	0.2861	0.5807	1.0657
6	0.4232	0.7767	1.3588
7	0.3080	0.7204	1.3995
8	0.3041	0.7756	1.5516
9	0.3304	0.8608	1.7338
10	0.4437	1.0329	2.0030

*: optimum lag selected
Source: Author's computation, 2019

The VAR model is fitted with lag one (VAR (1)) and the results are presented in Table 4.12. The lag 1 values for the production outputs of maize and rice were significant in the maize equation but insignificant in the rice equation. This means that in the maize equation, the lag 1 production outputs for maize and rice are useful in predicting future production values of maize. However, the maize lag 1 value negatively affects future production outputs of maize whiles the rice lag 1 value positively affects the maize production outputs. From the rice equation, it can also be seen that the lag 1 values for maize and rice are not useful in predicting future production outputs of rice. This finding is in agreement with the studies of Luguterah *et al*. (2013).

Table 4.12: Estimated parameters for the VAR (1) model

Equations	**Variables**	**Coefficient**	**Standard error**	**t-ratio**	***p*-value**
Maize	Maize.L1	-0.5102	0.1137	-4.4859	0.0000*
	Rice.L1	0.3357	0.1045	3.2124	0.0022*
Rice	Maize.L1	-0.0487	0.1478	-0.3294	0.7431
	Rice.L1	-0.1590	0.1358	-1.1708	0.2467

*: means significant at 5% significance level
Source: Author's computation, 2019

Table 4.13 revealed that the model was stable as all the eigenvalues had modulus less than one. That is all the eigenvalues fall within the unit circle. This affirms that all the series used in fitting the VAR (1) model are stationary as indicated by the ADF and PP tests.

Table 4.13: VAR (1) stability condition

Variable	Eigen-value	Modulus
Rice	-0.455	0.4550
Maize	-0.2142	0.2142

Source: Author's computation, 2019

The VAR (1) model was used to perform impulse response analysis to investigate how the maize and rice production outputs relate to each other when there is shock in the model. The response to a shock in production output is dependent on the history of the time series and the magnitude of the postulated shock. Positive shocks are those that affect the production outputs positively while negative shocks are those that affect the production outputs negatively. In Figure 4.6, ten-period (10 years) impulse response analyses are presented. When maize was considered as the impulse variable, the first period maize adjusts positively to a shock in its values, negatively at period two, positively at period three, negative at period four and quite stable for the remaining periods. Rice adjusted positively at periods one and two to a shock in the maize production outputs, negatively at period three, positively at period four, negatively at period five and quite stable for the remaining periods.

When rice is taken as the impulse variable, then maize adjusts positively at period one, negatively at period two, positively at period three and quite stable for the remaining periods. Rice adjusts positively at period one to shock in itself, negatively at period two, positively at period three and quite stable response for the remaining periods.

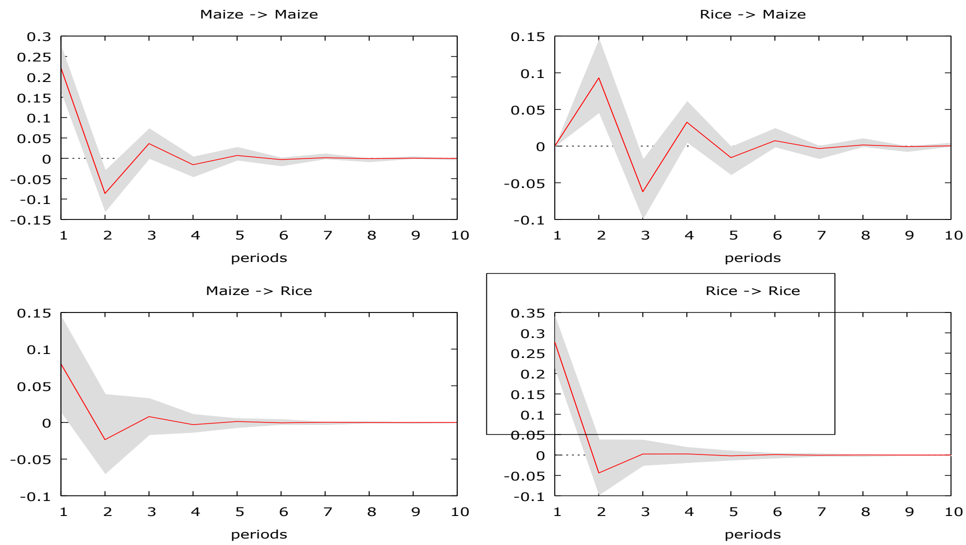

Figure 4.6: Impulse response analysis

The magnitude of the relationship between the variables cannot be determined using the impulse response analysis. Hence, the forecast error variance decompositions of the variables were studied to determine the magnitude. Table 4.14 presents the forecast error variance decomposition for maize. At period one, maize explained 100% of the forecast uncertainty in itself whiles at period 10 maize accounted for 80.75% and rice accounted for 19.25% of the forecast uncertainty in the maize. It can be seen that as the time period increases, the rice production outputs is able to account for an appreciable percentage of the forecast uncertainty in the maize production outputs.

Table 4.14: Forecast error variance decomposition for maize

Period	Standard error	Maize	Rice
1	0.2220	100.0000	0.0000
2	0.2557	86.7592	13.2408
3	0.2657	82.2385	17.7615
4	0.2681	81.0843	18.9157
5	0.2687	80.8188	19.1812
6	0.2688	80.7609	19.2391
7	0.2688	80.7486	19.2514
8	0.2689	80.746	19.2540
9	0.2689	80.7454	19.2546
10	0.2689	80.7453	19.2547

Source: Author's computation, 2019

Table 4.15 displays the forecast error variance decomposition for rice. At period 1, it can be seen that rice production outputs explains 92.33% of the forecast uncertainty in itself whiles maize accounts for 7.67% of the forecast uncertainty in the rice production outputs. At period 10, the rice production outputs accounts for 91.83% of the forecast uncertainty in itself whiles maize production outputs accounts for 8.17% of the forecast uncertainty in the rice production outputs.

Table 4.15: Forecast error variance decomposition for rice

Period	Standard error	Maize	Rice
1	0.2885	7.6687	92.3313
2	0.2928	8.0902	91.9098
3	0.2929	8.1573	91.8427
4	0.2930	8.1665	91.8335
5	0.2930	8.1677	91.8323
6	0.2930	8.1679	91.8321
7	0.2930	8.1680	91.8320
8	0.2930	8.1680	91.8320
9	0.2930	8.1680	91.8320
10	0.2930	8.1680	91.8320

Source: Author's computation, 2019

The VAR (1) model was used to investigate Granger causality between the production outputs of the maize and rice. The results of the Granger causality are presented in Table 4.16. The analysis indicated that rice Granger-cause maize but maize does not Granger-cause rice. Thus, there is a unidirectional relationship between the production outputs of maize and rice. This implies that the production outputs of rice can be used to predict that of maize but the production outputs of maize cannot be used to predict that of rice. The findings of this study concur with the result of Luguterah *et al*. (2013) on production growth rates of maize, rice and millet in Ghana.

Table 4.16: Granger causality Wald test

Equations	Excluded	χ^2	df	*p*-value
Maize	Rice	10.6950	1	0.0010*
	ALL	10.6950	1	0.0010*
Rice	Maize	0.1124	1	0.7370
	ALL	0.1124	1	0.7370

Source: Author's computation, 2019

4.5 Markov Chain Analysis of Maize Production Output

The results of the Markov chain analysis of maize production output values are presented in this section. Table 4.17 displays the frequency of transitions of the maize production output values from one state to another state for the period under study. The results showed that, if the previous year's production was in low state, then its transition frequencies to low, stable and high states in the current year are 10 years, 1 year and 14 years respectively. Again, if the previous year's production was in the stable state, then its transition frequencies to low state, stable state and high state in the current year were 1 year each respectively. This means that considering the whole production years for this study, maize production made equal transitions from the stable state in the previous year to other states in the current year only once. When the production state in the previous year was in high state, the transition frequencies from the high state to low state, stable state and high state in the current year were 13 years, 2 years and 14 years respectively. This implies that, when the previous year's production was in high state, it frequently changes to high state again in the current year than any other state.

Table 4.17: Frequency of transitions from previous production year to current year

		Current Year (j)			
		Low	**Stable**	**High**	**Total**
Previous Year (i)	**Low**	10	1	14	25
	Stable	1	1	1	3
	High	13	2	14	29

Source: Author's computation, 2019

Using the transition frequencies given in Table 4.17, the transition probabilities of moving from one state to another were estimated by dividing each given frequency with the corresponding row total. From the transition probability matrix $\boldsymbol{P}$ below, it was realized that the probability of making a transition to low state given that in the previous year the production was in a low state was $\frac{2}{5}$. Hence, there is 40% likelihood that if the production was in low state in the previous year then it will be in low state again in the current year. Similarly, if the production was in the low state in the previous year then, there is 4% chance that it will be in stable state and 56% chance that it will be in the high state in the current year. Thus, there is higher probability that when the production is in the low state in the previous year, then in the current year it will be in high state.

Also, if the production was in stable state in the previous year, then there is an equal likelihood of $\frac{1}{3}$ that the production will make transitions to low, stable and high states in the current year. This means that there is approximately 33.33% chance that the production will make transitions to low, stable and high states in the current year given that in the previous year it was in stable state. In addition, given that the production was in high state in the previous year, then it will make transitions to the low, stable and high states in the current year with probabilities $\frac{13}{29}, \frac{2}{29}$ and $\frac{14}{29}$ respectively. This implies that there is $44.8\%, 6.9\%$ and 48.3% likelihood of making transitions to low, stable and high states in the current year given that the previous year's production was in high state. Thus, if the previous year's production was high, then there is greater likelihood that the following year's production will be high.

$$\boldsymbol{P} = \begin{array}{c} \\ \text{Low} \\ \text{Stable} \\ \text{High} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ \begin{bmatrix} \frac{2}{5} & \frac{1}{25} & \frac{14}{25} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{13}{29} & \frac{2}{29} & \frac{14}{29} \end{bmatrix} \end{array}$$

The transition probabilities from the transition matrix **P** were used to plot the state transition diagram for the three production states of maize. From the state transition diagram shown in Figure 4.7, it can be seen that the three states communicate with each other. That is, if the production is in a particular state in the previous year then it can make a transition to the same state and other states in the current year. Hence, the states are ergodic in nature.

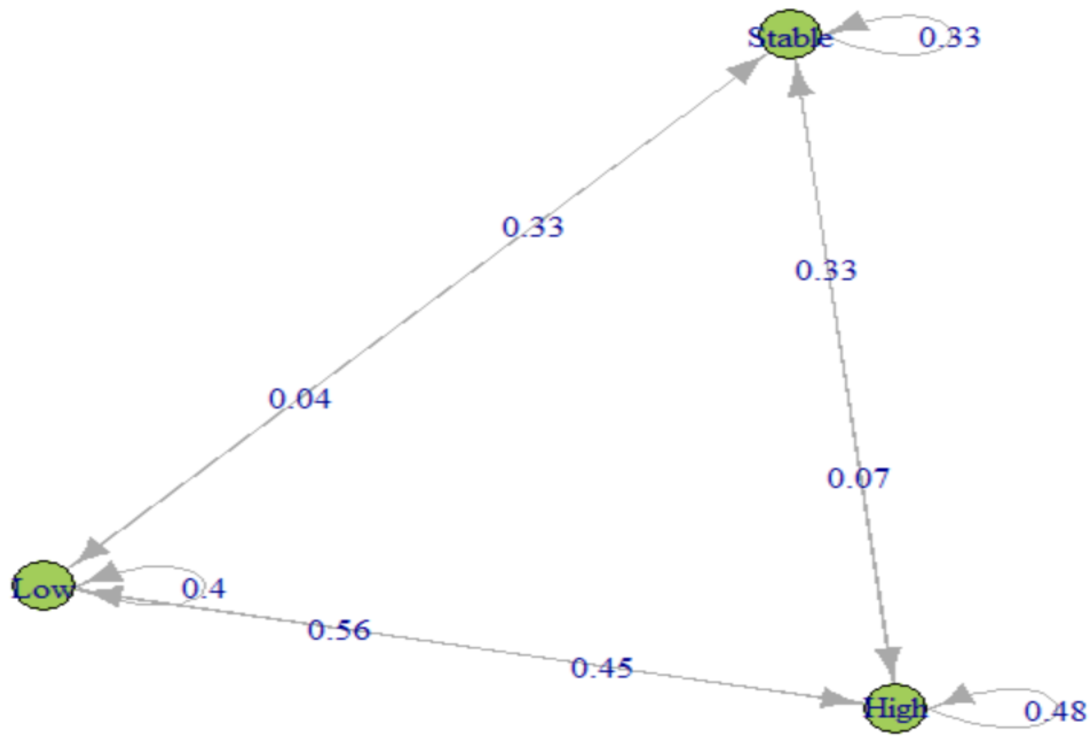

Figure 4.7: State transition diagram for maize production output

The equilibrium or long-run probabilities of the states were estimated to be $\left(\pi_{\text{Low}}, \pi_{\text{Stable}}, \pi_{\text{High}}\right) = (0.419, 0.077, 0.504)$. Hence, in 15 and above years to come, 41.9% of the production outputs are expected to move to low state, 7.7% of the production outputs are expected to move to stable state and 50.4% of the year's production outputs are expected to move to high state. The equilibrium state probability vector showed that, there is greater probability of maize production being in high state than any of the other states of production. Thus, the country stands a higher chance of recording high maize production outputs as the year progresses if all the necessary resources and policies are put in place. For instance, more subsidized farm inputs, easy access to credit facilities and improvement in agronomic practices. Also, farmers should be trained in other ways of harvesting or storing rain water in order for them to move from rain fed agriculture. This result concurs with that of Nasiru and Sarpong (2012a) who indicated in their research that there is greater likelihood that maize production in Ghana will increase.

To obtain the mean recurrent time for each production states, the reciprocals of each state equilibrium probability was estimated and the results revealed that the mean recurrent time for low state was approximately 2.387 years; that of stable state was approximately 12.987 years and that of high state was approximately 1.984 years. It is obvious from the mean recurrent time of the states that, it takes shorter years for the production to be in high state than the other states. This implies that, if proper measures are not put in place the production can easily move to a low state as it has the second least mean recurrent time.

Table 4.18 shows the expected length of low, stable and high production states. From Table 4.18, it can be observed that the expected lengths of low, stable and high states were 1.667 years, 1.5 years and 1.933 years respectively. However, it takes approximately two years to stay in each of the production states before leaving to another state. The expected production cycle was estimated to be 5.1 years. Thus, it takes approximately five years to complete the entire production cycle.

Table 4.18: Expected length of production states

State	Expected length (years)
Low	1.667
Stable	1.500
High	1.933

Source: Author's computation, 2019

Before forecasting the future values for the various states, the n-step transition probabilities were estimated and the results revealed that there is limiting probability that the production states will be in equilibrium or steady state after 15 steps (15 years). It implies that, after 15 transition steps, each row of the transition matrix **P** will have identical probabilities. This means that the production forecast for maize production after 15 years will remain the same. Below is the 1st step transition to the 16th step transition probability matrix of maize production.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Low} & \text{Stable} & \text{High}\end{array} \\ \boldsymbol{P}^1 = \begin{array}{r}\text{Low}\\ \text{Stable}\\ \text{High}\end{array} & \begin{bmatrix} \frac{2}{5} & \frac{1}{25} & \frac{14}{25} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{13}{29} & \frac{2}{29} & \frac{14}{29} \end{bmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc}\text{Low} & \text{Stable} & \text{High}\end{array} \\ \boldsymbol{P}^2 = \begin{array}{r}\text{Low}\\ \text{Stable}\\ \text{High}\end{array} & \begin{bmatrix} \frac{53}{125} & \frac{17}{250} & \frac{127}{250} \\ \frac{197}{500} & \frac{147}{1000} & \frac{459}{1000} \\ \frac{419}{1000} & \frac{37}{500} & \frac{507}{1000} \end{bmatrix}\end{array}$$

$$\vdots$$

$$\begin{array}{cc} & \begin{array}{ccc}\text{Low} & \text{Stable} & \text{High}\end{array} \\ \boldsymbol{P}^{15} = \begin{array}{r}\text{Low}\\ \text{Stable}\\ \text{High}\end{array} & \begin{bmatrix} \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \\ \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \\ \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \end{bmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc}\text{Low} & \text{Stable} & \text{High}\end{array} \\ \boldsymbol{P}^{16} = \begin{array}{r}\text{Low}\\ \text{Stable}\\ \text{High}\end{array} & \begin{bmatrix} \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \\ \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \\ \frac{419}{1000} & \frac{77}{1000} & \frac{63}{125} \end{bmatrix}\end{array}$$

Hence, the expected long-run production forecast is estimated as

$$F_{\text{Long}} = \begin{bmatrix} 0.419 & 0.077 & 0.504 \end{bmatrix} \begin{bmatrix} 699 \\ 1204 \\ 899 \end{bmatrix} = 837.786.$$

This implies that in the long-run, the overall average maize production forecast is estimated to be 837,786 metric tonnes. Thus, it is expected that all things being equal, the forecast values for each production state should be greater than or equal to 837,786 metric tonnes in the order to put Ghana's maize production output in an increasing state.

Five years short-run production output forecast for the various states of the maize production can be found in Table 4.19 which was computed by multiplying the n^{th} step transition matrix with the initial average production for each state. For instance, the forecasts for the years 2019 and 2020 were estimated using $\boldsymbol{P}^1\boldsymbol{\mu}_0$ and $\boldsymbol{P}^2\boldsymbol{\mu}_0$ respectively as:

$$F_{2019} = \begin{array}{c} \\ \text{Low} \\ \text{Stable} \\ \text{High} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ \begin{bmatrix} \frac{2}{5} & \frac{1}{25} & \frac{14}{25} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{13}{29} & \frac{2}{29} & \frac{14}{29} \end{bmatrix} \end{array} \begin{bmatrix} 699.000 \\ 1204.000 \\ 899.000 \end{bmatrix} = \begin{bmatrix} 831.200 \\ 934.000 \\ 830.379 \end{bmatrix}$$

$$F_{2020} = \begin{array}{c} \\ \text{Low} \\ \text{Stable} \\ \text{High} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ \begin{bmatrix} \frac{53}{125} & \frac{17}{250} & \frac{127}{250} \\ \frac{197}{500} & \frac{147}{1000} & \frac{459}{1000} \\ \frac{419}{1000} & \frac{37}{500} & \frac{507}{1000} \end{bmatrix} \end{array} \begin{bmatrix} 699.000 \\ 1204.000 \\ 899.000 \end{bmatrix} = \begin{bmatrix} 834.852 \\ 865.193 \\ 837.894 \end{bmatrix}.$$

From Table 4.19, the forecast result revealed that when the production is in a low state, there will be an increase in the production output of maize with all other things been equal since the forecast values for the years 2022 and 2023 are higher than the expected long-run forecast. Also, if the production is in a stable state, then the future production output of maize will be decreasing with time even though the five years forecast values are higher than the expected long-run forecast. This will affect the country's food security in the long term as well as increase our rate of importing maize which may lead to the depreciation of the Ghana cedis. In addition, if the production is in high state, then production pattern is expected to increase from 2020 onwards.

From the forecast results, it can also be inferred that, if the production output of maize is found to be in the low or high states then, there are higher hopes that future production output values will increase. This result is in agreement with Nasiru and Sarpong (2012a) who in their study concluded that all other things being equal there will be an increase in the production pattern of maize. The increase in the future production output from the Markov chain concurs with the one from the quadratic trend model. This increasing pattern could be linked to government interventions such as provision certified seeds, subsidized fertilizer among others (MoFA, 2017).

Table 4.19: Production output forecast for maize in thousand metric tonnes

State	Year				
	2019	**2020**	**2021**	**2022**	**2023**
Low	831.200	834.852	837.769	838.458*	838.654*
Stable	934.00*	865.193*	845.980*	840.721*	839.275*
High	830.379	837.894*	838.413*	838.646*	838.705*

* Means future production output of maize is higher than the expected long-run forecast value

Source: Author's computation, 2019

4.6 Markov Chain Analysis of Rice Production Output

This section focuses on the Markov chain analysis of rice production output. The transition frequencies of the three states of production were analysed as shown in Table 4.20. From Table 4.20, the frequency of a production previously in low state transiting to low state in the current year was 4 years, from low state to stable state was 1 year and moving from low state to high state was 14 years. Production previously in stable state neither transitioned to stable state nor high state in the current year, but rather to low state for 2 years.

Also, a previous year's production which was in high state moved to low, stable and high states in the current year by 13 years, 1 year and 22 years respectively. This shows that during the period of the study, rice production has experienced more transition from high to high state than the other transitioning states.

Table 4.20: Frequency of transitions from previous production year to current year

		Current Year (j)			
		Low	**Stable**	**High**	**Total**
Previous Year (i)	**Low**	4	1	14	19
	Stable	2	0	0	2
	High	13	1	22	36

Source: Author's computation, 2019

The transition probability matrix was also estimated for the various states by using their transition frequencies. From the transition probability matrix **P** below, it can be seen that if the rice production output value was in low state in the previous year then it can transition to low, stable and high states in the current year with probabilities $\frac{4}{19}, \frac{1}{19}$ and $\frac{14}{19}$ respectively. This means that there is 73.7% likelihood that rice production would transition to high state in the current year given that it was in low state in the previous year. Also, if the previous year's production was in stable state, then it can only transition to low state with 100% probability. Thus, when the previous year's production is in stable state there is 0% or no likelihood that it would transition to stable and high states in the current year.

In addition, given that the previous year's production was in high state then it can transition to low, stable and high states in the current year with probabilities $\frac{13}{36}, \frac{1}{36}$ and $\frac{22}{36}$ respectively. This also implies that there a higher chance of 61.1% that the current year's production would be in high state given that the previous year was in a high state. Furthermore, there is 36.1% and 2.8% likelihood that the production can change to low and stable states respectively given that it was in a high state previously.

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\
P = \begin{array}{c} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} & \begin{bmatrix} \frac{4}{19} & \frac{1}{19} & \frac{14}{19} \\ 1 & 0 & 0 \\ \frac{13}{36} & \frac{1}{36} & \frac{22}{36} \end{bmatrix}
\end{array}
$$

The transition probability matrix can be represented using a state transition diagram as shown in Figure 4.8. It can be seen from Figure 4.8 that, the production states of the rice are recurrent and communicates. This means that when production leaves a particular state, it can return to that state again.

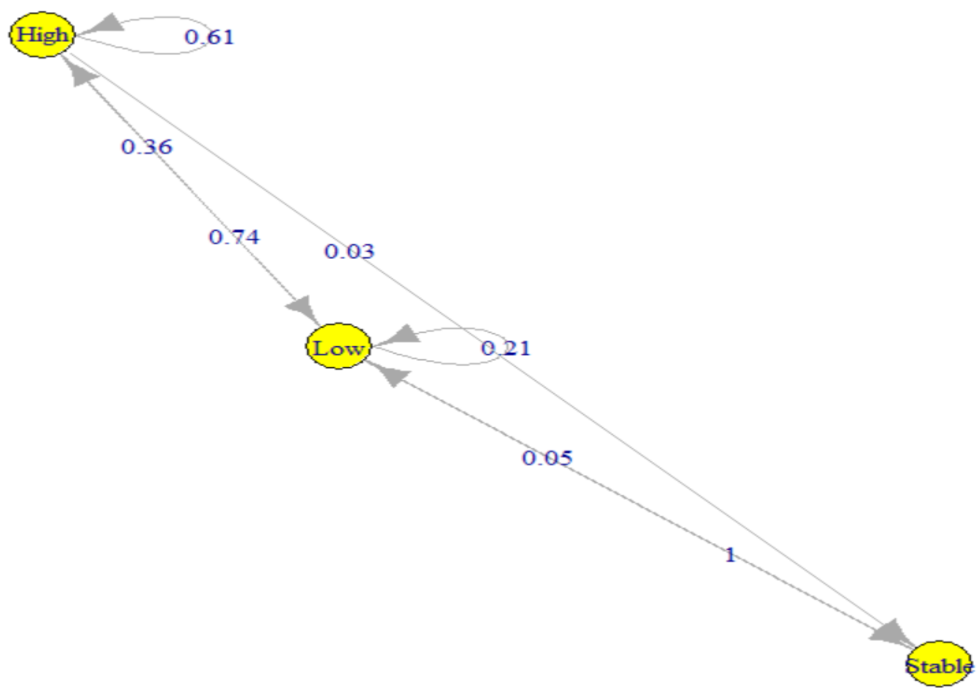

Figure 4.8: State transition diagram for rice production output

The steady state or equilibrium probabilities vector of the three states were estimated to be $\left(\pi_{\text{Low}}, \pi_{\text{Stable}}, \pi_{\text{High}}\right) = \left(0.333, 0.035, 0.632\right)$. This implies that in about 13 or more years to come, 33.3% of the production outputs are expected to move to a low state, 3.5% expected to move to a stable state and 63.2% are also expected to move to a high state. This shows that the prospects of rice production in the future are encouraging if only practical measures are kept in place in order to boost the industry. This finding concurs with that of Nasiru and Sarpong (2012b) and Awaab *et al*. (2018) on rice production in Ghana.

The mean recurrent time for low, stable and high states are expected to be approximately 3 years, 29 years and 2 years respectively which indicates that, it takes shorter years for production to move to high state as compared to the other states. The expected length of time for production to stay in low state is 1.267 years, stable state is 1 year and high state is 2.571 years as shown in Table 4.21. Hence, rice production is expected to stay longer in high state for about 3 years before transitioning to other states and the entire production cycle is expected to be completed in approximately 5years.

Table 4.21: Expected length of production states

State	Expected length (years)
Low	1.267
Stable	1.000
High	2.571

Source: Author's computation, 2019

The limiting probability for the production states were estimated to be at equilibrium after the 13^{th} transition (13 years). Thus, after the 13^{th} transition, each row of the transition probability matrix **P** is expected to be the same. Below is the transition probability matrix of rice from the 1^{st} step to the 14^{th} step.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ P^1 = \begin{array}{c} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} & \begin{bmatrix} \frac{4}{19} & \frac{1}{19} & \frac{14}{19} \\ 1 & 0 & 0 \\ \frac{13}{36} & \frac{1}{36} & \frac{11}{18} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ P^2 = \begin{array}{c} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} & \begin{bmatrix} \frac{363}{1000} & \frac{4}{125} & \frac{121}{200} \\ \frac{211}{1000} & \frac{13}{250} & \frac{737}{1000} \\ \frac{81}{250} & \frac{9}{250} & \frac{11}{25} \end{bmatrix} \end{array}$$

$$\vdots$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ P^{13} = \begin{array}{c} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} & \begin{bmatrix} \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \\ \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \\ \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\ P^{14} = \begin{array}{c} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} & \begin{bmatrix} \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \\ \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \\ \frac{333}{1000} & \frac{7}{200} & \frac{79}{125} \end{bmatrix} \end{array}$$

The expected long-run production forecast for rice was also estimated as:

$$F_{\text{Long}} = \begin{bmatrix} 0.333 & 0.035 & 0.632 \end{bmatrix} \begin{bmatrix} 83.000 \\ 37.500 \\ 153.600 \end{bmatrix} = 126.027.$$

This indicates that, the average rice production is expected to be equal or more than 126,027 metric tonnes in the short-run to enable the nation to meet the increasing demand for rice as well as reduce rice importation. The expected short-run production forecast for the years 2019 and 2020 were estimated as:

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\
F_{2019} = \begin{array}{r} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} &
\begin{bmatrix} \frac{4}{19} & \frac{1}{19} & \frac{14}{19} \\ 1 & 0 & 0 \\ \frac{13}{36} & \frac{1}{36} & \frac{11}{18} \end{bmatrix}
\begin{bmatrix} 83.000 \\ 37.500 \\ 456.000 \end{bmatrix} = \begin{bmatrix} 132.626 \\ 83.000 \\ 124.881 \end{bmatrix}
\end{array}
$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{Low} & \text{Stable} & \text{High} \end{array} \\
F_{2020} = \begin{array}{r} \text{Low} \\ \text{Stable} \\ \text{High} \end{array} &
\begin{bmatrix} \frac{363}{1000} & \frac{4}{125} & \frac{121}{200} \\ \frac{211}{1000} & \frac{13}{250} & \frac{737}{1000} \\ \frac{81}{250} & \frac{9}{250} & \frac{11}{25} \end{bmatrix}
\begin{bmatrix} 83.000 \\ 37.5000 \\ 456.000 \end{bmatrix} = \begin{bmatrix} 124.307 \\ 132.626 \\ 126.514 \end{bmatrix}
\end{array}
$$

Table 4.22 shows five years forecast of rice production output in Ghana. The results revealed that if the current production is in a low state, then only the years 2019 and 2021 forecast would be higher than the average long-run production of 126,027 metric tonnes. Also, when the current production is in a stable state, then the years 2019, 2021 and 2023 outputs are expected to be lower than the average long-run production forecast as compared to the other years forecasted.

In addition, if the current production is in a high state, then for the next five years to come, only the year 2020 output is expected to be more than the average production output. The undulating nature of the short-run forecast values for the rice production output depicts how unstable rice production in Ghana would be if new policies are not kept in place to improve rice production immediately. This unstable nature in the pattern of the rice production output from the Markov chain forecast can be attributed to poor market infrastructure and little incentive to invest in or adopt productivity enhancing technologies among others. The undulating short-run forecast pattern contradicts the forecasting results of the studies Nasiru and Sarpong (2012b) who just indicated an increasing pattern. Also, the forecast result of this study does not agree with that of the Awaab *et al*. (2018) finding that there will be an increasing pattern in the rice production.

Table 4.22: Production output forecast for rice production in thousand metric tonnes

State	Year				
	2019	**2020**	**2021**	**2022**	**2023**
Low	132.626*	124.307	126.371*	125.906	126.013
Stable	83.000	132.626*	124.307	126.371*	125.906
High	124.881	126.514*	125.887	126.018	125.987

* Means future production output of rice is higher than the expected long-run forecast value

Source: Author's computation, 2019

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter presents the summary of the study, the conclusions based on the general outcome of the research as well as the policy recommendations arising from the conclusions of the study.

5.1 Summary

The study investigated the production states of maize and rice, their probabilities, expected length, production cycle, expected recurrent times and forecasted the equilibrium condition the production outputs of maize and rice in Ghana using secondary data from 1960 to 2018. The data were obtained from the official website maintained by the United State of America department of agriculture.

To attain the goals of the study, the Markov chain analysis was employed to analyse the data on the production outputs of the maize and rice. Before the Markov chain analysis was carried out, preliminary analyses were performed using descriptive statistics. The results of the preliminary analyses revealed that maize had a higher average production output (823,400 metric tonnes) than rice (124,700 metric tonnes). The descriptive statistics further showed that the rice production output had a higher variability index than maize. The Pearson correlation indicated that there was strong positive significant relationship between the production outputs of maize and rice. The Kolmogorov-Smirnov and Anderson-Darling tests for normality were used to check whether the data were normally distributed and the result indicated that the production outputs of the two cereals were not normally distributed. This showed that it was not appropriate to analyse the data using any econometric model that requires the assumptions of normality as this will lead to wrong inferences. So, it was vital to use models that do not depend on the normality assumption. Hence, the use of the Markov chain to analyse the data.

Trend analyses were performed to investigate the nature of growth in the production outputs of the cereals. It was realized that the pattern of growth in the maize production output was best described by the quadratic trend model and that of the rice was best described by the exponential growth trend model. Five years forecast with the best trend models for each cereal showed an increase in the production outputs of maize and rice.

The unit root test was performed using ADF and PP tests to check for stationarity of the production outputs of maize and rice. The result showed that the production outputs of maize and rice were not stationary. However, the logarithmically transformed first differenced production outputs of the maize and rice were stationary. Hence, cointegration test was performed using the logarithmically transformed data and the result indicated that there is no long-run equilibrium relationship between the production outputs of the maize and rice. In order, to investigate the short-run dynamic relationship between the production outputs of the maize and rice, a VAR (1) model was fitted to the logarithmically transformed first differenced series and Granger causality test was also carried out. The findings showed that the production outputs of rice Granger-cause the production outputs of maize but not the reverse. Hence, there was a unidirectional relationship between the production outputs of maize and rice.

To perform the Markov chain analysis, the production outputs for each of the cereals were categorized into three states (low, stable and high) and the behaviour of the outputs in each of the states were investigated. The findings of the study revealed that at equilibrium, 41.9 % of the production output of maize are expected to be in low state, 7.7 % are expected to be in a stable state and 50. 4% are expected to be in a high state. This is an indication that Ghana stands the chance of recording high maize production if all things are equal. The result also revealed that the mean recurrent time for the maize production to return to low, stable and high states were 2.387 years, 12.987 years and 1.984 years respectively. Hence, it takes shorter time for the production output of maize to return to the high state after it has left the high state. However, if appropriate measures are not put in place, the production can easily move to the low state as it has the second least recurrent time. The expected length of time the production output of maize stays in the low state, stable state and high state were 1.667 years, 1.5 years and 1.933 years respectively. It implies that it takes approximately two years for the production output to stay in each of the states and the expected production cycle was found to be 5.1 years for the maize production output. The long-run production forecast was estimated at 837,786 metric tonnes. The short-run forecast results revealed that if the production outputs are in low or high states, then all other things being equal the country will experience increase in production output of maize.

For rice production, the results indicated that 33.3 % of the outputs are expected to be in low state, 3.5 % are expected to be in stable state and 63.2% are expected to be in high state. This indicates that the prospects for rice production in future are encouraging. The mean recurrent time for the low, stable and high states were found to be approximately 3 years, 29 years and 2 years respectively. Thus, it takes shorter years for the production to move to high states. The expected length of time for the rice production to stay in the low state, stable state and high state were found to be 1.267 years, 1 year and 2.571 years respectively. Hence, the rice production output is expected to stay longer in high state (approximately 3 years). The entire production cycle for the rice production output was found to be approximately five years. The expected long-run forecast for the rice production output was found to be 126,027 metric tonnes. This means that all other things being equal, the average production output is expected to be 126,027 metric tonnes. The five years short-run forecast revealed an undulating pattern in the future rice production output.

5.1 Conclusions

The objectives for the study were achieved as there are 50.4% and 63.2% likelihood that both maize and rice production outputs respectively will be in high state in the coming years all other things being equal. Also, it will take shorter years of approximately 1.98 and 1.58 for both maize and rice production outputs respectively to return to high state after leaving that state. In addition, maize and rice production outputs will stay 2 years and 3 years respectively longer in high state than the other states.

Furthermore, it will take approximately five years for the entire production cycle for both maize and rice production output to be completed. From the short-run forecast, it can be inferred that maize production output is likely to increase all other things being equal. However, despite the rice production output having greater likelihood to be in high state, the future rice production output may exhibit undulating pattern as revealed by the short-run forecast.

5.2 Recommendations

Although governments over the years have implemented many programs and interventions to increase production in both the maize and rice sectors, it appears they have been ineffective largely because of political interferences and changes in governments. Hence, from the conclusions of the study, the following are recommended:

1. For the production of maize and rice to continuously be in a high state, the regulatory bodies like the Ghana Commodities Exchange and Buffer Stock Exchange should strengthen their workshops and trainings on value addition, consumers' preference, packaging, improved technology among others in order to enhance the value chain process.
2. To help the production of maize and rice to stay longer in the high state, government policies like reducing their importation should be stable or firm to prevent maize and rice farmers from switching to other crops.
3. Government should intensify the current policies such as the fertilizer subsidy, certified seeds and extension service delivery in order to continuously increase productions in both the maize and rice sectors.
4. To reduce the high variability in these cereals, government should construct more irrigation dams, improve or ease access to formal agricultural credits facilities, further subsidized the cost of inputs like farm machinery among others.

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APPENDICES

Appendix A

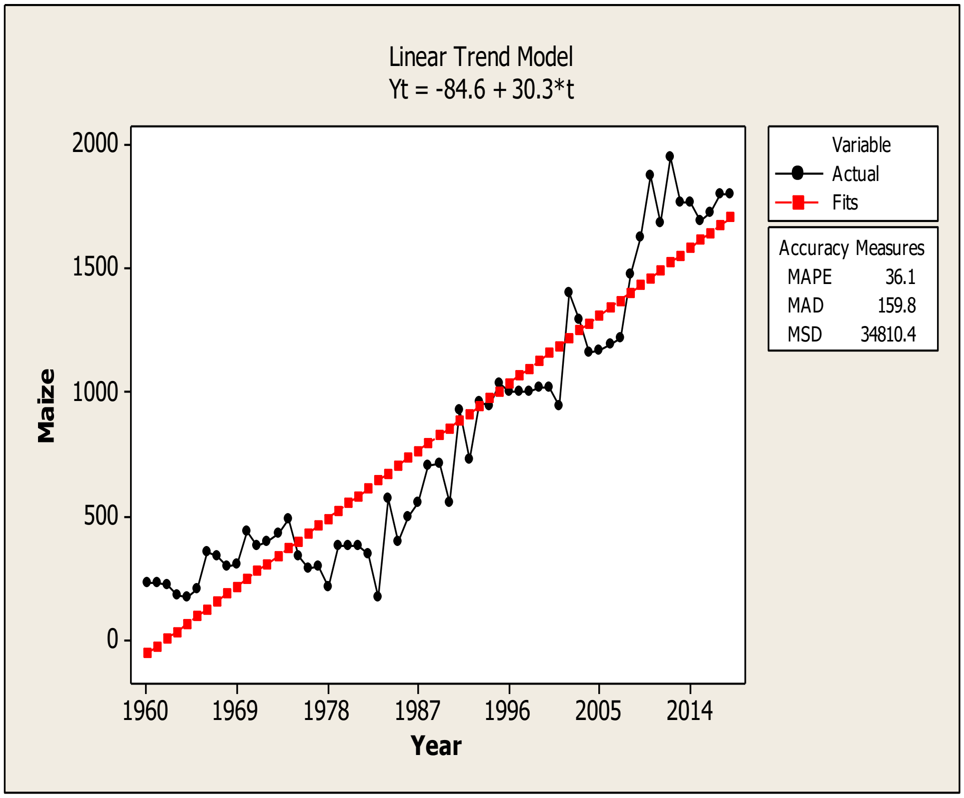


Figure A1: Linear trend analysis for maize production values

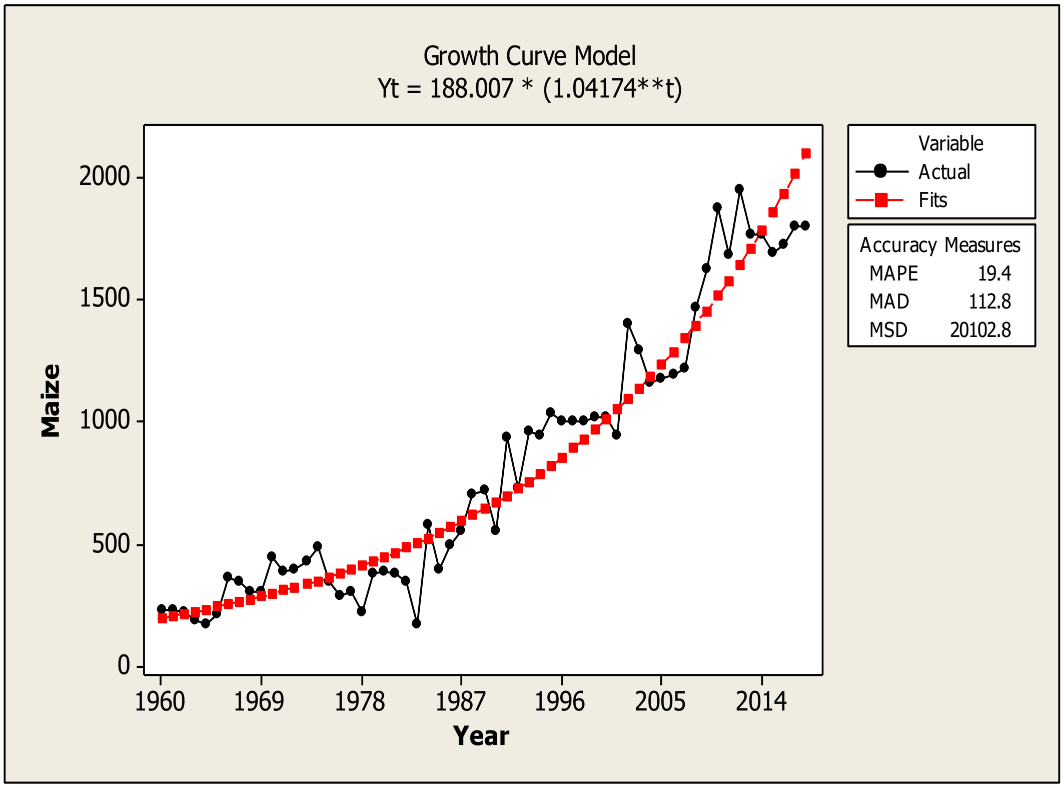


Figure A2: Exponential growth curve analysis for maize production values

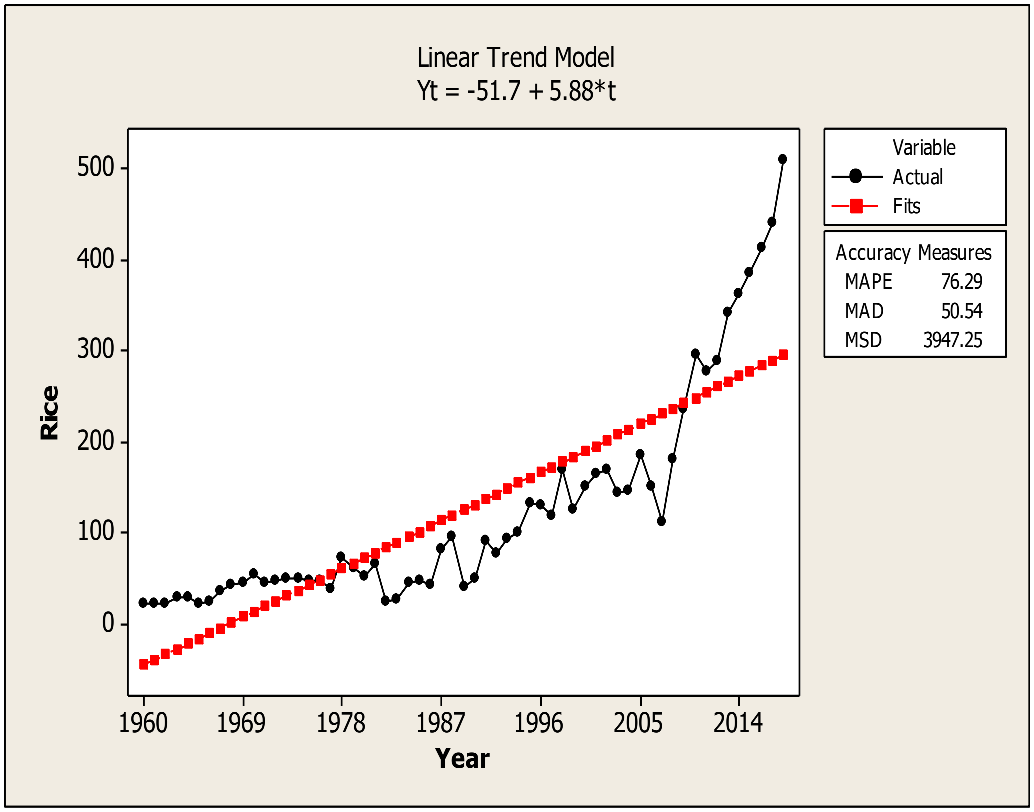


Figure A3: Linear trend analysis for rice production values

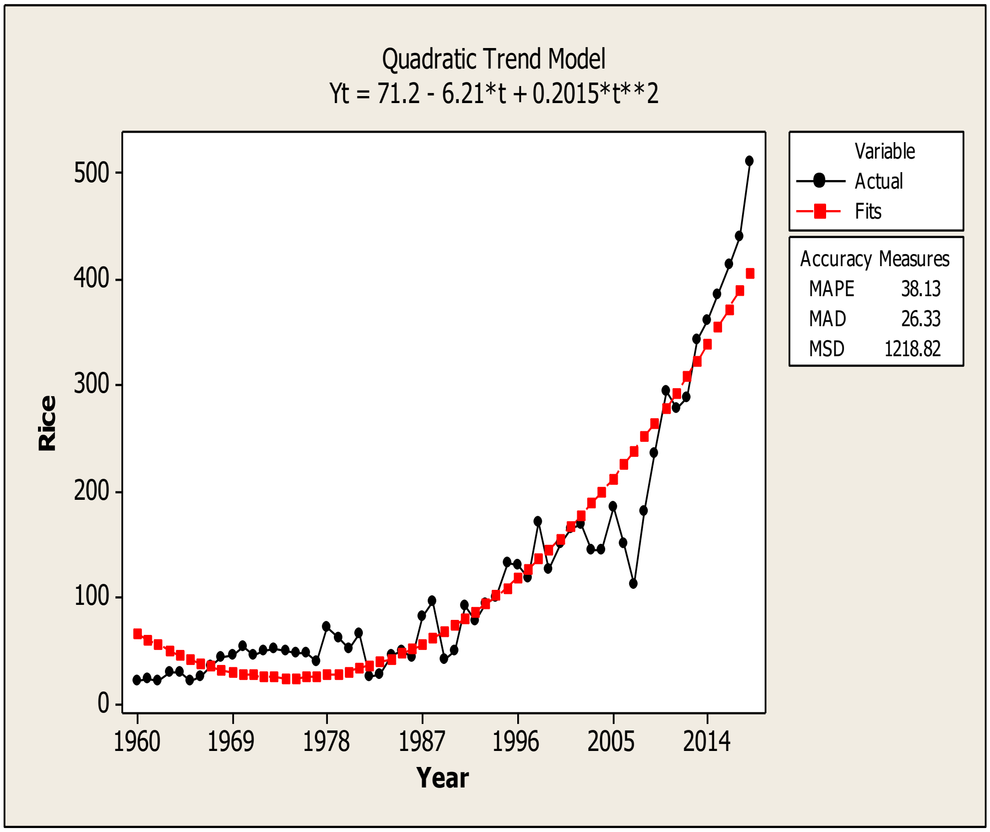


Figure A4: Quadratic trend analysis for rice production values

Appendix B

n-step transition matrix for maize

R Console

```
> P1<-matrix(c(2/5,1/25,14/25,1/3,1/3,1/3,13/29,2/29,14/29),nrow=3,byrow=TRUE)
> P1
          [,1]       [,2]      [,3]
[1,] 0.4000000 0.04000000 0.5600000
[2,] 0.3333333 0.33333333 0.3333333
[3,] 0.4482759 0.06896552 0.4827586
> P2<-P1%*%P1
> P2
          [,1]       [,2]      [,3]
[1,] 0.4243678 0.06795402 0.5076782
[2,] 0.3938697 0.14743295 0.4586973
[3,] 0.4187079 0.07421324 0.5070789
> P3<-P1%*%P2
> P3
          [,1]       [,2]      [,3]
[1,] 0.4199783 0.07463834 0.5053833
[2,] 0.4123151 0.09653340 0.4911515
[3,] 0.4195321 0.07645702 0.5040109
> P4<-P1%*%P3
> P4
          [,1]       [,2]      [,3]
[1,] 0.4194219 0.07653260 0.5040455
[2,] 0.4172752 0.08254292 0.5001819
[3,] 0.4192344 0.07702633 0.5037392
```

```
> P5<-P1%*%P4
> P5
          [,1]       [,2]      [,3]
[1,] 0.4192311 0.07704950 0.5037194
[2,] 0.4186439 0.07870062 0.5026555
[3,] 0.4191834 0.07718546 0.5036312
> P6<-P1%*%P5
> P6
          [,1]       [,2]      [,3]
[1,] 0.4191809 0.07719168 0.5036275
[2,] 0.4190194 0.07764519 0.5033354
[3,] 0.4191675 0.07722901 0.5036035
> P7<-P1%*%P6
> P7
          [,1]       [,2]      [,3]
[1,] 0.4191669 0.07723072 0.5036023
[2,] 0.4191226 0.07735529 0.5035221
[3,] 0.4191633 0.07724098 0.5035957
> P8<-P1%*%P7
> P8
          [,1]       [,2]      [,3]
[1,] 0.4191631 0.07724145 0.5035954
[2,] 0.4191509 0.07727566 0.5035734
[3,] 0.4191621 0.07724426 0.5035936
```

```
> P9<-P1%*%P8
> P9
          [,1]       [,2]      [,3]
[1,] 0.4191621 0.07724439 0.5035935
[2,] 0.4191587 0.07725379 0.5035875
[3,] 0.4191618 0.07724517 0.5035930
> P10<-P1%*%P9
> P10
          [,1]       [,2]      [,3]
[1,] 0.4191618 0.07724520 0.5035930
[2,] 0.4191609 0.07724778 0.5035913
[3,] 0.4191617 0.07724542 0.5035929
> P11<-P1%*%P10
> P11
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724542 0.5035929
[2,] 0.4191615 0.07724613 0.5035924
[3,] 0.4191617 0.07724548 0.5035928
> P12<-P1%*%P11
> P12
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724549 0.5035928
```

```
R Console

[2,] 0.4191616 0.07724568 0.5035927
[3,] 0.4191617 0.07724550 0.5035928
```

```
> P13<-P1%*%P12
> P13
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724550 0.5035928
[2,] 0.4191617 0.07724556 0.5035928
[3,] 0.4191617 0.07724551 0.5035928
> P14<-P1%*%P13
> P14
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724551 0.5035928
[2,] 0.4191617 0.07724552 0.5035928
[3,] 0.4191617 0.07724551 0.5035928
> P15<-P1%*%P14
> P15
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724551 0.5035928
[2,] 0.4191617 0.07724551 0.5035928
[3,] 0.4191617 0.07724551 0.5035928
> P16<-P1%*%P15
> P16
          [,1]       [,2]      [,3]
[1,] 0.4191617 0.07724551 0.5035928
[2,] 0.4191617 0.07724551 0.5035928
[3,] 0.4191617 0.07724551 0.5035928
```


n-step transition matrix for rice

```
R Console

> P<-matrix(c(4/19,1/19,14/19,1,0,0,13/36,1/36,11/18),nrow=3,ncol=3,byrow=TRUE)
> P
          [,1]       [,2]      [,3]
[1,] 0.2105263 0.05263158 0.7368421
[2,] 1.0000000 0.00000000 0.0000000
[3,] 0.3611111 0.02777778 0.6111111
> P2<-P%*%P
> P2
          [,1]       [,2]      [,3]
[1,] 0.3630348 0.03154817 0.6054171
[2,] 0.2105263 0.05263158 0.7368421
[3,] 0.3244802 0.03598116 0.6395387
```

```
> P3<-P%*%P2
> P3
          [,1]       [,2]      [,3]
[1,] 0.3265994 0.03592423 0.6374764
[2,] 0.3630348 0.03154817 0.6054171
[3,] 0.3352373 0.03484287 0.6299198
```

```
> P4<-P%*%P3
> P4
          [,1]       [,2]      [,3]
[1,] 0.3348818 0.03489712 0.6302211
[2,] 0.3265994 0.03592423 0.6374764
[3,] 0.3328902 0.03514184 0.6319680
> P5<-P%*%P4
> P5
          [,1]       [,2]      [,3]
[1,] 0.3329784 0.03513150 0.6318901
[2,] 0.3348818 0.03489712 0.6302211
[3,] 0.3334346 0.03507520 0.6314902
> P6<-P%*%P5
> P6
          [,1]       [,2]      [,3]
[1,] 0.3334148 0.03507768 0.6315076
[2,] 0.3329784 0.03513150 0.6318901
[3,] 0.3333101 0.03509058 0.6315993
> P7<-P%*%P6
> P7
          [,1]       [,2]      [,3]
[1,] 0.3333147 0.03509002 0.6315953
[2,] 0.3334148 0.03507768 0.6315076
[3,] 0.3333387 0.03508706 0.6315743
```

R Console

```
> P12<-P%*%P11
> P12
          [,1]       [,2]      [,3]
[1,] 0.3333333 0.03508772 0.6315789
[2,] 0.3333333 0.03508773 0.6315790
[3,] 0.3333333 0.03508772 0.6315790
> P13<-P%*%P12
> P13
          [,1]       [,2]      [,3]
[1,] 0.3333333 0.03508772 0.6315789
[2,] 0.3333333 0.03508772 0.6315789
[3,] 0.3333333 0.03508772 0.6315789
> P14<-P%*%P13
> P14
          [,1]       [,2]      [,3]
[1,] 0.3333333 0.03508772 0.6315789
[2,] 0.3333333 0.03508772 0.6315789
[3,] 0.3333333 0.03508772 0.6315789
```

Appendix C

Five years forecast for maize using Markov chain

```
R Console

> P<-matrix(c(2/5,1/25,14/25,1/3,1/3,1/3,13/29,2/29,14/29),nrow=3,byrow=T)
> P
          [,1]       [,2]      [,3]
[1,] 0.4000000 0.04000000 0.5600000
[2,] 0.3333333 0.33333333 0.3333333
[3,] 0.4482759 0.06896552 0.4827586
> M<-matrix(c(699,1204,899),nrow=3,byrow=T)
> M
     [,1]
[1,]  699
[2,] 1204
[3,]  899
> F1<-P%*%M
> F1
         [,1]
[1,] 831.2000
[2,] 934.0000
[3,] 830.3793
> P2<-P%*%P
> P2
          [,1]       [,2]      [,3]
[1,] 0.4243678 0.06795402 0.5076782
[2,] 0.3938697 0.14743295 0.4586973
[3,] 0.4187079 0.07421324 0.5070789
> F2<-P2%*%M
> F2
         [,1]
[1,] 834.8524
[2,] 865.1931
[3,] 837.8935
```

```
> P3
          [,1]       [,2]      [,3]
[1,] 0.4199783 0.07463834 0.5053833
[2,] 0.4123151 0.09653340 0.4911515
[3,] 0.4195321 0.07645702 0.5040109
> F3<-P3%*%M
> F3
         [,1]
[1,] 837.7690
[2,] 845.9797
[3,] 838.4130
> P4<-P%*%P3
> P4
          [,1]       [,2]      [,3]
[1,] 0.4194219 0.07653260 0.5040455
[2,] 0.4172752 0.08254292 0.5001819
[3,] 0.4192344 0.07702633 0.5037392
> F4<-P4%*%M
> F4
         [,1]
[1,] 838.4581
[2,] 840.7206
[3,] 838.6461
> P5<-P%*%P4
> P5
          [,1]       [,2]      [,3]
[1,] 0.4192311 0.07704950 0.5037194
[2,] 0.4186439 0.07870062 0.5026555
[3,] 0.4191834 0.07718546 0.5036312
> F5<-P5%*%M
> F5
         [,1]
[1,] 838.6539
[2,] 839.2749
[3,] 838.7049
```

Five years forecast for rice using Markov chain

```
> P<-matrix(c(4/19,1/19,14/19,1,0,0,13/36,1/36,11/18),byrow=TRUE,nrow=3)
> P
          [,1]       [,2]      [,3]
[1,] 0.2105263 0.05263158 0.7368421
[2,] 1.0000000 0.00000000 0.0000000
[3,] 0.3611111 0.02777778 0.6111111
> M<-matrix(c(83,37.5,153.6),byrow=TRUE,nrow=3)
> M
      [,1]
[1,]  83.0
[2,]  37.5
[3,] 153.6
> F1<-P%*%M
> F1
         [,1]
[1,] 132.6263
[2,]  83.0000
[3,] 124.8806
> P2<-P%*%P
> P2
          [,1]       [,2]      [,3]
[1,] 0.3630348 0.03154817 0.6054171
[2,] 0.2105263 0.05263158 0.7368421
[3,] 0.3244802 0.03598116 0.6395387
> F2<-P2%*%M
> F2
         [,1]
[1,] 124.3070
[2,] 132.6263
[3,] 126.5143
```

```
> P3<-P%*%P2
> P3
          [,1]       [,2]      [,3]
[1,] 0.3265994 0.03592423 0.6374764
[2,] 0.3630348 0.03154817 0.6054171
[3,] 0.3352373 0.03484287 0.6299198
> F3<-P3%*%M
> F3
         [,1]
[1,] 126.3713
[2,] 124.3070
[3,] 125.8870
> P4<-P%*%P3
> P4
          [,1]       [,2]      [,3]
[1,] 0.3348818 0.03489712 0.6302211
[2,] 0.3265994 0.03592423 0.6374764
[3,] 0.3328902 0.03514184 0.6319680
> F4<-P4%*%M
> F4
         [,1]
[1,] 125.9058
[2,] 126.3713
[3,] 126.0180
> P5<-P%*%P4
> P5
          [,1]       [,2]      [,3]
[1,] 0.3329784 0.03513150 0.6318901
[2,] 0.3348818 0.03489712 0.6302211
[3,] 0.3334346 0.03507520 0.6314902
> F5<-P5%*%M
> F5
         [,1]
[1,] 126.0130
[2,] 125.9058
[3,] 125.9873
```

Appendix D

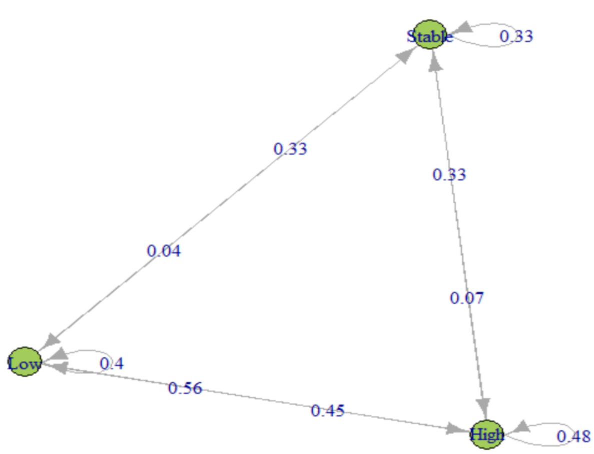
R program for the Markov chain analysis for maize

```
> library("markovchain")
Package:  markovchain
Version:  0.6.9.12
Date:     2018-08-23
BugReport: http://github.com/spedygiorgio/markovchain/issues

Warning message:
package 'markovchain' was built under R version 3.5.1
> maizeStates<-c("Low","Stable","High")
> maizeMatrix<-matrix(data=c(0.400,0.040,0.560,0.333,0.333,0.333,0.448,0.069,0.483),byrow=TRUE,nr
ow=3,dimnames=list(maizeStates,maizeStates))

> mcMaize<-new("markovchain",states=maizeStates,byrow=TRUE,transitionMatrix=maizeMatrix,name="Mai
ze")
> print(mcMaize)
             Low    Stable      High
Low    0.4000000 0.0400000 0.5600000
Stable 0.3333333 0.3333333 0.3333333
High   0.4480000 0.0690000 0.4830000

> plot(mcMaize)
```



```
> steadyStates(mcMaize)
          Low      Stable      High
[1,] 0.4190259 0.07727446 0.5036996
```

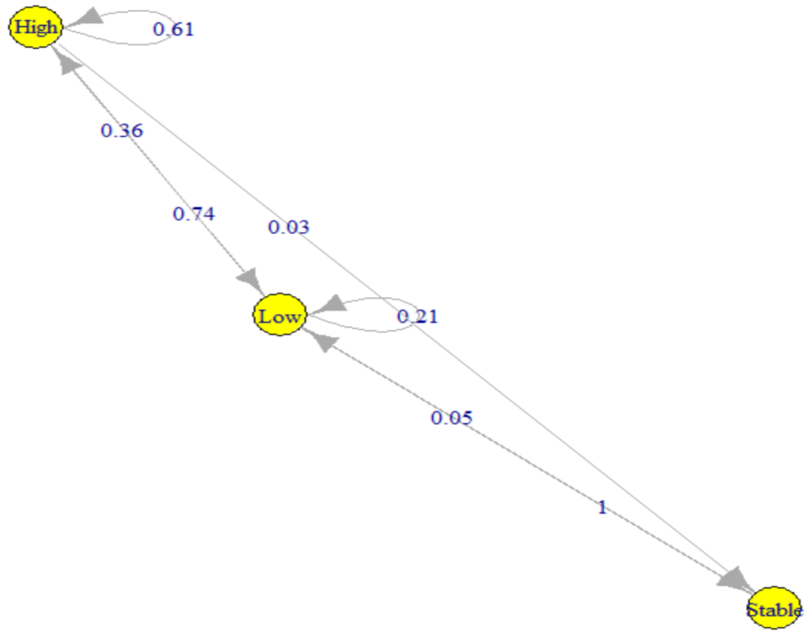
R program for the Markov chain analysis for rice

```
> library("markovchain")
Package:  markovchain
Version:  0.6.9.12
Date:     2018-08-23
BugReport: http://github.com/spedygiorgio/markovchain/issues

Warning message:
package 'markovchain' was built under R version 3.5.1

> riceStates<-c("Low","Stable","High")
> riceMatrix<-matrix(data=c(0.210,0.053,0.737,1.00,0.00,0.00,0.361,0.028,0.611),byrow=TRUE,nrow=3
,dimnames=list(riceStates,riceStates))
> mcRice<-new("markovchain",states=riceStates,byrow=TRUE,transitionMatrix=riceMatrix,name="Rice")
> mcRice
Rice
 A  3 - dimensional discrete Markov Chain defined by the following states:
 Low, Stable, High
 The transition matrix  (by rows)  is defined as follows:
         Low Stable  High
Low    0.210  0.053 0.737
Stable 1.000  0.000 0.000
High   0.361  0.028 0.611

> plot(mcRice,col="yellow")
```




```
> steadyStates(mcRice)
           Low     Stable      High
[1,] 0.3332611 0.03534195 0.631397
```